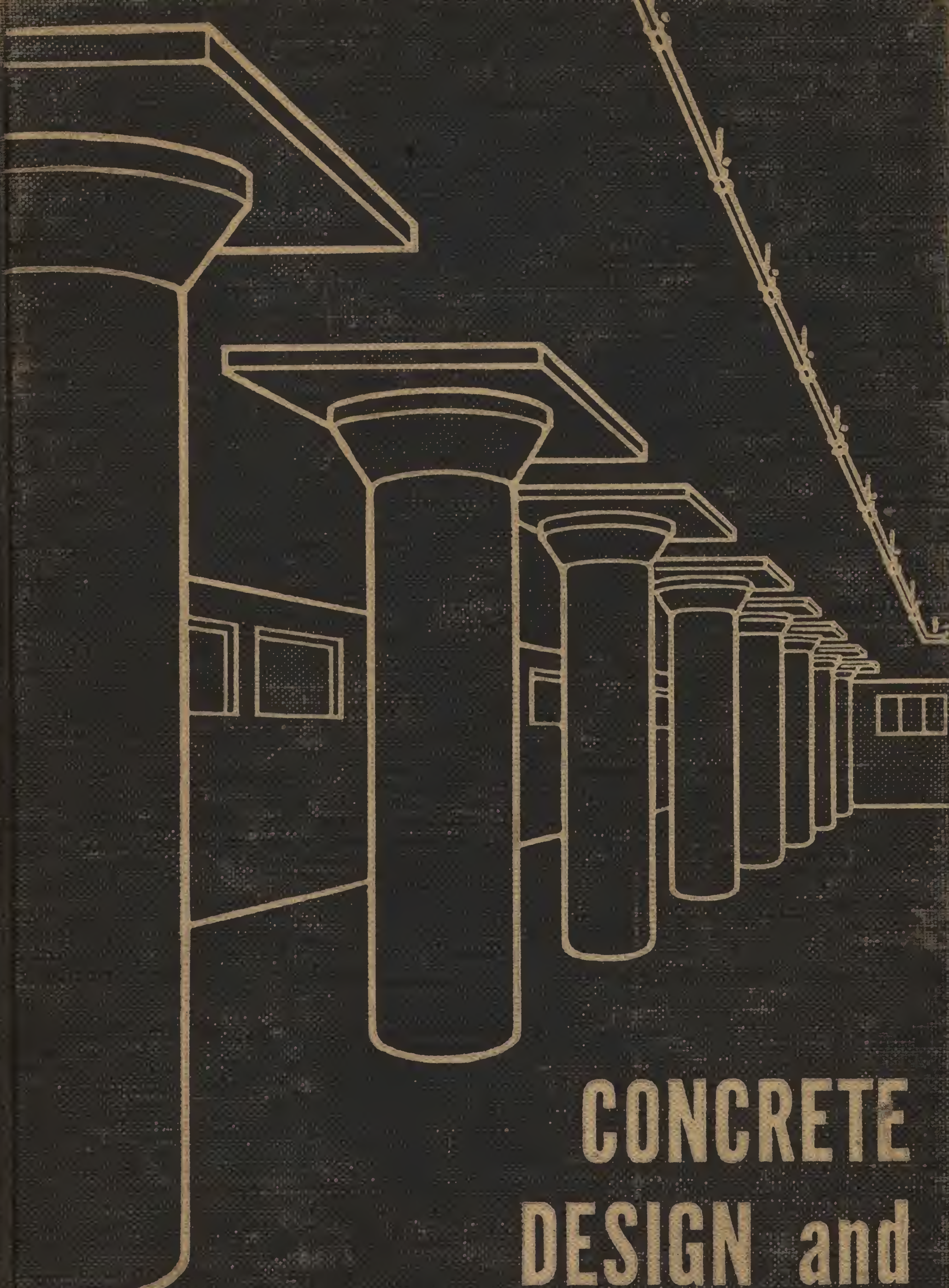

# CONCRETE DESIGN and CONSTRUCTION

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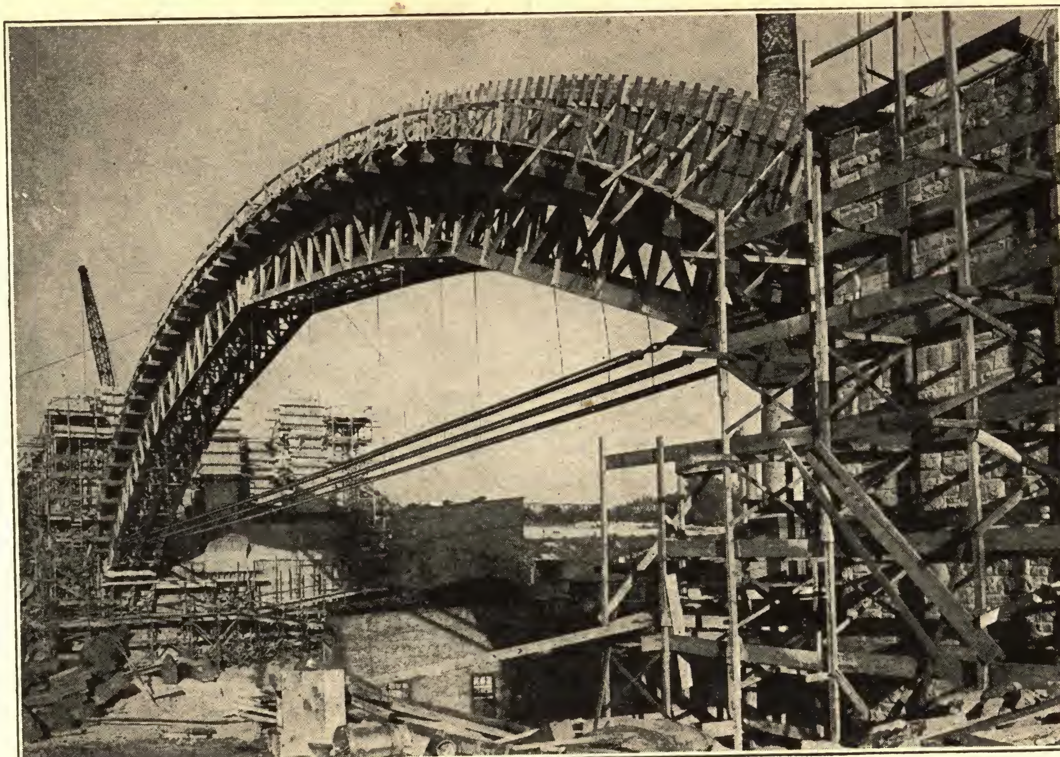

BLAW-KNOX STEEL CENTERING, 29TH STREET BRIDGE, BALTIMORE
POTTS & CALLAHAN, CONTRACTORS

# CONCRETE DESIGN AND CONSTRUCTION

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# INTRODUCTION

In a comparatively short time concrete has become one of the most valuable materials in the hands of the builders. Its use has a very wide range. In its simpler form, it takes the place of stone masonry, and in its more complicated designs, when reinforced with steel bars, it replaces structural steel and timber.

In the preparation of this volume the authors have endeavored to present the subject in a simple and concise manner, suitable alike for students and engineers.

The composition and treatment of the materials used in concrete, its fireproofing qualities, also the reinforcing steel used in it, are described in such a way as to give accurate knowledge of their relation to each other and to the general subject in hand.

The authors have made an effort to present a clear and simple text on the general theory and design of reinforced concrete, including the bonding of steel and concrete, vertical shear and diagonal tension and the allowable working stresses permitted in concrete and reinforcing steel.

The design of slabs, plain beams and T-beams is explained and illustrated by problems fully worked out. Many diagrams and tables have been inserted to assist in the design of these members in practical work. A chapter is devoted to flat slabs in which the theory is discussed and further explained by the solution of problems. The different types of columns and footings, including piling, commonly used in reinforced concrete structures, are presented and illustrated. Designs of plain and reinforced retaining walls, as well as other types of structures are demonstrated.

The mixing, transporting and placing of concrete is discussed. Modern machinery for this work is described and illustrated. The important subject of forms is treated and demonstrated.

This treatise supplies the information required by the engineer, contractor and student interested in the design and manufacture of concrete and reinforced concrete.

# ACKNOWLEDGMENTS

To the following societies that willingly supplied data for the text on Concrete, grateful appreciation is expressed:

National Cinder Concrete Products Association
American Society of Testing Materials
Portland Cement Association
American Concrete Institute
Architectural Concrete

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W. Herbert Gibson

*Part of the material of this text is included in the cyclopedia "Building, Estimating and Contracting."*

# CONTENTS

AN ATTRACTIVE SOUTHERN HOME, MADE OF CONCRETE

*Courtesy of Portland Cement Association*

# CONCRETE

## CHAPTER I

**Development in Concrete and Reinforced Concrete.** The design of reinforced concrete as used today is really a development of work done by Lambot, Joseph Moiner, and Francois Coignet. In 1850, Lambot constructed a boat and flower pots which were exhibited at the Paris Exposition in 1855. He regarded this material as being especially well adapted for shipbuilding, reservoirs, and work of that nature. In 1853, Francois Coignet constructed a roof which had a span of twenty feet. The total thickness of the construction was twelve inches. He patented this idea and it was used to some extent, but his work did not show a clear conception of the mechanics involved in making the design. Joseph Moiner was a Parisian gardener. He constructed flower pots, tubs, and tanks of concrete in 1861. He made a framework of wires and the concrete was placed around the wires.

Soon after this many other engineers took up reinforced concrete work, each one seeming intent upon developing his own ideas, securing patents on his system. At this time there did not seem to be a general development in the use of concrete, but it developed from individual ideas. This condition existed for many years, but in recent years engineers have developed a formula which follows the laws of mechanics and which applies to all systems of construction of reinforced concrete.

In the development of reinforced concrete in later years, about 1900 and prior to 1900, the names of Melan and Hennebique should be mentioned as leaders in the development of our present design and construction with this material. Hennebique believed that plain round bars furnished sufficient bond between the steel and the concrete, although he has been known to hook or split the ends of bars to secure better anchorage. This probably led up to the design of the deformed bars in general use today in this country. By deformed bar is meant one which furnishes mechanical bond

with the concrete by having projecting surfaces or "knobs" to form an anchorage with concrete. But even today, the French engineers use plain bars in most of their work. When they consider additional anchorage necessary, they hook the ends of the bars. To assist in making a good bond between the concrete and the steel, in this country, a higher yield point in the steel is used, as we believe if this bond is broken we would no longer have the type of structure on which our design is based.

In development of the bar with mechanical bond, the names of Ransome, Johnson, and Thacher should be mentioned. They were the leaders in this country in developing deformed bars, on which they secured patents.

In recent practice, engineers are not only using deformed bars but, in addition, are hooking the ends of bars on noncontinuous ends, which demonstrates that we believe the bond between the bar and the concrete must not be broken. Ransome and Thacher not only developed or patented bars but also did much to extend the use of reinforced concrete in this country.

**Plain Concrete.** The ingredients used in making concrete are cement, sand, stone, and water. The strength of the concrete depends largely on the proportions of these materials and, of course, on the materials themselves. To make good concrete, good materials must be used, well mixed together with a minimum of water to secure a plastic concrete. To mold the concrete in the shapes desired, forms must be constructed of sufficient strength to support the concrete when placed. Plain concrete is extensively used for dams, retaining walls, piers, abutments, foundations, and many other types of construction where loads are to be supported.

**Reinforced Concrete.** Concrete has a good compressive value but is weak in tensile strength and, therefore, steel bars are imbedded in the concrete in the proper places to resist the tensile stresses. In designing a beam, usually the top of the beam is in compression and is resisted by the compressive strength of the concrete, but the bottom of the beam will have a tensile stress to resist; and since the concrete has a low tensile value, the steel bars are placed near the bottom of the beam to resist the stress. The steel bars must be entirely imbedded in the concrete to form a bond with the concrete. The concrete on the bottom will protect the

steel from rusting and also protect it from heat in case of fires. The shear stresses in reinforced concrete are resisted by a combination of concrete and the steel.

Reinforced concrete is used for buildings, bridges, retaining walls, and in recent years it has been extensively used for highway construction. Concrete is a good material for fireproofing steel structures and also for protecting steel against rust. It is comparatively cheap material and the ingredients for making concrete are available nearly everywhere. By placing steel bars in the concrete in the correct percentage, the steel will greatly assist in reducing cracks in the concrete due to temperature stresses.

## CEMENTS

**Properties.** When cement is mixed with water, it is plastic for a short time, then sets in air or water and develops a considerable portion of its ultimate strength in a few days. It then possesses qualities of permanency to the extent that no material change in shape or volume will take place because of its inherent qualities or as a result of exterior agencies. There is more or less shrinkage in a volume of cement during the process of setting or hardening, and like many other building materials, there is a change in volume caused by change in temperature. Cement mixed with sand and water is called cement mortar. When stone and gravel or some other inert material is added to the cement, water, and sand, the resulting mixture is concrete.

**Portland Cement.** Portland cement is the most important of all cement products and is most extensively used in building work. In 1936, 112,396,000 barrels (376 pounds per barrel) were produced in the United States.

The American Society for Testing Materials defines Portland cement as follows: "Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum." The calcareous materials may consist of limestone, marl, oyster shell, chalks, etc., while the silica for the argillaceous materials may be cement rock, clay, shale, silical, blast furnace slag, or slate. The

composition of cement consists of approximately 80 per cent calcareous material and 20 per cent silica, with the addition of a small per cent of gypsum before the final grinding in order to control the setting of the cement.

**Natural Cement.** Natural cement is obtained by burning argillaceous or magnesium limestone which happens to have the proper chemical composition without the addition of any other material. The resulting clinker is finely ground and is at once ready for use. Such cement is often called Rosendale cement, owing to its having been produced first in Rosendale, Ulster County, New York. Suitable material is also found near Louisville, Kentucky. When cement was first made in this country, most of it was of this grade. But since great improvements have been made in the manufacture of Portland cement, the use of natural cement has decreased and now only a small amount is made. Natural cement sets more rapidly than Portland cement but does not attain such high strength and is not as uniform in composition and strength.

**Low-Heat Cement.** A low-heat cement has been developed and used in some of the large dams in the western part of this country. To produce this cement, the proportion of lime is decreased and a higher percentage of silica and iron is used. The lowering of the lime content decreases the amount of heat generated in the cement when setting up. This change will produce a somewhat weaker cement, but the deficiency is made up by finer grinding of the cement.

When large amounts of concrete are poured, much heat is generated in the hardening process and shrinkage cracks develop. By using low-heat cement, less time is consumed in cooling the previous pouring and work can proceed at a faster rate with fewer shrinkage cracks.

**Portland Puzzolan or Pozzolan Cement.** Portland Puzzolan cement is an interground mixture of Portland cement and a siliceous material known as Puzzolan. Puzzolan may be a natural or artificial product composed chiefly of lime, silica, and alumina. The proportion of the Puzzolan material usually is about 25 per cent and, with this proportion, it has about the same strength as straight Portland cement but requires slightly more time to develop its strength. The claims advanced for its use are: greater working ability with less mixing water; a reduction in the shrinkage of the

concrete in setting; and greater resistance against the action of sea water, sulphates, and acids. The cost is about the same as that of straight Portland cement.

**High-Early Strength Portland Cement.** This is a Portland cement produced by special burning and finer grinding. At the age of 24 hours it has about the same strength that the standard Portland cement produces in 7 days. At the age of 28 days it has about the same strength as the standard Portland cement. To secure the advantage of quick setting, the minimum amount of water should be used in the mixing. Quick setting cement costs about fifty cents per barrel more than the straight Portland cement.

**Bulk Cement.** Cement is often shipped loose where a large quantity is wanted; for example, at central mixing plants or on very large jobs. This cement requires careful handling and storage. It is usually pumped from the car to a tank or container that must be water-tight and weather-tight. The handling of bulk cement should always be done by mechanical means. It should be weighed when used, as it may become compact or, if exposed to the weather, it will become fluffy.

**Manufacture of Cement.** The manufacture of Portland cement may be classed as one of the big businesses of this country. There are about 160 mills scattered over 35 states. The manufacture of cement consists of taking an inert material and producing a powdered, chemically active material that will pass through a sieve having 40,000 holes per square inch. Portland cement is the principal cementing material used in construction work today. The inert material is composed of a mixture of lime, silica, aluminum, iron oxide, gypsum, etc., which are prepared in the proper proportions, crushed, brought to incipient fusion in a rotary kiln under intense heat, cooled, and ground ready for use. The manufacture of Portland cement is a complicated and exacting process requiring much testing of materials, and the different processes of manufacture can be outlined only briefly in this text.

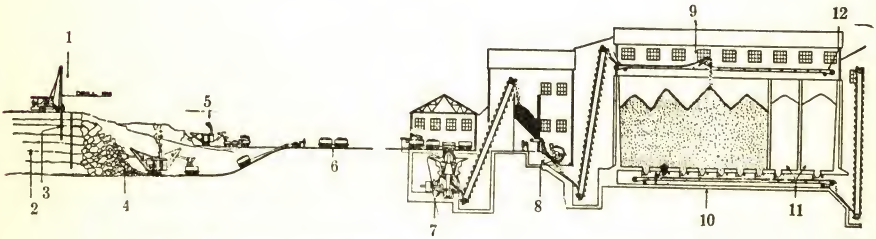
The essential operations in the manufacture of Portland cement are the quarrying of the stone, crushing and pulverizing the stone, burning these materials to clinkers, and grinding the clinkers.

The first operation in the manufacture of cement is the quarrying of the stone and other materials. This operation is accomplished

by drilling holes about 5 inches in diameter and from 50 to 80 feet deep. They usually are located about 25 feet from the quarry face and are about 15 feet on centers. When the stone is required, the holes are charged with dynamite and the blast set off. As much as 400,000 tons have been loosened in a single blast. The rock is loaded by power shovels and is transported to the crushers by locomotive and cars or trucks. When clay or marl is used, usually it is scooped and loaded on the cars with a steam shovel.

Usually the crushing is accomplished in two operations. In the first operation, rock which may weigh several tons is reduced to sizes ranging from 8 inches downward by jaw or gyratory crushers having a capacity of 200 tons or more per hour. The stone is discharged from the crushers onto conveyers by which it is carried to a hammer mill crusher where it is reduced to 1 inch or less. This material is then conveyed to bins for storing coarse materials.

The next step in the manufacture of cement is the fine grinding of the raw materials. This may be done by the dry or the wet process. In the dry process, the limestone, clay, shale, etc., pass from the bins in which coarse materials are stored to dryers and, after being dried, are conveyed to the raw grinding mills. From the grinding mill, the mixture is carried by conveyers to raw mix bins for storing fine raw material and from there is fed into rotary kilns. In the wet process, sufficient water is added to the limestone, clay, shale, etc., when they are taken from the coarse material




1. Drill Rig—Drilling Samples Taken
2. Solid Limestone
3. Drill Hole in Progress
4. Blasted Limestone
5. Steam Shovel, Clay or Shale, Quarry Samples Taken
6. Cars in Transit
7. Gyratory Crusher
8. Hammer Mills
9. Hourly Sample Taken Here
10. Rock Storage
11. Clay, Shale or Slag Storage
12. Hourly Sample Taken Here

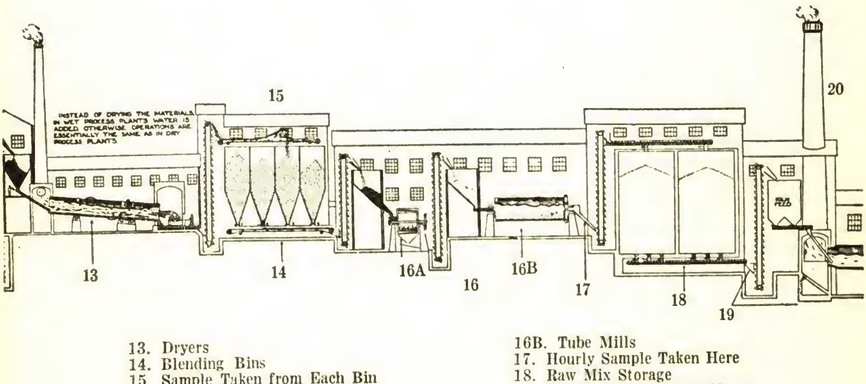
CEMENT MANUFACTURING PLANT

storage bins so that they can be ground into a creamy mixture called "slurry." The slurry is then pumped from the storage bins containing fine raw materials and passes to the rotary kilns. This grinding produces a powder that passes through a 200-mesh sieve.

From the fine raw material storage bins the mixture is transported by conveyers into the upper end of a rotary kiln. These kilns are made of steel cylinders, 6 to 12 feet in diameter and 125 to 400 feet long, which are lined with a refractory material. They are set at a slight horizontal angle and, as they revolve, the material feeds downward to the discharge end. The kiln is revolved about ¾ turn per minute by means of special gears. The fuel is fed through a pipe at the discharge end. The temperature in the kiln at the discharge is about 2800°F., the contents of the kiln being in incipient fusion. The intense heat is obtained by burning powdered coal, gas, or fuel oil.

The clinker, on passing from the kiln, may be cooled and ground or it may be conveyed to the storage piles. The grinding of the clinker is done in about the same way as the grinding of the raw materials. From the finishing mills the cement is conveyed to storage bins and held in bulk or weighed and packed for shipment.

The cement mills have complete laboratory facilities for testing the raw materials and cement. At all stages, beginning with the raw product before it is quarried and continuing to the finished product, tests are made.




13. Dryers
14. Blending Bins
15. Sample Taken from Each Bin
16. Raw Grinding
16A. Ball Mills
16B. Tube Mills
17. Hourly Sample Taken Here
18. Raw Mix Storage
19. Hourly Sample Taken Here
20. Stack

CEMENT MANUFACTURING PLANT

## CEMENT TESTING

All cement should be tested. On large operations a testing laboratory can be fitted up and all cement tested at the site of operation. On the smaller jobs tests generally are made by professional laboratories. The cost of these tests is small. The professional laboratories keep men at all the big cement plants in their vicinity so they can secure samples when the shipments are being made. Often by the time the cement is received at the job and unloaded, the report of the seven-day test is already on hand.

In the following pages will be described the physical tests for cement based on the requirements of the American Society for Testing Materials, as found in their *Standard Methods of Sampling and Physical Testing of Portland Cement,* designation C77-37. A few omissions have been made regarding the requirements of sieves, and the chemical analysis will be entirely omitted as chemical tests should be made by chemical experts in a laboratory especially fitted for the purpose.

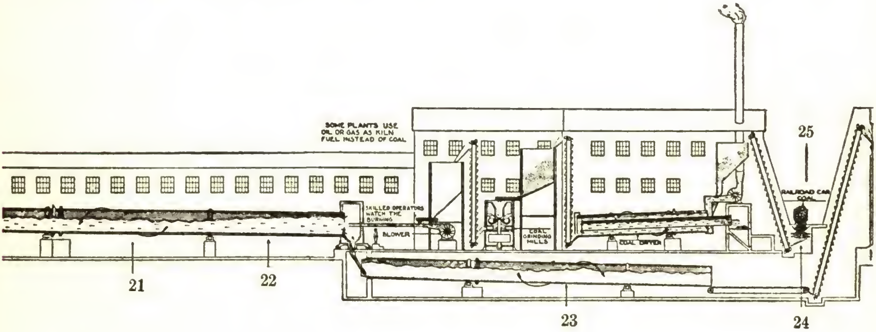
The designations for specifications for Portland cement and high-early Portland cement, which in this text follow "Cement Testing," are C9-38 and C74-38, respectively.

### SAMPLING

#### Size and Number of Samples

1. Samples for purpose of tests shall weigh at least 4 lb.

2. Test samples shall be either individual or composite samples, as may be ordered, and one test sample shall represent not more than 300 bbl., unless otherwise specified by the purchaser.




21. Kilns
22. Clinker Zone, 2500°–3000°F.
23. Clinker Coolers
24. Roll Crusher
25. Railroad Car Coal Sample Taken

CEMENT MANUFACTURING PLANT

### Methods of Sampling

3. When cement is sampled in cars, trucks, boats, warehouses, etc., one sample shall be taken from one sack in each 40 sacks (one sample per 10 bbl.), and combined to form one test sample. In the case of truck samples where the cement is being trucked from one mill, it is permissible to combine the samples from several trucks to form a test sample representing not more than 300 bbl. When sampling bulk shipments, representative samples shall be taken from well distributed points.

4. The cement may be sampled at bulk storage points by any of the following methods:

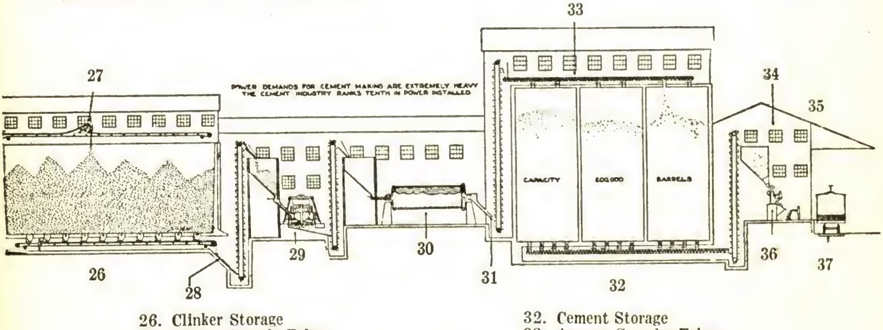
(*a*) *From the Conveyor Delivering to Storage.*—At least 4 lb. of cement shall be taken from at least each 300 bbl. passing over the conveyor. This may be secured by taking the entire test sample at a single operation, known as the "Grab Method," or by combining several portions taken at regular intervals, known as the "Composite Method." When obtaining a composite sample this shall be secured by combining approximately equal weights taken at regular intervals, each portion representing approximately 10 bbl. Automatic samplers may be used in obtaining samples.

(*b*) *From Storage by Means of Proper Sampling Tubes.*—Tubes inserted vertically may be used for sampling cement to a maximum depth of 10 ft. Tubes inserted horizontally may be used where the construction of the storage permits. Samples shall be taken at points well distributed over the storage.

(*c*) *From Storage at Points of Discharge.*—Sufficient cement shall be drawn from the discharge openings to obtain samples representative of the cement, as determined by the appearance at the discharge openings of indicators placed on the surface of the cement directly above these openings before the drawing of the cement is started. One 4-lb. sample shall be taken for at least each 300 bbl.

### Preparation of Sample

5. The sampling shall be done by or under the direction of a responsible representative of the purchaser.




26. Clinker Storage
27. Clinker Sample Taken
28. Gypsum Added Here
29. Centrifugal Mills
30. Tube Mills—Finish Grinding
31. Mill Sample Taken
32. Cement Storage
33. Average Samples Taken
34. Final Samples Taken Here
35. Sacks Cleaned and Mended
36. Packing House, Sacking Machines
37. Shipping

CEMENT MANUFACTURING PLANT

6. Samples shall be shipped and stored in moisture-proof, airtight containers. Before testing, samples shall be thoroughly mixed and then passed through a No. 20 sieve in order to break up lumps and remove foreign materials.

## MIXING CEMENT PASTES AND MORTARS

### Apparatus

25. The scales used in weighing materials for neat cement and mortar mixes shall conform to the following requirements: On scales in use the tolerance at a load of 1000 g. shall be ± 1.0 g. The tolerance on new scales shall be one-half of this value. The sensibility reciprocal shall not be greater than twice the tolerance.

### Glass Graduates

26. Glass graduates of 100-ml. to 200-ml. capacities used for measuring the mixing water shall be made to deliver the indicated volume at 20 C. (68 F.). The tolerance on these graduates shall be ± 1.0 ml.

### Method of Mixing

27. The quantities of dry materials to be mixed at one time shall be 500 g. for neat cement mixtures and not less than 1000 g. nor more than 1200 g. for mortar mixtures. The proportions of cement or cement and sand shall be stated by weight in grams of the dry materials; the quantity of water shall be expressed in milliliters. The dry materials shall be weighed, placed upon a smooth non-absorbent surface, thoroughly mixed dry if sand is used, and a crater formed in the center, into which the proper percentage of clean water shall be poured; the material on the outer edge shall be turned into the crater within 30 seconds by the aid of a trowel. After an additional interval of 30 seconds for the absorption of the water the operation shall be completed by continuous, vigorous mixing, squeezing and kneading with the hands for 1½ minutes. During the operation of mixing, the hands shall be protected by rubber gloves.

### Temperature

28. The temperature of the room and dry materials shall be maintained at not less than 20 C. (68 F.) and not more than 27.5 C. (81.5 F.). The temperature of the mixing water, moist closet, and water in the briquet storage tank shall not vary from 21 C. (70 F.) more than 1.7 C. (3 F.).

## NORMAL CONSISTENCY

### Apparatus

29. The Vicat apparatus shall consist of a frame *A* (Fig. 1) bearing a movable rod *B*, weighing 300 g., one end *C*, the plunger end, being 1 cm. in diameter for a distance of at least 5 cm., the other end having a removable needle *D*, 1 mm. in diameter and 5 cm. long. The rod *B* is reversible, and can be held in any desired position by a screw *E*, and has an adjustable indicator *F* which moves over a scale (graduated to millimeters) attached to the frame *A*. The paste is held in a rigid conical ring *G*, resting on a glass plate *H* about 10 cm. square. The ring shall be made of a non-corroding, non-absorbent material,

and shall have an inside diameter of 7 cm. at the base, 6 cm. at the top, and a height of 4 cm.

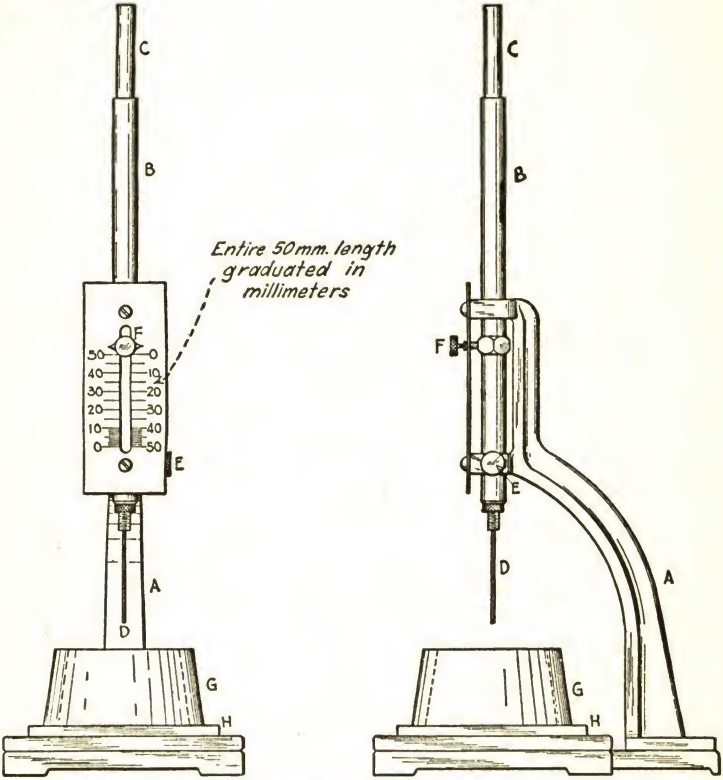



Fig. 1. Vicat Apparatus

In addition to the above, the Vicat apparatus shall conform to the following requirements:

| | | | | |
|---|---|---|---|---|
| Weight of plunger | 300 g. | ± 0.5 g. | (0.661 lb. | ± 8 grains) |
| Diameter of larger end of plunger | 1 cm. | ± 0.005 cm. | (0.394 in. | ± 0.002 in.) |
| Diameter of needle | 1 mm. | ± 0.05 mm. | (0.039 in. | ± 0.002 in.) |
| Inside diameter of ring at bottom | 7 cm. | ± 0.3 cm. | (2.75 in. | ± 0.12 in.) |
| Inside diameter of ring at top | 6 cm. | ± 0.3 cm. | (2.36 in. | ± 0.12 in.) |
| Height of ring | 4 cm. | ± 0.1 cm. | (1.57 in. | ± 0.04 in.) |
| Graduated scale | The graduated scale, when compared with a standard scale accurate to within 0.1 mm. at all points, shall not show a deviation at any point greater than 0.25 mm. | | | |

**Method**

30. *Normal Consistency, Neat Cement.*—In making the determination, 500 g. of cement, with a measured quantity of water, shall be kneaded into a paste, as described in Section 27, and quickly formed into a ball with the gloved hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in. apart; the ball resting in the palm of one

hand shall be pressed into the larger end of the conical ring *G,* Fig. 1, held in the other hand, completely filling the ring with paste; the excess at the larger end shall then be removed by a single movement of the palm of the hand; the ring shall then be placed on its larger end on a glass plate *H,* and the excess paste at the smaller end sliced off at the top of the ring by a single oblique stroke of a sharp-edged trowel held at a slight angle with the top of the ring, and the top smoothed, if necessary, with a few light touches of the pointed end of the trowel. During these operations care shall be taken not to compress the paste. The paste confined in the ring, resting on the plate, shall be centered under the rod *B,* Fig. 1, the plunger end *C,* of which shall be brought in contact with the surface of the paste and the set-screw *E* tightened. Then the movable indicator *F* shall be set to the upper zero mark of the scale or an initial reading taken and the rod released 30 seconds after completion of mixing. The apparatus shall be free from all vibrations during the test.

The paste shall be of normal consistency when the rod settles to a point 10 mm. below the original surface in 30 seconds after being released. Trial pastes shall be made with varying percentages of water until the normal consistency is obtained. Each trial shall be made with fresh cement. The amount of water required shall be expressed in percentage by weight of the dry cement.

TABLE I

**Percentage of Water for Standard Mortars**

| Percentage of Water for Neat Cement Paste of Normal Consistency | Percentage of Water for Mortar of One Cement, Three Standard Ottawa Sand | Percentage of Water for Neat Cement Paste of Normal Consistency | Percentage of Water for Mortar of One Cement, Three Standard Ottawa Sand |
|---|---|---|---|
| 15 | 9.0 | 23 | 10.3 |
| 16 | 9.2 | 24 | 10.5 |
| 17 | 9.3 | 25 | 10.7 |
| 18 | 9.5 | 26 | 10.8 |
| 19 | 9.7 | 27 | 11.0 |
| 20 | 9.8 | 28 | 11.2 |
| 21 | 10.0 | 29 | 11.3 |
| 22 | 10.2 | 30 | 11.5 |

31. *Normal Consistency, Standard Mortar.*—The consistency of standard mortar shall depend on the amount of water required to produce a paste of normal consistency from the same sample of cement. Having determined the normal consistency of the cement sample, the consistency of standard mortar made from the same sample shall be as indicated in Table I, the values being in percentage of the combined dry weights of the cement and standard sand.

## SOUNDNESS

### Apparatus

32. A steam apparatus, which can be maintained at a temperature between 98 and 100 C., or one similar to that shown in Fig. 2, is recommended. The capacity of this apparatus may be increased by using a rack for holding the pats in a vertical or inclined position.

### Method

33. A pat from cement paste of normal consistency about 3 in. in diameter, ½ in. in thickness at the center, and tapering to a thin edge, shall be made on a

flat, clean glass plate about 4 in. square. In molding the pat, the cement paste shall first be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center to give a rounded surface. The pats used for the time of setting tests by the Gillmore method (Section 40) may be used for soundness tests. After making, pats shall be stored in the moist closet for 24 hours.

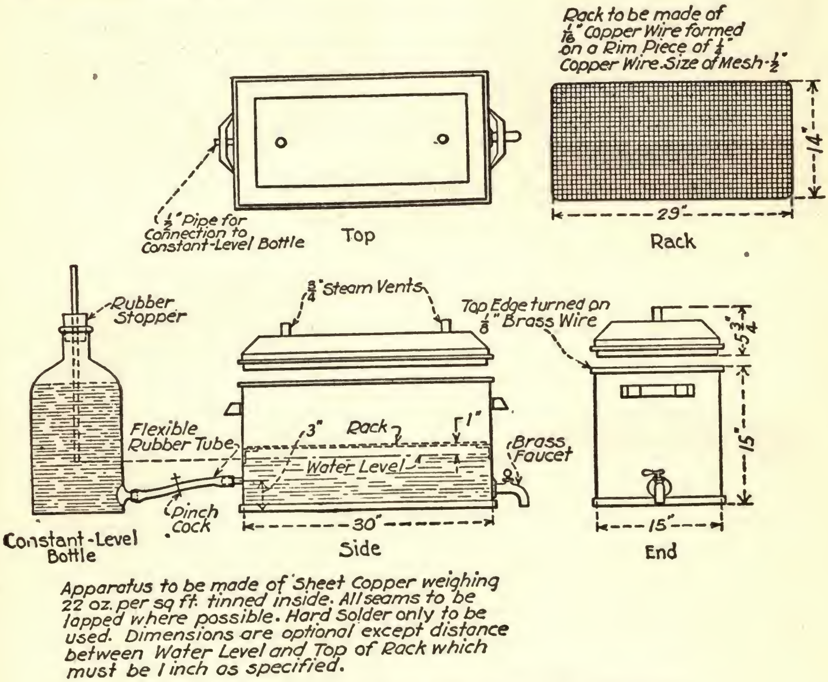



Fig. 2. Apparatus for Making Soundness Test of Cement

34. The pat on the glass plate shall then be placed in an atmosphere of steam at a temperature between 98 and 100 C., upon a suitable support 1 in. above boiling water for 5 hours.

**Soundness**

35. The pat shall be examined for unsoundness within 1 hour after removal from the steam apparatus. Unsoundness is usually manifested by change in volume which causes distortion, cracking, checking, or disintegration, as illustrated in Fig. 3 (*b*).

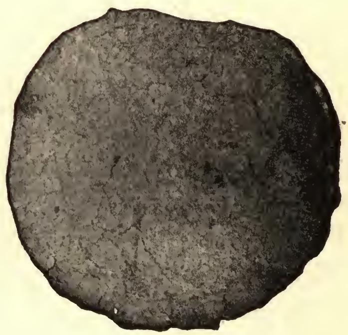
Should the pat leave the plate, distortion may be detected best with a straight edge applied to the surface which was in contact with the plate.

## TIME OF SETTING

36. The following are alternate methods, either of which may be used as specified:

(*a*) Shrinkage Cracks Due to Exposure of Pats to Dry Air During Setting

Distortion



Checking

Cracking

Disintegration

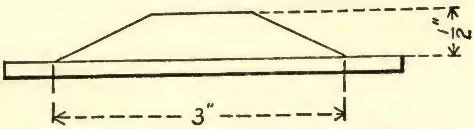
(*b*) Typical Failures in Soundness Test

Fig. 3. Soundness Test Pats

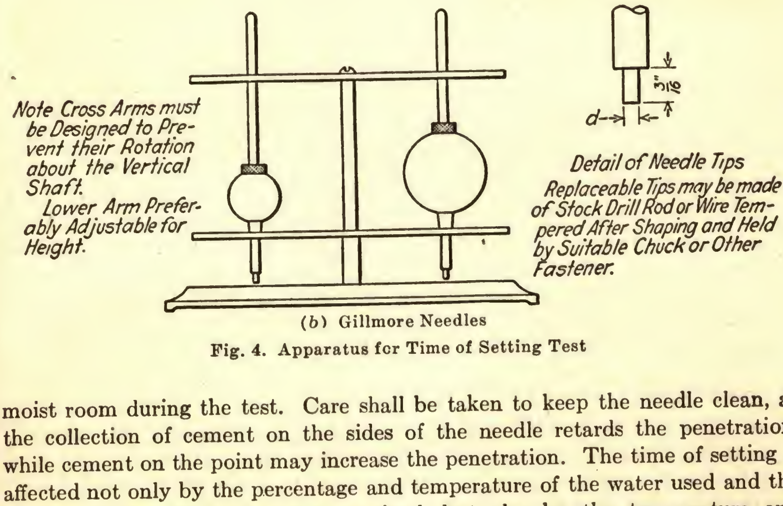
### Vicat Method

37. The time of setting shall be determined with the Vicat apparatus described in Section 29. (See Fig. 1.)

38. A paste of normal consistency shall be molded in the ring *G*, Fig. 1, as described in Section 30, and placed on the glass plate *H*. The needle *D* of the rod *B*, Fig. 1, shall then be lowered until it rests on top of a portion of the glass plate which projects beyond the ring *G*, and the adjustable indicator *F* set to the lower zero mark of the scale or an initial reading taken. The rod *B* shall then be raised, the needle *D* carefully brought in contact with the surface of the paste, and the rod quickly released. The initial set shall be said to have occurred when the needle ceases to pass a point 5 mm. above the glass plate in 30 seconds after being released; and the final set, when the needle does not sink visibly into the paste. The test pieces shall be kept in moist closet or




(*a*) Pat with Top Surface Flattened for Determining Time of Setting by Gillmore Method




(*b*) Gillmore Needles

Fig. 4. Apparatus for Time of Setting Test

moist room during the test. Care shall be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste received, but also by the temperature and humidity of the air, and its determination is therefore only approximate.

### Gillmore Method

39. The time of setting shall be determined by the Gillmore needles. The Gillmore needles should preferably be mounted as shown in Fig. 4 (*b*). The needle tips shall be cylindrical for a distance of about 3/16 in. The needle

ends shall be plane and at right angles to the axis of the rod and shall be maintained in a clean condition.

In addition to the above, the Gillmore needles shall conform to the following requirements:

Initial Needle:

| | | | | |
|---|---|---|---|---|
| Weight | ¼ lb. | ± 8 grains | (113.4 g. | ± 0.5 g.) |
| Diameter | 1/12 in. | ± 0.002 in. | (2.12 mm. | ± 0.05 mm.) |

Final Needle:

| | | | | |
|---|---|---|---|---|
| Weight | 1 lb. | ± 8 grains | (453.6 g. | ± 0.5 g.) |
| Diameter | 1/24 in. | ± 0.002 in. | (1.06 mm. | ± 0.05 mm.) |

40. A pat from cement paste of normal consistency about 3 in. in diameter, ½ in. in thickness at the center with a flat top and tapering to a thin edge, Fig. 4 (*a*), shall be made on a flat, clean glass plate about 4 in. square. In molding the pat the cement paste shall first be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center, then flattening the top. The pat shall be kept in moist closet or moist room at a temperature of 21 C. ± 1.7 C. (70 F. ± 3 F.) and a relative humidity of at least 90 per cent. The cement shall be considered to have acquired its initial set when the pat will bear, without appreciable indentation, the Gillmore needle 1/12 in. in diameter, loaded to weigh ¼ lb. The final set has been acquired when the pat will bear, without appreciable indentation, the Gillmore needle 1/24 in. in diameter, loaded to weigh 1 lb. In making the test, the needles shall be held in a vertical position and applied lightly to the surface of the pat.

## TENSILE STRENGTH TESTS

### Test Specimen

41. The form of test specimen shown in Fig. 5 shall be used. The molds shall be made of non-corroding metal and have sufficient material in the sides to prevent spreading during molding. Gang molds when used shall be of the type shown in Fig. 6. Molds shall be oiled with a mineral oil. The dimensions of the briquet molds shall conform to the following requirements: width of mold, between inside faces, at waist line of briquet, 1 in. with tolerances of ± 0.01 in. for old molds and ± 0.005 in. for new molds; thickness of new molds, measured at point of greatest thickness on either side of mold at waist line, 1 in. with tolerances of + 0.004 in. and — 0.002 in.

### Standard Sand

42. The sand to be used shall be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve. This sand may be obtained from the Ottawa Silica Co., Ottawa, Ill.

43. This sand shall be considered standard when not more than 15 g. are retained on the No. 20 sieve, and not more than 5 g. pass the No. 30 sieve, after 5 minutes continuous sieving of a 100-g. sample, in the manner specified for sieving cement on the No. 200 sieve.

44. The No. 20 and 30 sieves shall conform to the requirements for these sieves as given in the Standard Specifications for Sieves for Testing Purposes (A.S.T.M. Designation: E 11) of the American Society for Testing Materials.

**Molding Specimens**

45. The standard mortar shall be mixed in accordance with the methods for mixing cement pastes and mortars as described in Sections 25 to 28. Im-

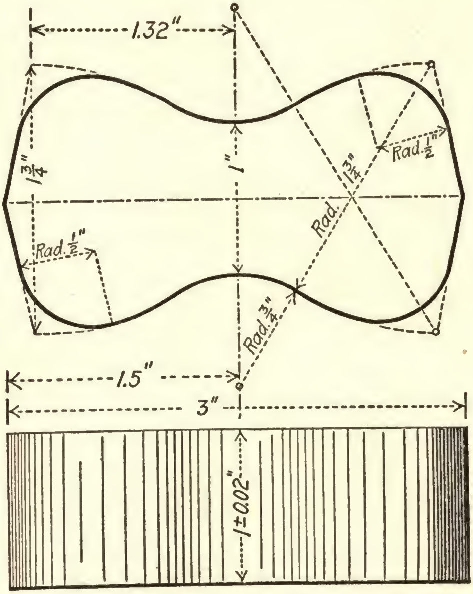



Fig. 5. Diagram Showing Form and Dimensions of Standard Cement Briquet Used for Testing

mediately after mixing, the molds, resting on unoiled glass or metal plates, shall be filled heaping full without compacting. Then the mortar shall be pressed in firmly with the thumbs, applying pressure 12 times to each briquet, at

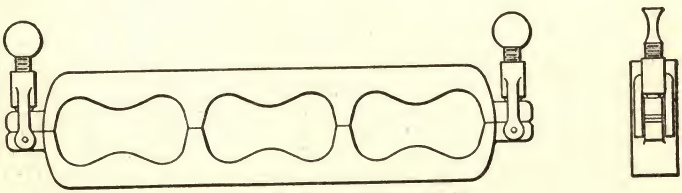
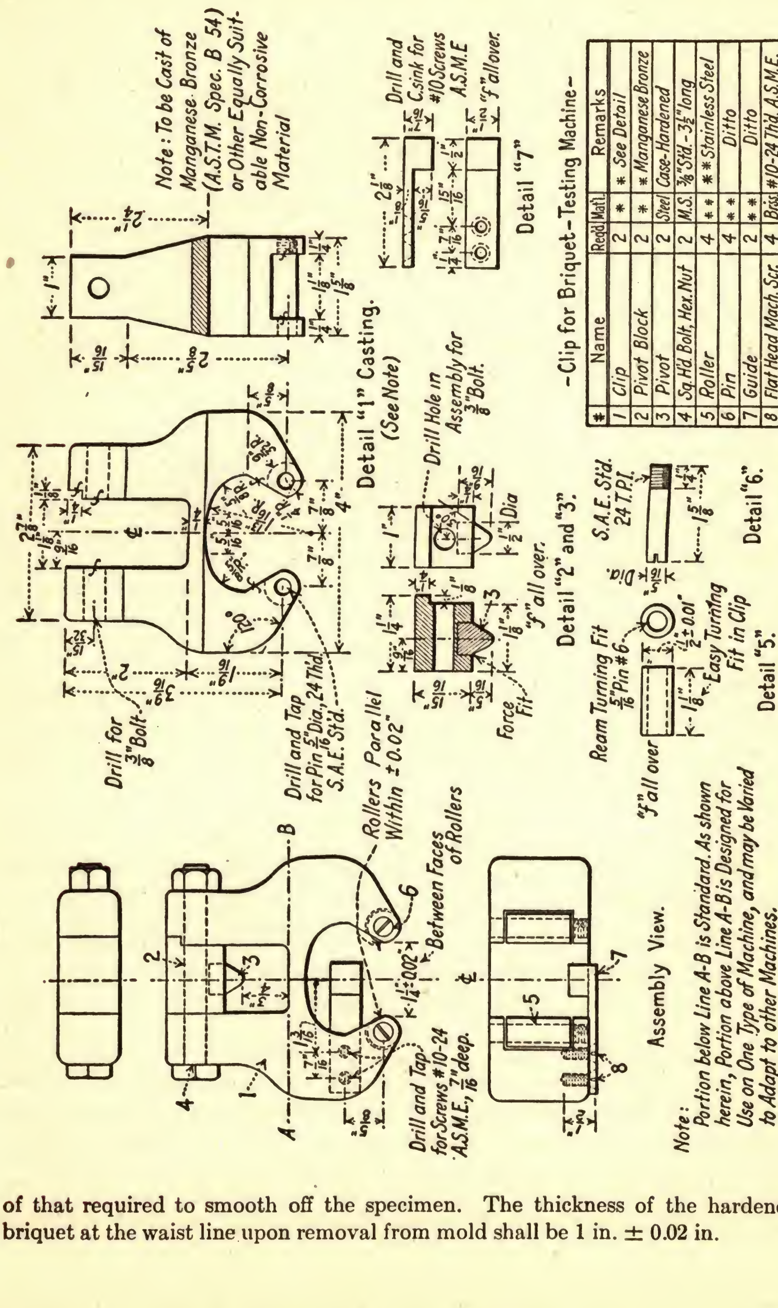


Fig. 6. Briquet Gang Mold

points to include the entire surface. The pressure shall be such that the simultaneous application of both thumbs will register a pressure of between 15 and 20 lb. Each application of the thumbs shall be maintained not longer than sufficient to attain the specified pressure. Then the mortar shall be heaped above the mold and smoothed off with a trowel. The trowel shall be drawn over the mold in such a manner as to exert a pressure of not more than 4 lb. The mold shall then be turned over upon a plane plate oiled with mineral

oil, and the operation of heaping, thumbing, heaping and smoothing off repeated. No ramming or tamping shall be used, nor any troweling in excess




—Clip for Briquet-Testing Machine—

| # | Name | Req'd | Mat'l | Remarks |
|---|---|---|---|---|
| 1 | Clip | 2 | * | * See Detail |
| 2 | Pivot Block | 2 | * | * Manganese Bronze |
| 3 | Pivot | 2 | Steel | Case-Hardened |
| 4 | Sq. Hd. Bolt, Hex. Nut | 2 | M.S. | 3/8" Std. - 3 1/2" long |
| 5 | Roller | 4 | ** | ** Stainless Steel |
| 6 | Pin | 4 | ** | Ditto |
| 7 | Guide | 2 | ** | Ditto |
| 8 | Flat Head Mach. Scr. | 4 | Brass | #10-24 Thd. A.S.M.E. |

Fig. 7. Clips for Briquet Testing Machine

of that required to smooth off the specimen. The thickness of the hardened briquet at the waist line upon removal from mold shall be 1 in. ± 0.02 in.

**Storage of Test Specimens**

46. All test specimens, immediately after molding, shall be kept in the molds on plane plates in the moist closet or moist room for from 20 to 24 hours in such manner that the upper surfaces shall be exposed to the moist air, but protected from dripping water.

47. The moist closet or moist room shall be so constructed as to provide storage facilities for test specimens at a relative humidity of not less than 90 per cent and at a temperature of 21 C. ± 1.7 C. (70 F. ± 3 F.).

48. The briquets shall then be removed from the molds and immersed in clean water in storage tanks constructed of non-corroding material. The storage water shall be kept clean by frequent changing. If briquets should be removed from molds before 24 hours, they should be replaced on shelves of moist closet until expiration of the 24-hour period.

**Tensile Strength Test**

49. The briquets shall be tested as soon as they are removed from the water. Tests may be made with any machine meeting the following requirements: The error for loads of not less than 100 lb. shall not exceed ± 1.0 per cent for new machines or ± 1.5 per cent for used machines. The clips for holding the tension test specimens shall be in accordance with Fig. 7. The bearing surfaces of the clips and briquets shall be free from sand or dirt, and the roller bearings shall be well oiled and maintained so as to insure freedom of turning. The briquets shall be carefully centered in the clips and the load applied continuously at the rate of 600 lb. ± 25 lb. per minute.

50. Testing machines shall be frequently calibrated in order to determine their accuracy.

**Faulty Briquets**

51. Briquets that are manifestly faulty, or which give strengths differing more than 15 per cent from the average value of all test specimens made from the same sample and tested at the same period, shall not be considered in determining the tensile strength.

## TEST MACHINES

There are many varieties of testing machines on the market. One very common type of machine is illustrated in Fig. 9. A reservoir contains a supply of shot, which falls through the pipe closed by means of a valve at the bottom. The briquet is carefully placed between the clips, as shown in the figure, and the wheel below is turned until the indicators are in line. A hook lever is moved so that a screw worm is engaged with its gear. Then the valve of the shot reservoir is opened so as to allow the shot to run into the cup, a small valve regulating the flow of shot into the cup. Better results will be obtained by allowing the shot to run slowly into the cup. The crank is then turned with just sufficient speed so that the scale

beam is held in position until the briquet is broken. Upon the breaking of the briquet, the scale beam falls and automatically closes the

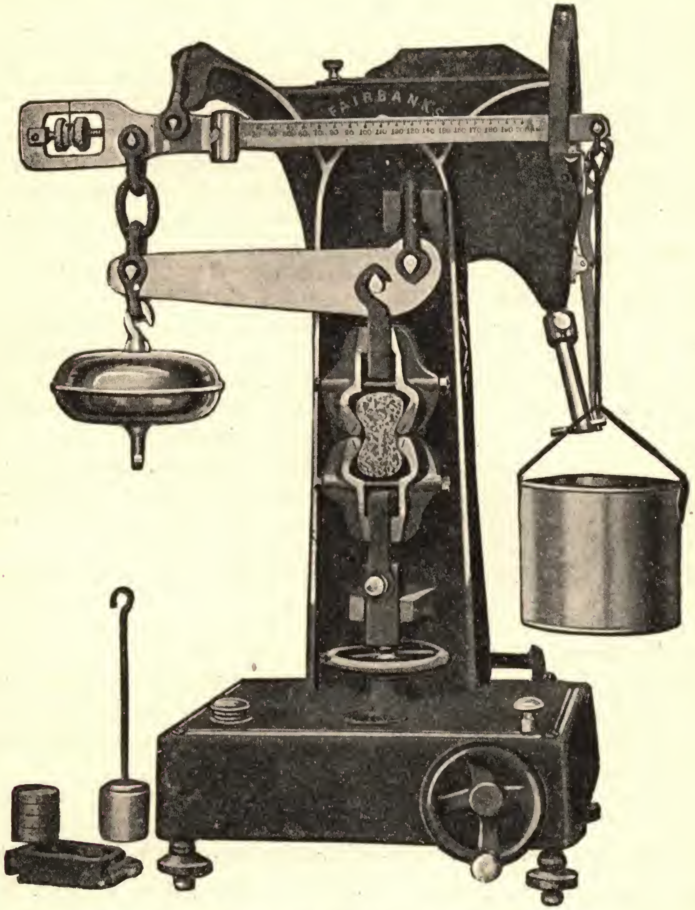


Fig. 9. Cement Testing Scales with Briquet in Position
*Courtesy of Fairbanks, Morse, and Company*

valve. The weight of the shot in the cup then indicates, according to some definite ratio, the stress required to break the briquet.

## SPECIFICATIONS FOR CEMENT

The standard specifications for Portland cement and high-early-strength Portland cement quoted are according to the standard specifications issued by the American Society of Testing Materials.

## PORTLAND CEMENT

### Definition

1. Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

### Chemical Limits

2. The following maximum limits shall not be exceeded by amounts greater than the respective tolerances indicated as allowable in the chemical determinations:

| | Limits | Tolerances |
|---|---|---|
| Loss on ignition, per cent | 4.00 | 0.25 |
| Insoluble residue, per cent | 0.85 | 0.15 |
| Sulfuric anhydride ($SO_3$), per cent | 2.00 | 0.10 |
| Magnesia (MgO), per cent | 5.00 | 0.40 |

### Soundness

3. A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

### Time of Setting

4. The cement shall not develop initial set in less than 45 min. when the Vicat needle is used or in less than 60 min. when the Gillmore needle is used. Final set shall be attained within 10 hr.

### Tensile Strength

5. The average tensile strength in pounds per square inch of not less than three standard mortar briquets (see Section 11) composed of one part of cement and three parts of standard sand, by weight, shall be equal to or higher than the following:

| Age at Test, Days | Storage of Briquets | Tensile Strength lb. per sq. in. |
|---|---|---|
| 7 | 1 day in moist air, 6 days in water | 275 |
| 28 | 1 day in moist air, 27 days in water | 350 |

6. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

### Packaging and Marking

7. The cement shall be delivered in packages as specified with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. When shipped in bulk, this information shall be contained in the shipping advices accompanying the shipment. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net. All packages shall be in good condition at the time of inspection.

### Storage

8. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

### Inspection

9. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work as may be specified by the purchaser. At least 12 days from the time of sampling shall be allowed for completion of the 7-day test, and at least 33 days shall be allowed for completion of the 28-day test. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test need not be made if waived by the purchaser.

### Rejection

10. (*a*) The cement may be rejected if it fails to meet any of the requirements of these specifications.

(*b*) Cement remaining in storage prior to shipment for a period greater than 6 months after completion of the tests shall be retested and shall be rejected if it fails to meet any of the requirements of these specifications.

(*c*) Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter. The provisional acceptance of the cement at the mill shall not deprive the purchaser of the right of rejection on a retest of soundness and time of setting at the time of delivery of cement to the purchaser.

(*d*) Packages varying more than 5 per cent from the specified weight may be rejected; and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

### Methods of Testing

11. The cement shall be sampled and tested in accordance with the Standard Methods of Sampling and Testing Portland Cement.

## HIGH-EARLY-STRENGTH PORTLAND CEMENT

### Chemical Limits

1. The following limits shall not be exceeded by amounts greater than the respective tolerances indicated as allowable in the chemical determinations:

| | Limits | Tolerances |
|---|---|---|
| Loss on ignition, per cent | 4.00 | 0.25 |
| Insoluble residue, per cent | 0.85 | 0.15 |
| Sulfuric anhydride ($SO_3$), per cent | 2.50 | 0.10 |
| Magnesia (MgO), per cent | 5.00 | 0.40 |

Portland cement is the product obtained by pulverizing clinker consisting essentially of calcium silicates, to which no additions have been made subsequent to calcination other than water and/or untreated calcium sulfate except that additions not to exceed 1 per cent of other materials may be added, provided such materials have been shown not to be harmful by tests prescribed and carried out by Committee C-1 on Cement.

Note.—Tests to determine whether a proposed addition is harmful will be carried out by Committee C-1 on Cement, for those making requests, through its Cement Reference Laboratory or other laboratory which the committee may select. As such tests are completed the committee will make known those additions which have been found not to be harmful. (For details regarding the conditions under which the tests will be made, address Technical Assistant, Committee C-1, Cement Reference Laboratory, National Bureau of Standards, Washington, D. C.)

**Soundness**

3. A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

**Time of Setting**

4. The cement shall not develop initial set in less than 45 min. when the Vicat needle is used or 60 min. when the Gillmore needle is used. Final set shall be attained within 10 hr.

**Strength**

5. (*a*) The average strength in pounds per square inch of not less than three standard specimens shall be equal to or higher than the following:

| Age at Test, Days | Storage of Specimens | Option No. 1 Tensile Strength lb. per sq. in. | Option No. 2 Compressive Strength, lb. per sq. in. |
|---|---|---|---|
| 1 | 1 day in moist air | 275 | 1300 |
| 3 | 1 day in moist air, 2 days in water | 375 | 3000 |

(*b*) If, at the option of the purchaser, a 28-day test (with storage of 1 day in moist air and 27 days in water) is required, the average strength obtained at 28 days shall be higher than the strength obtained at 3 days.

(*c*) When either of the optional strength tests is not specified by the purchaser, the tensile strength test shall be used.

**Packaging and Marking**

6. When, as specified, the cement is delivered in packages the name and brand of the manufacturer of the cement and the nature and amount of the material (found not to be harmful by Committee C-1 on Cement) added to the clinker other than water and/or untreated calcium sulfate shall be plainly marked thereon. When, as specified, the cement is delivered in bulk shipments this information shall be contained in the shipping advices accompanying the shipment. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net. All packages shall be in good condition at the time of inspection.

**Storage**

7. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

**Inspection**

8. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 6 days from the time of sampling shall be allowed for completion of the 1-day tests, at least 8 days shall be allowed for completion of the 3-day tests, and at least 33 days shall be allowed for completion of the 28-day tests. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test need not be made unless specified by the purchaser.

**Rejection**

9. (*a*) The cement may be rejected if it fails to meet any of the requirements of these specifications.

(*b*) Cement remaining in storage prior to shipment for a period greater than 6 months after completion of the tests shall be retested and shall be rejected if it fails to meet any of the requirements of these specifications.

(*c*) Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100 C. for 1 hr. it meets this requirement.

(*d*) Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter. The provisional acceptance of the cement at the mill shall not deprive the purchaser of the right of rejection on a retest of soundness and time of setting at the time of delivery of cement to the purchaser.

(*e*) Packages varying more than 5 per cent from the specified weight may be rejected; and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

**Methods of Testing**

10. The cement shall be sampled and tested in accordance with the Standard Methods of Sampling and Testing Portland Cement (A.S.T.M. Designation: C 77) of the American Society for Testing Materials, with the following exceptions:

(*a*) *Fineness.*—The specific surface shall be determined in accordance with the Tentative Method of Test for Fineness of Portland Cement by Means of the Turbidimeter (A.S.T.M. Designation: C 115—34 T) of the American Society for Testing Materials.

(*b*) *Compressive Strength.*—Compressive strength, when specified (Section 5, Option 2), shall be determined in accordance with the Tentative Method of Test for Compressive Strength of Portland-Cement Mortars (A.S.T.M. Designation: C 109—34 T) of the American Society for Testing Materials.

## AUTOCLAVE TEST

The autoclave test is an additional test for determining the soundness of cement. It is used to some extent and probably will come into general use. This test is made as an assurance against excessive expansion in concrete. It consists of subjecting the test specimens to a high steam pressure, gradual cooling, etc. The increase in the length of the test specimen should not be more than 0.50 per cent to be accepted.

# CHAPTER II

## AGGREGATES FOR CONCRETE

### SAND

Sand is a constituent part of mortar and concrete. The strength of the masonry is dependent to a considerable extent on the qualities of the sand and it is therefore important that the desirable and the defective qualities should be understood, and that these qualities be always investigated as thoroughly as are the qualities of the cement used. There have been many failures of structures due to the use of poor sand.

**Object.** Sand is required in mortar or concrete for economy and to prevent the excessive cracking that would take place in neat lime or cement without the use of sand. Mortar made without sand would be expensive and the neat lime or cement would crack so badly that the increased strength, due to the neat paste, would be of little value, if any, on account of it contracting and cracking very badly.

**Essential Qualities.** The word "sand" as used above is intended as a generic term to apply to any finely divided material which will not injuriously affect the cement or lime, and which is not subject to disintegration or decay. Sand is almost the only material which is sufficiently cheap and which will fulfill these requirements, although stone screenings (the finest material coming from a stone crusher) and powdered slag have occasionally been used as substitutes. Specifications usually demand that the sand shall be "sharp, clean, and coarse," and such terms have been repeated so often that they are accepted as standard, notwithstanding the frequent demonstration that modifications of these terms are not only desirable but also economical. These words also ignore other qualities which should be considered, especially when deciding between two or more different sources of sand supply.

**Geological Character.** Quartz sand is the most durable and unchangeable. Sands which consist largely of grains of feldspar,

mica, hornblende, etc., which will decompose upon prolonged exposure to the atmosphere, are less desirable than quartz, although, after being made up into the mortar, they are virtually protected against further decomposition.

**Coarseness.** A mixture of coarse and fine grains, with the coarse grains predominating, is found very satisfactory, as it makes a denser and stronger concrete with a less amount of cement than when fine-grained sand is used with the same proportion of cement.

The grains of the sand may be either round or angular. Many specifications state that the sand shall be "sharp." By this is meant angular-grained with rough edges. Usually there are less voids in a round, worn sand than in a sand containing angular grains. Laboratory tests made on both of these sands show little difference in ultimate strength. In one case, namely, the round grains, the concrete is denser than in the other. In the case of angular-grained sand, the cement adheres to the roughness of the grains better than it does to the smooth grains and produces a good concrete. In many sections of the country, there is not a great choice from which a selection of sand can be made for concrete work and one must be satisfied with a well-graded clean sand. The shape of the grains of sand, round or angular, is not nearly as important as the soundness of the grains and the grading from coarse to fine.

**Cleanness.** In all specifications for concrete work is found the clause: "The sand shall be clean." Very few natural deposits of sand, even when dredged from a river bed, are absolutely clean. *Organic impurities,* if excessive, will reduce the strength of mortar. The degree of organic impurity is approximately indicated by a *"color test."* "A 12-ounce graduated glass prescription bottle shall be filled to the 4½-ounce mark with the sand to be tested. A 3 per cent solution of sodium hydroxide ($NaOH$) in water shall be added until the volume of sand and liquid after shaking gives a total value of 7 liquid ounces. The bottle shall be stoppered and shaken thoroughly and then allowed to stand for 24 hours. A standard color solution shall be prepared by adding 2.5 cu. cm. of a 2 per cent solution of tannic acid in 10 per cent alcohol to 22.5 cu. cm. of a 3 per cent sodium hydroxide solution. This shall be placed in a 12-ounce prescription bottle, stoppered, and allowed to stand for 24 hours, then 25 cu. cm. of water added. The color of the clear liquid above

the sand shall be compared with the standard color solution prepared as above." If the tested sand shows a darker color than the standard color, it shows that the sand is at least doubtful and that it should be physically tested by a mortar-strength test, comparing it with a similar test made with standard Ottawa sand.

Any soluble matter in the sand is usually deleterious. Its percentage may be determined by the *decantation test.* A sample of sand is dried at a temperature of 100°C. (=230°F.) until the weight remains constant. It is then thoroughly washed until the effluent water is clear, being careful to pour the water through a 200-mesh

**Fig. 10. Test for Silt**
*Courtesy of Portland Cement Association*

sieve and returning to the sample any material retained on the sieve. Then the percentage of silt, clay, loam, etc.,

$$=\frac{\text{original dry weight—dry weight after washing}}{\text{original dry weight}}\times 100$$

If this percentage exceeds 3 per cent, the sand is doubtful and should be tested physically by comparison, as above, with standard Ottawa sand.

**Tests for Silt.** In Fig. 10 is shown a standard test for determining the amount of silt in the sand. In the first bottle is shown a sample of sand to which water has been added. The liquid above

the sand is clear. This indicates that the sand is free of organic matter and is safe to use if other requirements are fulfilled. In the second bottle, the liquid above the sand is slightly colored, which shows that organic matter is present, but not enough to be injurious. In the third bottle, the material is dark for the full height and should not be used for concrete work unless it is washed. If the sand is clear after being washed, it may then be used. The bottles used in making this test should be 32-ounce graduated prescription bottles and should be filled with sand to the 14-ounce mark. Clear water is added until the 28-ounce mark is reached. After shaking well, the materials should be allowed to stand for at least one hour, when, if more than one ounce of sediment is found on top, the sand should be washed and retested. If the sand at this time fulfills the requirements, it may be used.

**Grading.** While the general requirement for sand is that it shall have the greatest density, the best results are usually found when sieve testing will show figures about as follows:

Passing through a No. 4 sieve, not less than 85%
Passing through a No. 50 sieve, not more than 30%
nor less than 10%

Sand containing loam or earthy material is cleansed by washing with water, using a machine specially designed for the purpose. This is not an expensive operation. In general, all sand should be washed.

Fig. 11 shows a sample of well-graded sand before and after it has been separated into various sizes. Particles vary from fine up to ¼ inch in size.

Very fine sand may be used alone, but it makes a weaker concrete than either coarse sand or coarse and fine sand mixed. A mortar consisting of very fine sand and cement will not be so dense as one of coarse sand and the same cement, although, when measured or weighed dry, both contain the same proportion of voids and solid matter. In a unit measure of fine sand, there are more grains than in a unit measure of coarse sand and, therefore, more points of contact. More water is required in gaging a mixture of fine sand and cement than in a mixture of coarse sand and the same cement. The water forms a film and separates the grains, thus producing a larger volume having less density.

The screenings of broken stone are sometimes used instead of

sand. Tests frequently show a stronger concrete when screenings are used than when sand is used. This is perhaps due to the variable sizes of the screenings, which would have a less percentage of voids.

**Percentage of Voids.** As before stated, a mortar is strongest when composed of fine and coarse grains mixed in such proportion

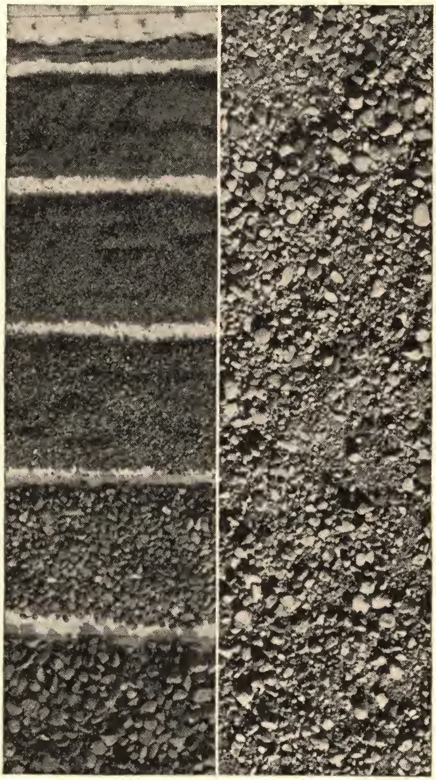


Fig. 11. Grading of Sand
*Courtesy of Portland Cement Association*

that the percentage of voids shall be the least. The simplest method of comparing two sands is to weigh a certain gross volume of each, the sand having been thoroughly shaken down. Assuming that the stone itself of each kind of sand has the same density, then the heavier volume of sand will have the least percentage of voids. The actual percentage of voids in packed sand may be approximately determined by measuring the volume of water which can be added to a given volume of packed sand. If the water is poured into the

sand, it is quite certain that air will remain in the voids in the sand, which will not be dislodged by the water, and the apparent volume of voids will be *less* than the actual. The precise determination involves the measurement of the specific gravity of the stone of which the sand is composed, and the percentage of moisture in the sand, all of which is done with elaborate precautions. The measured volume of a given quantity or weight of sand is variable, depending on moisture conditions. Adding about 6 per cent, by weight, of water to sand causes it to "bulk" 20 per cent or possibly 30 per cent. Soaking it still more will decrease the bulking, and when the sand is "inundated" and the voids are all filled with water, the volume will be substantially the same as dry sand. Very wet sand will carry from 10 per cent to 13 per cent of water, and even "moist" sand will carry about 3 per cent, which makes its volume quite uncertain when it is measured "by volume" while making mortar or concrete. A quick and approximate calculation of moisture may be made by weighing a specimen quantity of damp sand and then drying the sand by heating it until the weight is constant—no more moisture to drive off. The loss of weight is the amount of moisture in that weight of sand. But this includes the moisture of "absorption" within the sand grains, which for average sand may be considered to be 1 per cent by weight. If the total loss of weight on drying was 8 per cent, then the "surface water" on the surface of the sand grains is 7 per cent of the weight of the sand.

## BROKEN STONE

This term ordinarily signifies the product of a stone crusher in which either large stone or pebbles are reduced to their desired size for use in concrete.

**Classification of Stone.** Trap rock is the hardest and most durable stone that can be used in concrete work. The stone is dark, heavy, close-grained, and of igneous origin. The term "granite" usually is made to include not only true granite, but also gneiss, syenite, etc. These materials make good concrete and usually are less expensive than trap rock. Hard limestone is suitable for many classes of concrete work. This stone is not as strong as granite or trap rock, and is often more affected by fire, but the calcium particles can readily be cleaned off and the surface plastered. The value

of sandstone for concrete is variable according to its texture. Some grades are hard, compact, and make good concrete; other grades are friable, and, like shale or slate, are practically unfit for use. Gravel consists of pebbles of various sizes produced from stones which have been broken up and worn smooth. Generally they are hard and close-textured. Large pebbles, when crushed, make a good, dense concrete.

**Size of Crushed Stone.** In general, stone is crushed in sizes that vary from 1/4 inch to 3 inches. In crushing the stone, some of the material will be much smaller than the 1/4-inch size usually used in concrete work. This material, when properly graded, may be used as the fine aggregate for concrete work. After crushing, the stone or gravel is screened and the different sizes are discharged from the crusher into their respective bins.

For reinforced concrete where the concrete members are small and the bars close, the stone is usually graded from 1/4 to 3/4 of an inch. This size should also be used for fireproofing structural steel. Where the members are larger and the steel not so close, the stone or gravel may be graded from 1/4 inch to 1 1/4 inches. The best grading does not necessarily consist of equal amounts of the various sizes, and such grading is not often practicable. A combination of aggregates made up largely of coarse particles presents less total surface to be coated with cement than do aggregates of fine particles and so is more economical. The lowest proportion of fine aggregate that will properly fill the voids is desirable for a dense concrete.

For mass concrete, the grading for the stone or gravel may be made as large as 2 1/2 inches. Some specifications allow this material to be graded from 1/4 inch to 2 1/2 inches, but most of them will be graded from 1 1/4 to 2 1/2 inches.

When the stone and gravel are crushed, the particles should be cubic in shape. Flat, elongated, thin material should not be used. One of the essential features of good concrete is that the concrete shall have the greatest density possible. To assist in securing the percentage of voids in stone, a simple test as described herein can be used. In making this test, care must be taken to eliminate air bubbles as much as possible. If the stone is dry, it is recommended that the surfaces of the stone be wetted before the experiment is made. In like manner, tests can be made on sand.

*Illustrative Example.* A pail having a mean inside diameter of 10 inches and a height of 14 inches is filled with broken stone well shaken down; a similar pail filled with water to a depth of 8 inches is poured into the pail of stone until the water fills up all the cavities and is level with the top of the stone; there is still 2¼ inches depth of water in the pail. This means that a depth of 5¾ inches has been used to fill up the voids. The area of a 10-inch circle is 78.54 square inches and therefore the volume of the broken stone was $78.54 \times 14 = 1{,}099.56$ cubic inches. The volume of the water used to fill the pail was $78.54 \times 5.75$, or 451.6 cubic inches. This is 41 per cent of the volume of the stone, and is in this case the percentage of voids. The accuracy of the above computation depends largely on the accuracy of the measurement of the *mean inside diameter* of the pail. If the pail were truly cylindrical, there would be no inaccuracy. If the pail is flaring, the inaccuracy might be considerable; and if a precise value is desired, more accurate methods should be chosen to measure the volume of the stone and of the water.

**Screening of Stone.** The product of a stone crusher is a mixture of all sizes, varying from "sand" to pieces which are perhaps undesirably large. The output should be screened through revolving screens having graded sizes of openings. From this output are selected the various sizes, in quantity and proportion, which have been found by test to give the densest mixture of aggregate. Sometimes the finest of this crushed material is a "sand" which is equal to the best natural bank sand.

## CINDERS

The cinders from the burning of anthracite and bituminous coal are used for making concrete. The physical properties of the cinders from the two kinds of coal vary, and some engineers will permit the use of only anthracite coal cinders. Good material may be secured from both kinds of coal.

Cinders should be well burned, free from foreign matter and ashes. Large clinkers should be crushed. The amount of the unconsumed coal in the cinders should not be more than 35 per cent of the weight of the dry material. The small amount of sulphur in the cinders in general has not been found injurious.

Cinders should be obtained from gas works, industrial plants,

or similar sources where a large amount of coal is burned at a high temperature. Such plants usually buy their coal under definite specifications, the method of firing is uniform and, therefore, the cinders should not vary greatly from any one source of supply. Cinders or ashes from domestic furnaces should never be used for cinder concrete. Sometimes the qualities of cinders for concrete work can be improved by washing. Washing removes some of the impurities found in cinders made from soft coal.

### BLAST-FURNACE SLAG

Blast-furnace slags are chiefly composed of silica, alumina, magnesium, lime and a small percentage of sulphur. The weight of crushed slag varies from about 30 pounds to 100 pounds per cubic foot. Well-graded crushed slag may be substituted for stone or gravel when it weighs 70 pounds or more per cubic foot when thrown loose into a measuring box.

### HAYDITE

Haydite is made by burning shale which produces a light weight cellular, inert material for use when a light weight concrete is desirable. This material weighs about 55 pounds per cubic foot and may be used as a fine or coarse aggregate. To produce a satisfactory concrete when haydite is used as the coarse aggregate, the amount of sand or fine haydite should be nearly equal in volume to the coarse aggregate.

## CONCRETE

**General Requirements.** Concrete work, to be satisfactory, must possess the qualities of strength, safety and durability. To secure these qualities the concrete must be dense. To fill these requirements the materials must be well graded, of a good quality, well mixed, and the concrete must be properly placed and protected until it is cured. In securing a dense concrete the amount of water used is important. In mixing concrete, water is required for chemical reaction with the cement, wetting the surfaces of the aggregates and making the concrete into a sufficiently fluid state to be workable. The water-cement ratio law, now used extensively, has been of great assistance in securing a better grade of concrete.

**Durability.** Concrete has proved to be durable when made of good materials, well mixed, and properly cured. Failures can be found in concrete work, but the trouble is usually caused by poor material, faulty foundations, lack of knowledge of the properties of concrete, or poor workmanship. For example, some cements will give better results in sea water than others. This fact had to be established by experience and experiments.

It is more difficult to secure durable reinforced concrete than mass concrete. This is due to the reinforcing steel and the additional water required to make the concrete flow around the steel bars. When moisture reaches the steel, it will rust and the expansion caused by the rust will crack the concrete, resulting in an unsightly structure and necessary repairs. In all structures exposed to the weather the reinforcing steel must be carefully placed and well secured so that it cannot be displaced while concreting. No metal should project to the surfaces. Small wires will soon cause rust spots on the surface of the concrete if they are exposed.

Concrete, to be durable, must be made of good materials, uniform in quality, mixed with a minimum amount of water, and properly placed and protected while curing. Concrete exposed to sea water and the rise and fall of water levels, especially in cold climates where ice forms on the structures, requires special attention in the selection of the cement, aggregates, mixing, placing and curing.

**Water-Cement Ratios.** The term *water-cement ratio* is the ratio of the weight of water used in a mix of concrete to the weight of cement used in the mix. The ratio is usually expressed as so many gallons of water per bag of cement. The weight of water includes the surface water on the aggregates as well as the added water, but does not include water absorbed by the aggregate prior to mixing.

Cement mixed with water is the bonding material in concrete used to unite the sand and stone. It is readily seen that the thinner this material is made, the less strength it will have when it hardens. Likewise, a wet concrete will have less strength than a drier mix unless the amount of cement is increased. This means that the water-cement ratio must be maintained, but the concrete must always be workable. To produce a concrete that is uniform in strength and density it is necessary to maintain a constant water-cement ratio

with the best combination of aggregates. The minimum amount of water required to make concrete workable will always be in excess of the amount required for crystallization. The excess water remains uncombined within the concrete. When this water evaporates, the space it occupied will be found to be air voids.

In Fig. 12 are curves showing the effect of the quantity of mixing

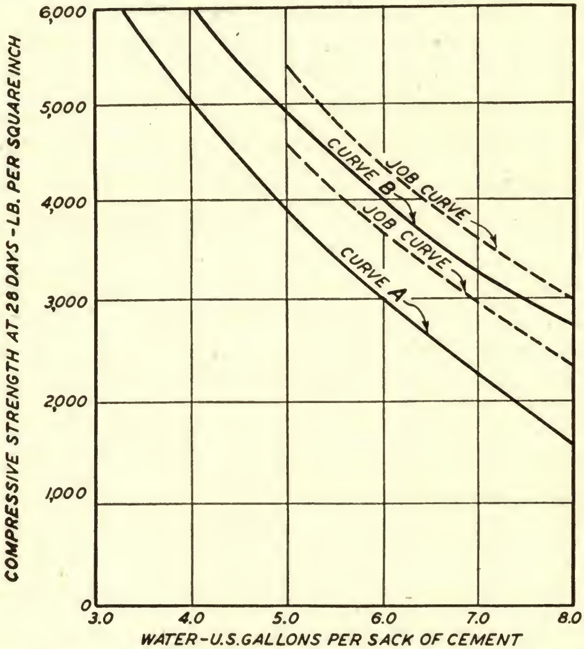



Fig. 12. Effect of Quantity of Mixing Water on the Strength of Cement

*Courtesy of Portland Cement Association*

water on the strength of concrete. Curve $A$ represents the results of tests which formed the basis for the water-cement ratio strength law which was published in 1918. Curve $B$ represents the values obtained from the cements that are in use today. The higher values of curve $B$ are due to improvements in the manufacture of cement. For purpose of design, the values shown in curve $B$ should not be exceeded unless tests on the materials to be used indicate that a higher value can be safely used.

**Proportionings.** On the general principle that the voids in ordi-

nary broken stone are somewhat less than half of the volume, it is common practice to use one-half as much sand as the volume of the broken stone. The proportion of cement is then varied according to the strength required in the structure, and according to the desire to economize. On this principle we have the familiar ratios 1:2:4, 1:2½:5, 1:3:6, and 1:4:8. It should be noted that in each of these cases (in which the numbers give the relative proportions of the cement, sand, and stone respectively) the ratio of the sand to the broken stone is a constant, and only the ratio of the cement is variable, for it would be just as correct to express the ratios as follows: 1:2:4; 0.8:2:4; 0.67:2:4; 0.5:2:4.

The proportions used in concrete work vary, depending largely on the use of the concrete, the climate, and the exposure of the finished work. For example, for concrete at the water line along a river, lake front, etc., where the water line is changing and the concrete is alternately wet and dry, the mix should be a rich one and water content should be kept low so that a dense concrete will be secured. Such a mixture should not be leaner than one part cement, two parts sand and three parts stone, and the water content should not be more than 5 to 6 gallons of water per bag of cement. This is especially true in sea water and where alternate freezing and thawing takes place.

For general building work, a mix of 1:2:4 or 1:2½:3½ is used. In either case the mix is 1:6; that is, one part cement to 6 parts aggregate. The 1:2:4 mix is satisfactory where all materials are weighed, but for the average job the 1:2½:3½ mix is more satisfactory due to the bulking of the sand. Sand with an ordinary amount of moisture will bulk about 20 per cent. Therefore, in the 1:2:4 mix there is a shortage of sand of about 20 per cent, while in the 1:2½:3½ mix the proportion of the sand is increased and the stone decreased. This may result in the over-sanding of the mix, but test cylinders taken from many jobs show a higher crushing strength and less honeycomb surface than when less sand is used.

To secure the best results in concrete it is necessary to use uniform materials made into a workable concrete, with the water-cement ratio maintained. In Table II are shown a number of suggested proportions.

Table II shows that, for the interior work in buildings, where

## TABLE II

### Classes of Concrete for Different Degrees of Exposure

| Type of Structure | Degree of Exposure | U. S. Gal. of Water per Sack of Cement |
|---|---|---|
| Walls, dams, piers and other structures exposed to sea or alkali waters | Extreme | 5½ |
| Walls, dams, piers, reservoir linings, etc., exposed to alternate wetting and drying in fresh water in northern climate. Watertight structures. Sewers, pressure pipe, tanks, piles, athletic stadia, pavements, all thin structural members exposed to severe weather and frost action | Severe | 6 |
| Walls, dams, piers, reservoir linings exposed to fresh water in southern climate. Exterior columns and beams of reinforced concrete buildings. Basement walls. Thin structural members of all types exposed to moderate weather and frost action | Moderate | 6¾ |
| Ordinary enclosed structural members. Heavy piers and retaining walls in moderate exposure. Pavement bases. Mass concrete, footings, etc., protected from alternate wetting and drying and from severe weather conditions | Protected | 7½ |

the structural members are protected, the water-cement ratio 7½ may be used. This ordinarily will mean a concrete having a compressive strength in 28 days of a little over 2,000 pounds, and when special care is taken to select sand and stone so as to give the maximum density, the strength will be still more. But if a strength of 2,500 pounds in 28 days is needed, the water-cement ratio should be cut to 6¾. The exterior columns and wall girders of a building, where not protected by a facing of brick or tile, should use the ratio 6¾. As the degree of exposure to weather, frost, or chemicals increases, the number of gallons of water per sack of cement should diminish. A "sack" of cement is officially 94 pounds. While its volume may be varied slightly by compacting, the volume of a sack is usually estimated as exactly one cubic foot, whenever it is necessary or desirable to consider its volume. The ratio of "gallons of water per sack of cement" is the most convenient for actual working.

The $7\frac{1}{2}$ ratio, for example, means $7\frac{1}{2}$ gallons=1732.5 cubic inches= 1 cubic foot of water to 1 cubic foot of cement.

**Quantities of Materials.** In a publication of the Portland Cement Association, a method is given to determine accurately the amount of cement, sand, stone and water in a concrete mix "by making use of the fact that the volume of concrete produced by any combination of materials, so long as the concrete is plastic, is equal to the sum of the *absolute* volume of the cement plus the *absolute* volume of the aggregate plus the volume of water. The absolute volume of a loose material is the actual total volume of solid matter in all the particles." This statement is demonstrated from the weight of unit volume of surface-dry materials. The specific gravity or the weight of water is taken as $62\frac{1}{2}$ pounds per cubic foot, or $8\frac{1}{3}$ pounds per gallon.

$$\text{Absolute volume} = \frac{\text{unit weight}}{\text{specific gravity} \times \text{unit weight of water}}$$

A further explanation can be made by the use of an example in which one bag of cement is taken as 94#, 1 cu. ft. of surface-dry sand as 110# and 1 cu. ft. of surface-dry stone at 105#. This is mixed with a water-cement ratio of 7 gallons per bag. The specific gravity of the cement is usually 3.1 and of sand and stone about 2.65, but for granite it would be about 2.70 and 2.95 for trap rock.

The calculation for the volume of concrete produced by a $1:2\frac{1}{2}:3\frac{1}{2}$ mix is as follows.

$$\text{Cement} = 1 \text{ cu. ft.} \times \frac{94}{3.1 \times 62.5} = .49 \text{ cu. ft. abs. vol.}$$

$$\text{Fine aggregate} = 2.5 \text{ cu. ft.} \times \frac{110}{2.65 \times 62.5} = 1.66 \text{ cu. ft. abs. vol.}$$

$$\text{Coarse aggregate} = 3.5 \text{ cu. ft.} \times \frac{105}{2.65 \times 62.5} = 2.22 \text{ cu. ft. abs. vol.}$$

$$\text{Volume of water} = \frac{7.0}{7.5} = .93 \text{ cu. ft. abs. vol.}$$

$$\text{Total volume of concrete produced} \quad \overline{5.30} \text{ cu. ft.}$$

Neglecting such losses as absorption, etc., one bag of cement will produce 5.30 cu. ft. of concrete with the $1:2\frac{1}{2}:3\frac{1}{2}$ mix. The quantity of material for one cubic yard of concrete depends on the number of cubic feet of material used with each bag of cement.

$$\text{Cement} = \frac{27}{5.30} = 5.09 \text{ bags or } 5.09 \div 4 = 1.27 \text{ barrels}$$

$$\text{Sand} \quad = \frac{5.09 \times 2.5}{27} = 0.47 \text{ cu. yds.}$$

$$\text{Stone} \quad = \frac{5.09 \times 3.5}{27} = 0.66 \text{ cu. yds.}$$

**Bulking of Aggregates.** The bulking of sand and stone or gravel has already been discussed. This bulking must be watched, especially for sand. In Table III is given a method for the correction of bulking.

**TABLE III**

**Bulking of Aggregate**

| Item | Sand | Coarse Aggregate |
|---|---|---|
| 1. Weight of damp sample, ounces | 35.0 | 32.5 |
| 2. Weight of oven-dried sample, ounces | 33.0 | 31.7 |
| 3. Weight of water in damp sample (Item 1—Item 2) | 2.0 | 0.8 |
| 4. Per cent of total moisture in terms of dry aggregate, (Item 3 $\div$ Item 2) $\times$ 100 | 6 | 2.5 |
| 5. Per cent of absorption (assumed) | 1 | 1 |
| 6. Per cent of surface moisture (Item 4—Item 5) | 5 | 1.5 |
| 7. Weight per cu. ft., damp, loose, pounds | 97.2 | 94.4 |
| 8. Weight of surface dry aggregate in 1 cu. ft. of damp loose material, pounds, Item 7 $\times \frac{100}{100 + \text{Item } 6}$ | 92.6 | 93.0 |
| 9. Weight of water in 1 cu. ft. of damp loose material, pounds (Item 7 − Item 8) | 4.6 | 1.4 |
| 10. Weight per cu. ft. of surface dry compact aggregate, pounds by test | 112.0 | 99.0 |
| 11. Bulking factor, (Item 10 $\div$ Item 8) | 1.21 | 1.06 |

Cement should always be used from the original packages. A bag contains one cubic foot and weighs 94 pounds. If the cement is shipped loose, it should be proportioned by weight when used. It may bulk much more than sand when loose.

**Correction for Moisture and Bulking.** "The correction for bulking is made by adding proportionately larger amounts of the bulked aggregate to secure any desired actual volume of dry compact aggregate." A typical numerical example will readily explain the method.

Thus, if a dry compact mix of 1:2¼:3½ is to be used with the above aggregates in Table III, the proportion based on volume of loose sand will be $2\frac{1}{4} \times 1.21 = 2.7$ and of the coarse aggregate $3\frac{1}{2} \times 1.06 = 3.7$, giving a field mix of 1:2.7:3.7. To maintain uniform amounts of aggregate at the mixer, allowance for changes in bulking due to changes in moisture content should be made by corresponding changes in the measured proportions. Thus if the bulking factor of the above sand increases to 1.25, the relative volume of sand in the field mix should be changed to $\frac{1.25}{1.21} \times 2.7 = 2.8$, or, by direct calculation, $2\frac{1}{4} \times 1.25 = 2.8$.

**Compressive Strength of Concrete.** The compressive strength of concrete depends on the mix used, materials, curing, etc. A 1:3:6 mix should produce a concrete with an ultimate strength of

TABLE IV

Compressive Strength of Concrete

| Dry and Rodded Prop. | Dry Aggre. Cu. Ft. | Cement | | Wet Wt. Sand and Gravel | Water Gal. | | Slump Consistency | Comp. Strength at 28 Days | Assumed Strength at 28 Days |
|---|---|---|---|---|---|---|---|---|---|
| | | Sacks | Lb. | | Per Sack | Per Yd. | | | |
| 1–4.5 | 30.0 | 6.75 | 635 | 1100 | 5.0 | 33.7 | 3½–4½ | 4200 | |
| | | | | 2150 | 5.5 | 37.2 | 6–7 | 3950 | |
| 1–4.7 | 30.6 | 6.5 | 610 | 1150 | 5.5 | 35.7 | 3½–4½ | 3850 | 3000 |
| | | | | 2125 | 5.9 | 38.2 | 6–7 | 3500 | |
| 1–5.2 | 30.8 | 6.0 | 564 | 1200 | 5.75 | 34.7 | 3½–4½ | 3500 | |
| | | | | 2120 | 6.25 | 37.8 | 6–7 | 3250 | |
| 1–5.4 | 31.0 | 5.75 | 540 | 1225 | 6.2 | 35.6 | 3½–4½ | 3250 | 2500 |
| | | | | 2100 | 6.7 | 38.5 | 6–7 | 3000 | |
| 1–5.75 | 31.2 | 5.4 | 507 | 1240 | 6.5 | 35.0 | 3½–4½ | 3150 | |
| | | | | 2100 | 7.0 | 37.8 | 6–7 | 2700 | |
| 1–6.1 | 31.4 | 5.16 | 485 | 1280 | 6.8 | 35.1 | 3½–4½ | 2850 | 2000 |
| | | | | 2085 | 7.4 | 38.2 | 6–7 | 2475 | |
| 1–6.35 | 31.6 | 4.95 | 465 | 1300 | 7.1 | 35.2 | 3½–4½ | 2600 | |
| | | | | 2085 | 7.5 | 38.0 | 6–7 | 2450 | |
| 1–6.88 | 31.9 | 4.68 | 440 | 1320 | 7.6 | 35.5 | 3½–5 | 2300 | 1500 |
| | | | | 2087 | 8.25 | 38.6 | 6–7 | 1950 | |
| 1–7.3 | 32.2 | 4.36 | 410 | 1335 | 8.15 | 35.6 | 3½–5 | 2000 | |
| | | | | 2100 | 8.7 | 38.0 | 6–7 | 1700 | |
| 1–7.9 | 32.4 | 4.1 | 385 | 1350 | 8.7 | 35.7 | 4–5 | 1700 | |
| | | | | 2100 | 9.3 | 38.2 | 6–7 | 1300 | |
| 1–8.8 | 32.6 | 3.72 | 350 | 1375 | 9.2 | 34.3 | 4–5 | 1250 | |
| | | | | 2120 | 9.8 | 36.5 | 6–7 | 1000 | |

1500#/□″, but even in ordinary jobs it usually will have a crushing strength of 2000#/□″. While a 1:2:4 or 1:2½:3½ mix is rated at 2000#/□″, a crushing strength of 2600 to 2800#/□″ is reached without difficulty; in fact, this result is expected, but the trouble is to maintain this crushing strength on the average job. In laboratories or well organized jobs the higher limit is easily maintained.

Table IV gives the results of a number of tests made by H. J. Knopel, Engineer for the Warner Co., Central Mixed Plant, Philadelphia, Pennsylvania. The sand used in these tests was the regular washed and graded material used in their mixing plant. The coarse aggregate was gravel which was washed and graded and was of the ¾-inch commercial size. In the first column, the cement and the total aggregate is given and in the fifth column the weight of the sand and gravel are separated.

The results shown in Table IV are interesting in that all of the details as to the amount of materials used are shown, the amount of water used, the slump, and the breaking strength at 28 days. The assumed strength of the different mixes used is given in the last column.

## TESTING CONCRETE MIXTURES

**Small Batch Tests.** A fairly accurate test of a mixture can be made by small batch tests. If it is desirable to make preliminary tests of certain aggregates, this may be done by small batches. A few pounds of cement are thoroughly mixed with the amount of aggregate desired to be used and the product is dumped out and tested with a trowel, as shown in Fig. 13. In Fig. 13A it will be noted that there is a lack of mortar. Such a mix as shown here would produce a concrete that would have rough surfaces and would be honeycombed. In Fig. 13B it will be noted that with a small amount of troweling, a smooth surface is secured and even the edges of the batch have a good appearance. This mix is workable and will produce a satisfactory concrete. In Fig. 13C the specimen has too much cement and sand. While it is plastic, workable, and will produce smooth surfaces, the concrete is likely to be porous and the materials will produce a poor yield of concrete.

**Slump Tests.** The slump test is an approximate way of determining the consistency of concrete. Fig. 14 shows a mold for mak-

ing these tests. It is a frustrum of a cone, 12 inches high, with both ends open. The ends are parallel to each other and at right angles to the axis of the cone. The mold is made of number 16 gage metal,

**Fig. 13A. Concrete Mixture Which Lacks Sufficient Mortar**

**Fig. 13B. Concrete Mixture of Correct Proportions**

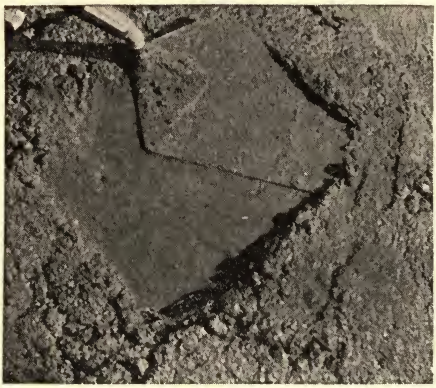


**Fig. 13C. Concrete Mixture Having Excess Cement and Sand**
*Courtesy of Portland Cement Association*

is provided with foot holds and handles and may be used for either field or laboratory testing.

When the test is made at the mixer, the sample should be secured as soon as the concrete is discharged into a hopper. When mixed at a central plant or when transit mixed, the sample should be taken as soon as the concrete is discharged.

The mold is filled at three operations. About one-third of the volume is placed and rodded 25 times with a rod $\frac{5}{8}$-inch in diameter.

A second lot of concrete is added and rodded 25 times. As a third operation, the mold is filled and rodded 25 times. In filling the mold, a uniform distribution of the concrete must be obtained; that is, the concrete must be deposited by moving the scoop around the top edge. This detail should be watched closely at each filling.

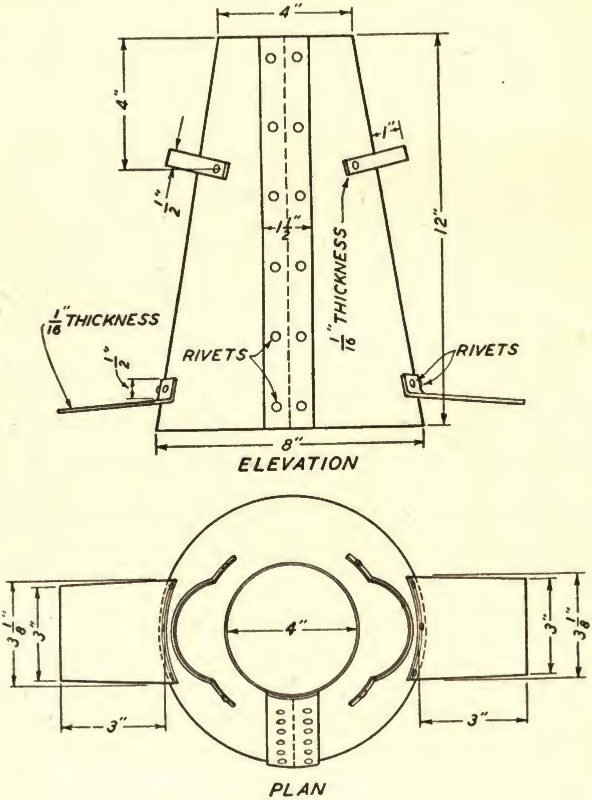



Fig. 14. Slump Test Apparatus

When the first lot of concrete is placed, it should be rodded to the bottom in a uniform manner over the entire surface and each of the following layers should be rodded into the layer below. Following this operation, the top is struck off so the mold is exactly filled. The mold is removed at once. Care must be taken to raise the mold in a vertical direction. Immediately the slump is measured by securing the difference in height between the top of the mold and the top of the concrete being tested.

The consistency is known by the number of inches the concrete settled when the mold was removed. That is, if it settled 5 inches below the top of the mold, then the slump is 5 inches.

Fig. 15 shows the method of taking the measurement in making a slump test. A straightedge is placed over the top of the cone and the difference between the bottom of the straightedge and the top of the concrete is the slump.

Slump tests must be made carefully or false results may be obtained. If the specimen slides off sidewise something is wrong

Fig. 15. Measuring the Slump
*Courtesy of Portland Cement Association*

and the test should be remade with fresh concrete. A concrete properly proportioned and of a workable mix will slump gradually and retain its original appearance. A poor mix will separate, crumble, and fall apart.

**Frequency of Making Tests.** "The frequency of making tests will depend largely upon the uniformity of the aggregates. At the beginning of operations it may be advisable to make these tests two or more times a day, but as the work progresses they may be made at longer intervals. When the aggregates are uniform and are properly handled to maintain this uniformity, it may be necessary to make some of the tests only at infrequent intervals. Usually moisture tests are made once or twice a day. After the inspector has made a few tests, he will be able to judge the moisture content fairly accu-

rately and it will be necessary to make a test only when the change is apparent. The slump test is made whenever the appearance of the concrete indicates a change in consistency or as a matter of routine inspection for record."

**Proportions of Cement and Aggregates.** Where the magnitude of the work will justify it, the proportioning is best done by selecting the aggregates several weeks before any concreting is done. This allows time for testing several combinations of cement and aggregates, using at least four different water-cement ratios and plotting the several results as to density and compressive strength. From

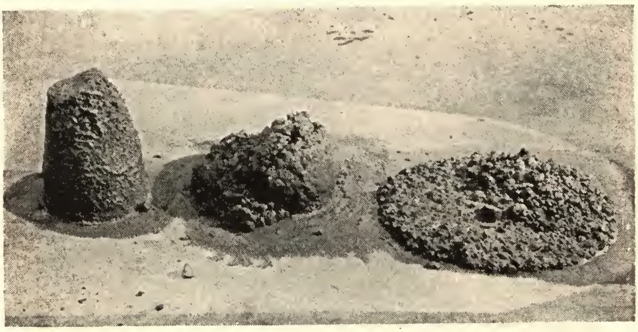


Fig. 16. Slump Tests of Three Types of Concrete Mix
*Courtesy of Portland Cement Association*

the curves thus plotted, the particular combination which will furnish a concrete having the required density and strength, but which will also be most economical of cement, may be selected. And even this combination may need to be varied somewhat, as the work proceeds, in case there is any material change in the character (size or quality) of the sand or coarse aggregate which is brought to the work.

Fig. 16 shows the results of three slump tests; the first figure to the left shows a stiff mix of concrete. Such a mix is used where it can be well tamped. The second test is that of a medium mixture such as would be used in floor construction, tanks, etc. The third test is a wet mixture and would be used in thin walls, etc.

**Test Cylinders.** On all work of any importance, test cylinders are required. These cylinders are usually 6 inches in diameter and 12 inches long, that is, the height is twice the diameter. But the concrete to be tested must not have aggregates larger than one-

third of the diameter of the cylinder, that is, concrete to be tested in a cylinder 6 inches in diameter must not have stone larger than 2 inches. The concrete placed in these cylinders should be a representative sample of the concrete used in the work. The samples usually are taken after the concrete has been poured. The cylinders are made of a nonabsorbent material such as steel or stiff waxed paper. The mold is placed on a smooth, even surface such as a piece of glass or steel. The concrete is placed in the mold in three layers of equal depth. Each layer is rodded 25 times with a $\frac{5}{8}$-inch rod 24 inches long. The strokes must be well distributed over the entire section of the specimen and penetrate into the underlying

**Fig. 17. Typical Breaks of Concrete Test Cylinders**
***Courtesy of Portland Cement Association***

layer. When the cylinder is filled and the rodding completed, the specimen will be struck off smooth on top and covered with a glass plate. After it has set from two to four hours and the shrinkage has taken place, the cylinder is filled or capped with a stiff mortar and tightly covered. If the capping is not done at the field, it may be done in a laboratory before testing.

Cylinders usually are tested at the age of 7 and 28 days. To get a fair test, at least two cylinders for each breaking should be made. They are kept usually at the site of the work for 24 hours under the conditions which obtain for the freshly poured concrete. At the end of the period they are taken into the laboratory or storage room and kept in moist air at a temperature of 70°F. until they are broken. The handling and curing of cylinders is an important item.

In Fig. 17 are shown typical fracture lines in tested cylinders.

The 7-day test should show about half the strength of the 28-day test.

**Measuring Equipment.** Concrete of definitely known strength cannot be made unless the absolute quantities of all the materials, including the water, are known and are used in definite proportions. The foregoing pages have shown the variation in strength produced by a change in proportions, which are in turn due to the change in volume on account of bulking and to the amount of water already in the aggregate when it comes to the site of the work. Uncertainty regarding these quantities makes equally uncertain the real strength of the concrete. On work of any importance this justifies the use of equipment which will, more or less automatically, measure definite *quantities* (or weights, not volumes) of the materials. There are devices for "inundating" the sand, which automatically avoid the necessity for making any allowance for bulking. Water tanks are made with automatic shut-offs, to operate when the required amount of water has been added to the mixture. These devices are locked so that no extra water can be used except by direct authorization of the engineer or inspector.

**Weighing.** At the present time, the method in favor for proportioning aggregates and loosely shipped cements is to weigh them. The amount of moisture in the aggregates is determined by tests and the necessary compensation for water is made in the aggregates.

**Time of Mixing.** Tests show that the strength of concrete and also its workability and imperviousness are increased by increasing the time of mixing. There is increase in strength up to two minutes, and an additional percentage of increase when the time of mixing is made five minutes. The added strength, workability, and imperviousness are cheaply bought by increasing the time of mixing. Time-locking devices have been used to insure full-time mixing. The time should never be less than two minutes; three or four minutes are justified by the ten per cent increase in strength. When imperviousness is a vital quality of the work, two or three minutes' mixing should be required in the specifications.

**Water-Cement Ratio for Average Materials.*** "Where no preliminary tests of the materials to be used are made, the water-cement

*From the Joint Code, Building Regulations for Reinforced Concrete, as adopted by the American Concrete Institute, and also by the Reinforcing Steel Institute.

ratios shall not exceed the values in Table V. The mixes shown in the table are approximate only, and may require adjustment to give

### TABLE V
### Assumed Strength of Concrete Mixtures

| Water-cement ratio U. S. gallons per 94-lb. sack of cement | Approximate Mix. Volume of Portland cement to sum of separate volumes of fine and coarse aggregate as measured dry | Assumed compressive strength at 28 days in pounds per square inch |
|---|---|---|
| | Plastic Concrete | |
| 8¼ | 1:7 | 1,500 |
| 7½ | 1:6 | 2,000 |
| 6¾ | 1:5¼ | 2,500 |
| 6 | 1:4½ | 3,000 |
| | Moderately Wet Concrete | |
| 8¼ | 1:6½ | 1,500 |
| 7½ | 1:5½ | 2,000 |
| 6¾ | 1:4¾ | 2,500 |
| 6 | 1:4 | 3,000 |

"Surface water" must be included when computing mixing water.

proper workability." The amounts of materials used per cubic yard for various proportions are given in Tables VI, VII, VIII.

**Proper Proportions Determined by Trial.** An accurate and simple method to determine the proportions of concrete is by trial batches. The apparatus consists of a scale and a cylinder which may be a piece of wrought-iron pipe from 10 to 12 inches in diam-

### TABLE VI
### Materials Required for One Cubic Yard of Mortar

| Mortar Mixture | Theoretical units based on loose dry materials, using a water-cement ratio of 6 gallons per sack for the mortar mixes | | Practical estimating units based on damp sand with an average amount of waste, and a water-cement ratio of 6 gallons per sack | |
|---|---|---|---|---|
| | Well-Graded Sand | | Well-Graded Sand | |
| | Cement, Barrels | Sand, Cubic Yards | Cement, Barrels | Sand, Cubic Yards |
| 1:1½ | 3.61 | 0.80 | 3.87 | 0.86 |
| 1:2 | 3.02 | 0.90 | 3.21 | 0.95 |
| 1:2½ | 2.60 | 0.96 | 2.74 | 1.01 |
| 1:3 | 2.28 | 1.01 | 2.39 | 1.06 |

*Courtesy of the North American Cement Corporation*

## TABLE VII

### Materials Required for One Cubic Yard of Concrete

| Concrete Mixture | Theoretical units based on loose dry materials, using a water-cement ratio of 6 gallons per sack for the concrete mixes — Well-graded Sand and Stone or Gravel | | | Practical estimating units based on damp sand* and stone, with an average amount of waste, and a water-cement ratio of 6 gallons per sack — Well-graded Sand and Stone or Gravel | | | Average Gradation of Sand and Gravel | | |
|---|---|---|---|---|---|---|---|---|---|
| | Cement, Barrels | Sand, Cubic Yards | Stone or Gravel, Cubic Yards | Cement, Barrels | Sand, Cubic Yards | Stone or Gravel, Cubic Yards | Cement, Barrels | Sand, Cubic Yards | Coarse Aggregate, Cubic Yards |
| 1:1:2 | 2.30 | 0.35 | 0.70 | 2.57 | 0.39 | 0.78 | 2.63 | 0.40 | 0.80 |
| 1:1½:3 | 1.71 | 0.39 | 0.78 | 1.85 | 0.42 | 0.84 | 1.90 | 0.43 | 0.87 |
| 1:2:3 | 1.54 | 0.47 | 0.70 | 1.70 | 0.52 | 0.77 | 1.73 | 0.53 | 0.79 |
| 1:2½:3½ | 1.33 | 0.49 | 0.68 | 1.44 | 0.53 | 0.75 | 1.47 | 0.54 | 0.76 |
| 1:2:3½ | 1.44 | 0.44 | 0.77 | 1.57 | 0.48 | 0.83 | 1.61 | 0.49 | 0.85 |
| 1:2:4 | 1.34 | 0.41 | 0.81 | 1.46 | 0.44 | 0.89 | 1.48 | 0.45 | 0.90 |
| 1:2½:4 | 1.24 | 0.47 | 0.75 | 1.35 | 0.52 | 0.82 | 1.38 | 0.53 | 0.84 |
| 1:2½:5 | 1.10 | 0.42 | 0.83 | 1.19 | 0.46 | 0.91 | 1.21 | 0.46 | 0.92 |
| 1:3:5 | 1.03 | 0.47 | 0.78 | 1.11 | 0.51 | 0.85 | 1.14 | 0.52 | 0.87 |
| 1:3:6 | 0.92 | 0.42 | 0.84 | 1.01 | 0.46 | 0.92 | 1.02 | 0.47 | 0.93 |

*Sand is normally delivered to the job in a damp state. "Practical estimating" figures based on 20 per cent bulking of sand and 6 per cent bulking of stone.

*Courtesy of the North American Cement Corporation.*

eter capped at one end. Measure and weigh the cement, sand, stone, and water and mix on a piece of sheet steel, the mixture having a consistency the same as to be used in the work. The mixture is

## TABLE VIII

### Suggested Concrete Mixes

(Sand assumed to be damp, about 5 per cent moisture, 20 per cent bulking)

| Type of Work | Size of Coarse Aggregate, Inches | Mix | Consistency | Compressive Strength, 28 Days Lb. per Sq. In. |
|---|---|---|---|---|
| Plain footings or foundations (not exposed to weather)......... | 2 | 1–3–5 | Medium and Stiff | 1500 |
| Reinforced footings, retaining walls, columns, beams, slabs, and tanks........... | 1 | 1–2½–3½ | Medium | 2000 |
| Large columns, beams and slabs, and tanks. | ¾ to 1 | 1–1¾–3 | Medium | 2500 |
| Pavements............ | 1½ | 1–2½–3½ | Stiff | 2000 |
| Thin walls............ | ¾ | 1–2½–3½ | Wet | 2000 |

placed in the cylinder, carefully tamped, and the height to which the pipe is filled is noted. The pipe should be weighed before and after being filled so as to check the weight of the material. The cylinder is then emptied and cleaned. Mix up another batch using the same amount of cement and water, slightly varying the ratio of the sand and stone but having the same total weight as before. Note the height in the cylinder, which will be a guide to other batches to be tried. Several trials are made until a mixture is found that gives the least height in the cylinder, works well while mixing (all the stones being covered with mortar) and makes a good appearance. This method gives good results, but it does not indicate the changes in the physical sizes of the sand and stone so as to secure the most economical composition, as would be shown in a thorough mechanical analysis.

## VOLUME CHANGES IN CONCRETE

Concrete changes in volume due to variations in temperature and the amount of moisture in the concrete. Concrete exposed to dry air for some time will shrink, but on being immersed in water it will expand. When drying is resumed the concrete will again shrink. Volume changes are also caused by temperature changes. High temperatures cause it to expand and low temperatures cause it to contract. Therefore, with moisture and temperature changes, volume changes are to be anticipated. These changes are small.

When concrete is placed in the forms, the rate of evaporation of moisture will depend on the size and mass of concrete poured and the condition of the atmosphere. If a thin floor slab is poured on a hot dry day, the water in the concrete will evaporate rapidly and the top of the slab will check badly unless it is promptly and well protected.

In construction work, cracks can be reduced by reinforcing steel and expansion joints. The steel will at least cause a better distribution of cracks, generally resulting in many small cracks instead of a few large ones. Expansion joints should be provided to allow for changes in volume. Pavements should be laid with joints 4 to 6 feet apart. They should be large enough to allow for an expansion in hot weather. This is only one example of many that might be cited.

## CURING OF CONCRETE

As mentioned above, concrete should be carefully protected during the time it is setting up. This protection should be maintained for several days, especially in hot weather. It may be done by covering the concrete with burlap or sand, which should be kept wet. Canvas may be used to protect the concrete from the sun's rays and it will also assist in retarding the evaporation of the water. Sometimes a heavy paper made of several plies containing asphalt or coal tar between the layers is used. This material must be put down as soon as the concrete can be walked on and is glued on the edges to hold the paper in place and retain the moisture in the concrete so that it can harden thoroughly. The paper must be left in place until it wears out or it is necessary to remove it for other operations. One fault generally found in curing concrete is that the curing process is not started for some hours after the concrete has been poured, much moisture has evaporated, and the surface of the concrete has started to craze.

WOODWARD & LOTHROP WAREHOUSE, WASHINGTON, D. C., UNDER CONSTRUCTION
ABBOTT, MERKT & COMPANY, ENGINEERS

# CHAPTER III

## METHODS OF MIXING, TRANSPORTING, AND DEPOSITING CONCRETE

**Mixing Concrete.** There are four general methods of mixing concrete which are as follows:

1. Mixed by machine at site of work
2. Central-mixed
3. Transit-mixed
4. Hand-mixed

*Mixing Machine at Site of Work.* This method includes a complete mixing plant at the site of the work. All materials are trucked to the site, measured, and mixed. Mixers, in size, may vary from small portable machines which mix only one-half of a bag of cement in a batch, to mixers with a capacity of three cubic yards or more. The small-sized mixers usually are mounted on trucks and can be moved easily to different points on the site. This reduces the distance of wheeling the concrete when the materials can be trucked readily to the points desired for locating the mixer. For ordinary jobs, the sand and stone or gravel may be measured in carefully calibrated wheelbarrows and dumped into the hopper of the mixer. After mixing the materials for the desired time, the concrete can be conveyed to the forms or hoisted and placed.

For large jobs or where a better controlled concrete is required, a more precise plant than described above is needed. In such a plant, the sand and stone must be measured or weighed. Storage bins for the aggregates usually are built several feet above the ground and the materials hoisted to the bins. If measured by volume, the sand and stone are permitted to pass through a gate and drop to their respective measuring vessels and are then dropped into the mixer where the cement and water are added. Likewise, if the aggregates are to be weighed, which is the favored method, they pass through the gates to weighing hoppers and to the mixer which is located below.

*Central-Mixed Concrete.* The term "central-mixed" defines a plant permanently located at some definite point. All materials for each batch of concrete are weighed and the concrete mixed and dumped into specially built water-tight trucks to convey it to the site of the building operation. These trucks usually have a capacity of two to five yards and are equipped with agitators to keep the concrete slowly moving. Engineers vary on the limit of time allowed from the time the concrete is mixed until it must be deposited in the forms. This time varies from one hour to one and one-half hours.

The central-mixed concrete has been extensively used in larger cities, especially where it is difficult to operate a standard mixer and store the necessary materials at the site of the building. A few yards can be ordered to be delivered at a set time.

One criticism about central-mixed concrete concerns the control of the water content. The amount of moisture in the sand and stone is supposed to be ascertained at least three or four times a day so that the amount of water to be added will be known definitely. If too much water is added, then a sloppy load, probably two to four yards of concrete, is delivered and must be used in that state or rejected. On important jobs, an inspector should be kept at the central plant at all times when the concrete is being mixed.

*Transit-Mixed Concrete.* The term "transit-mixed" is understood to include concrete in which the stone, gravel, and cement are placed in a truck, the cement being placed on top and the water put in a separate compartment. When the truck arrives at the job, the mixer, located in the cylinder of the truck, is put in operation and the concrete is mixed, or it can be mixed while in transit if the concrete is to be dumped immediately. The plant where the materials are loaded is called a batching plant.

The truck that has an opening in the center of the length of the cylinder appears to give more uniform concrete than one that is loaded at the end. The center-loading truck has a shorter cylinder but is greater in diameter and the concrete is more uniformly mixed.

The transit-mixed has somewhat better control on the amount of water used than the central-mixed. If the concrete is stiff, then additional water can be added and mixed thoroughly into the concrete. Advantages for city work mentioned under central-mixed

concrete also applies to the transit-mixed. Transit-mixed concrete seems to be slightly less in cost than the central-mixed.

*Hand-Mixed Concrete.* Hand-mixed concrete is used only where there is a small amount of concrete to be used. The sand and stone must be carefully measured, mixed, and turned over several times with square shovels. The piles should be leveled off, cement added, and all turned over several times. When this material has been well mixed, the water should be added and turned over a sufficient number of times to secure a uniform mixture without streaks of brown.

**Controlled Concrete.** The term "controlled concrete" describes a concrete made of selected materials, accurately measured, thoroughly mixed, having a constant water-cement ratio and consistency, all batches being as nearly uniform as possible. The concrete is to be transported by suitable means and placed in the forms without segregation. Compression tests are made to prove the quality of the concrete.

**Transporting and Depositing Concrete.** Concrete usually is deposited in layers 6 inches to 12 inches in thickness. In handling and transporting concrete, care must be taken to prevent the separation of the stone from the mortar. The usual method of transporting concrete is by wheelbarrows, "buggies," or through chutes. On larger operations it is loaded into large skips, which are raised by power hoists as high as desired and then dumped into chutes which carry it by gravity either directly to the points of deposition or into wheelbarrows or buggies, which distribute it as needed. Of course the hoists must be extended and the chutes rearranged as the work proceeds. In any case the essential requirement is that the concrete shall *flow* in a steady stream and not *fall* in such a way that the mixture disintegrates and separates the mortar from the aggregate. Skill and ingenuity are required to so design a distributing system (which is necessarily quite expensive) that the maximum amount of concrete will be distributed for each shift of the distributing system, and also that the concrete mixture will *not* disintegrate.

**Ramming Concrete.** Immediately after concrete is placed, it should be rammed or puddled, care being taken to force out the air bubbles. The amount of ramming necessary depends upon how much water is used in mixing the concrete. If a very wet mixture

is used, there is danger of too much ramming, which results in wedging the stones together and forcing the cement and sand to the surface. The chief object in ramming a very wet mixture is simply to expel the bubbles of air.

The style of rammer used depends on whether a dry, medium, or very wet mixture is used. A rammer for dry concrete is shown in Fig. 18, and one for wet concrete in Fig. 19. In thin walls, where a wet mixture is used, often the tamping or puddling is done with a part of a reinforcing bar. A common spade is often employed

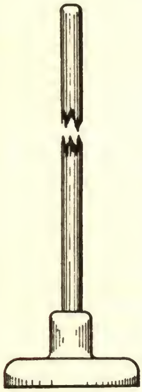


Fig. 18. Rammer for Dry Concrete

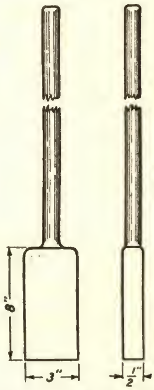



Fig. 19. Rammer for Wet Concrete

for the face of work to push back stones that may have separated from the mass and also to work the finer portions of the mass to the face, the method being to work the spade up and down the face until it is thoroughly filled. Care must be taken not to pry with the spade, as this will spring the forms unless they are unusually strong.

**Placing Concrete by Vibration.** There are two types of vibrators for placing concrete. In one type, the vibrator is attached to the forms. The other type is known as the internal vibrator in which the vibratory action is transmitted to the concrete through a rod or a spud. The latter type seems to be more generally used than the one attached to the form. Vibrators are electrically or pneumatically driven. A high frequency seems to be the most effective and the amplitude may be adjusted to suit the conditions of the work.

The advantage obtained by vibration is a reduction in the amount of water required and a denser concrete secured than if placed in the ordinary way. The shrinkage in the concrete is slightly reduced and a greater crushing strength is obtained in the concrete. This is an advantage if the finish for a floor is to be applied immediately after placing the concrete, as a reduction in the amount of water will give a stiffer mix and will permit the finishers to start their work sooner. Too, a better bond will be secured between the base and finish. The mix should not be reduced, but it may be found that the amount of the fine aggregate may be reduced slightly and the amount of the coarse aggregate increased, but the ratio of cement to the total aggregate should not be changed. The vibration should be applied uniformly throughout to the concrete being placed. It should not be over-vibrated as this treatment drives the coarse material to the bottom and the fine material to the surface, and when the forms are removed, a rough surface is found on the bottom. Stronger forms are required for concrete to be vibrated than if placed in the ordinary way.

The vibrator requires skillful handling, as over-vibration segregates the fine and coarse aggregates. It can be applied only for a few seconds at a point. It is doubtful if the additional expense required for vibration is justified for ordinary building jobs. If a building operation is to be vibrated, it may cause a complete revision of the mixing plant and transportation system to handle the stiffer mixes that are used when the concrete is compacted by vibration.

## BONDING CONSTRUCTION JOINTS

The bonding of concrete at construction joints requires careful workmanship to make the bond satisfactory. When bonding new concrete to old concrete, the old surfaces must be roughened sufficiently to expose the aggregate, carefully cleaned of all sand or other waste material, and thoroughly drenched with water for several hours before the concrete is to be poured. When ready to concrete, the surfaces should be damp but not containing drops of water. The surfaces should be painted over with a thin mortar composed of one part cement and one part sand. The concrete should be well slushed against the old concrete and carefully worked.

When construction joints are made in new work, the forms

should be removed as soon as possible. The surface should be cleaned of all laitance or soft concrete and the coarse aggregate slightly exposed, but care must be exercised not to loosen the coarse aggregate. This may be done by wire brushing, picking, or sand blasting, followed by washing with a hose under pressure. When ready to resume pouring concrete, the surfaces should be treated with the cement mortar and the fresh concrete worked well against the concrete poured previously. The cleaning of the surface of the concrete is especially required for a wet concrete as the amount of laitance will be much greater than when a dry mix is used. In heavy construction work where the concrete is in compression, at least the top surface should be well cleaned. By so doing, future disintegration of the joints should be avoided. In constructing some of the large dams in the western part of this country, special attention was given to the cleaning of the surface of the concrete. When a large block of concrete is poured, as soon as the forms can be removed, all exposed sides are thoroughly cleaned. Different methods have been used such as hand cleaning with wire brushes, sand blasting, chipping with hand picks, using water jets, etc.

If a joint must be water-tight, then other methods must be used in addition to those already described. A horizontal joint is difficult to make water-tight. Therefore, it is necessary to do something to avoid the straight joint. This is done by placing a stick of timber 3 by 4 inches, 2 by 6 inches, or 2 by 8 inches in the top of the pouring being completed. This timber is bedded so that the top will be flush with the concrete, and if the sides are slightly beveled it will be easier to remove it. A slight amount of oil on the timber will assist in the removal. These timbers should be placed in both the horizontal and vertical joints. As soon as the concrete has set sufficiently for the removal of the timber, the surface should be treated as already explained. In Fig. 20, at (*A*), is shown a horizontal joint in a wall in which the timber is used. In addition to building timber in the top of the wall, a further step may be taken that will assist greatly in securing a tight joint. This consists of placing a piece of copper or zinc in the joint. This material should be about 8 inches wide and extend 4 inches into the concrete just poured and 4 inches into the future pouring. In Fig. 20, at (*B*), is shown a joint in which the metal is used. Some-

times the top and bottom edges are bent, and the ends should be jointed the same as when this material is used for roofs. The tightness of construction joints can be aided often by the use of reinforcing steel. In plain walls, bars about 6 feet long with hooked ends may be placed near the surface on both sides, and in reinforced concrete walls, floors, etc., additional short bars with hooked ends may be added.

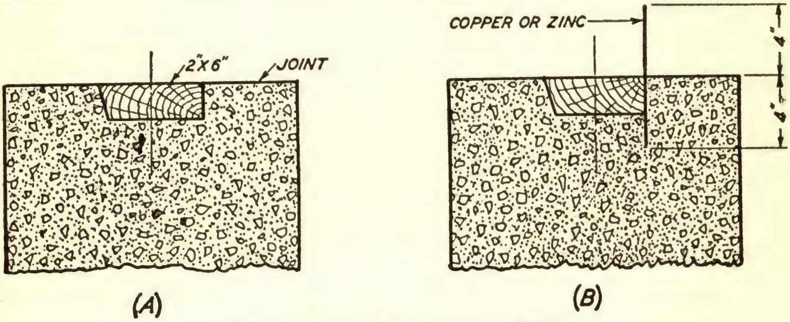



Fig. 20. Joints in New Concrete Work

## CONCRETING IN COLD WEATHER

To obtain the best results in concrete work, it must not be permitted to freeze. If the concrete is allowed to freeze, the water crystallizes and is not available for combination with the concrete. If the concrete should become frozen, it should be thawed out slowly and kept warm for several days until it has set hard. It should not be allowed to freeze and thaw alternately as the results will prove disastrous usually.

The setting of concrete is greatly retarded by cold weather. At a temperature below 50°Fahrenheit, the concrete sets slowly and must not be disturbed or have loads placed on it. To secure good results adequate protection against frost must be taken. During the fall months in the northern states, the temperature in the day time may be warm, but at night it is cool and may even go near the freezing point. Therefore, the average temperature for the 24 hours is low and the concrete will not gain strength readily unless it is heated. The heat is required especially if the forms are to be removed promptly for re-use. To secure the proper chemical combination, for the best results in concrete, a temperature of 70°F. should be maintained day and night for a period of several days. Also, in

building construction during cold weather, sufficient shores should be kept in place below the floor being concreted to transmit the load to a firm bearing which may consist of the ground or lower floors that have been properly cured and are of sufficient strength to carry the load.

The protection of concrete in cold weather may be divided under the headings of "Frost Protection" and "Winter Protection." When the temperature drops to 35° or 40°F. at night, the temperature of the concrete may be raised by heating the water. A large storage tank for the mixing of water must be placed convenient to the mixer and live steam used to heat this water so that the temperature of the concrete, when placed in the forms, will be at least 70°F. To retain this heat in concrete, it should be covered with swamp hay or canvas. A steam boiler should be installed to supply the steam for heating the water.

For winter mixing, it is necessary to heat the sand and gravel as well as the water, to enclose the structure, and to supply temporary heat. The sand and stone are heated by projecting steam jets into or under the materials stored, or by placing strong sheet iron pipes about 18 inches in diameter under the piles of materials to be used and then build a fire in these tubes. At no time should any sand or stone be used that contains frost. In freezing weather, the stone or gravel may have dry surfaces, but by applying heat it will be found that the material contains frost. In a wall bearing building, the openings can be covered with heavy canvas and the entire building below the floor being poured can be heated by steam or coke fires in salamanders, the temperature being kept as near 70°F. as possible. The top of the concrete which has just been poured should be covered with marsh hay and canvas. A building with a complete structural frame of reinforced concrete should be entirely covered with heavy canvas or at least the section which has not reached its proper strength. This canvas can be constructed outside of the columns so that the heat can reach the exterior sides.

Forms and shores must be kept in place until the concrete has attained sufficient strength to support itself and all loads placed on it. In constructing shores for the first floor of a building in winter, the builder must watch the condition of the soil on which he places the load. If the ground is frozen, it may thaw from the heat used

to warm the concrete and let the shores settle, but if the ground freezes there will be a tendency to rise. With precaution, good concrete can be secured in winter weather. In fact, the cracking in fresh concrete that occurs in hot weather will be avoided. The additional cost for winter concreting, compared with summer work, is not a great expense.

## WATER-TIGHT CONCRETE

**Requirements.** To secure a water-tight concrete, a rich, dense, and workable mix is required. The materials must be well graded. It is preferable that 15 per cent of the fines in the sand pass a 50-mesh sieve and if it is not practicable to secure sand in which at least 10 per cent does pass, then additional cement should be used. The coarse aggregate should not contain a lot of material of one size, but it should consist of several sizes of particles. Nonporous aggregates must be used.

The concrete mix should never be leaner than 1 part Portland cement, 2½ parts of sand and 3½ parts stone for a structure to be water-tight. The concrete must be workable whatever mix is used and the water-cement ratio must be carefully determined and strictly followed. A change in the amount of water causes change in the amount of shrinkage in the concrete and, therefore, a tendency to crack. The amount of water used in the mix should be a minimum (not over 6½ gallons per bag of cement) to secure a well-spaded concrete. Many structures have been constructed of a leaner mixture than that mentioned above and have been water-tight, but they were fortunate in having a well-graded mixture. The surfaces of concrete walls must be dense to resist the penetration of water as the alternate freezing and thawing in cold climates will cause the surfaces to scale off.

The mixing is an important point in securing water-tight concrete. After the cement, aggregates, and water have been placed in the mixer, it should be kept there for at least two minutes, and the mixer kept revolving at the rate specified by the manufacturer. The mixer should not be overloaded and the entire contents should be discharged at one operation and immediately placed in the forms by methods that will not cause it to separate. Then it should be carefully spaded.

Much care must be exercised in the placing of concrete in a structure that is to be water-tight. A large quantity of concrete should not be placed at one point and permitted to flow a long distance. It should be placed in small quantities and at many points.

A great difficulty in securing a water-tight job is to make tight construction joints. Straight joints should never be permitted. In constructing both horizontal and vertical joints, a groove at least two inches deep and four inches wide should be secured by building a piece of wood of these dimensions in the form work. A piece of copper or zinc, six to eight inches wide, placed in the joints, will prove of assistance. The copper or zinc is to be placed so that one half of it will be in the section being poured and the other half projecting into the future section.

For concrete to be water-tight, it must be carefully cured. A floor may be protected by placing two inches of sand on it, or the concrete may be covered with burlap. Either covering will require frequent water sprinklings. The walls will be protected by the forms if permitted to stay in place, and by spraying the forms with water, the concrete will be given a good opportunity to cure properly. If it is necessary to remove the forms, the walls should be protected with canvas or burlap for a period of at least one week and kept well sprinkled with water.

**Cinder Concrete.** Cinder concrete has been used extensively in the eastern part of the United States for fireproofing structural steel frame work for buildings and for floor slabs. Usually cinders from the burning of anthracite coal have been used. In recent years, cinder concrete has been extensively used for building blocks. Cinder concrete makes an excellent material for fireproofing structural steel from the fire hazard standpoint. It is much lighter than stone concrete. The weight of cinder concrete will vary from 106 to 118 pounds. Often the cinders are used without the addition of sand, that is, one part of cement is used to about five parts of cinders. A more uniform product can be secured by using sand for part of the fine aggregate. This may be mixed in a proportion of 1 part Portland cement, 2 parts sand and 4 to 5 parts of cinders. Cinder concrete is porous and will absorb much water. Therefore, it should not be used as a protection for structural steel below the ground level or where there is a great deal of moisture.

The compressive strength of cinder concrete is from 800 to 900#/□″ and, therefore, cannot be used as a compression member for great loads.

An example of work completed with cinder concrete is the group composing Rockefeller Center, New York City. The structural steel frame work for these buildings was fireproofed with cinder concrete, and the floors and roof slabs also were constructed with this material.

**Rubble Concrete.** Rubble concrete is a concrete in which large stones are placed, and will be discussed further on.

## WATERPROOFING METHODS

**Steel Reinforcement.** Reinforcing steel properly proportioned and located both horizontally and vertically in walls for buildings, reservoirs, etc., will greatly assist in rendering the concrete impervious to water by reducing the number and size of the cracks. The amount of steel in the longitudinal direction for indoor structures should have a minimum ratio of 0.0025 of reinforcement area to the concrete area. For outside areas, this ratio should be increased to 0.003. Deformed bars should be used for this work so as to assist in securing a better distribution of the stresses. The bars should be placed in both faces of the wall and be well distributed.

**Plastering of Walls.** Waterproofing should be applied to the side of the wall or structure where the pressure exists. For example, if a basement wall is to be water-tight, the waterproofing material should be applied to the exterior side of the wall. However, by expert means, it may be done on the inside. In either case, if plaster is to be applied to a concrete, brick, or stone wall, the surface must be well roughened, joints cleaned out to a depth of at least one inch for masonry walls, and washed down with clean water before the plaster is applied. The mortar for such work consists of 1 part Portland cement, 2½ parts of well-graded sand, and 25 pounds of lime for each bag of cement used. The plaster should be applied in two coats or more, the first coat being well scratched before the second is applied. Both coats should be applied with pressure so as to secure a plaster dense and well bonded to the masonry wall.

Tanks, reservoirs, and swimming pools are constructed for the storage of water, oil, or other liquids, and therefore the waterproofing

is applied on the inside surface. The waterproofing is applied directly to the concrete surface which should be so designed that it can be properly waterproofed.

**Cement Wash.** Often tanks with a moderate head can be made water-tight by washing the interior surface with a coat of mortar composed of 1 part cement and 2 parts of fine sand. Such a wash will not be effective for large areas or porous concrete. Surfaces containing large stone pockets or porous spots should be treated by one of the other methods given in this text.

**Portland Cement Paints.** Portland cement paints may be applied to the interior and exterior surfaces to make the concrete surfaces more dense. These paints are to be applied when the surfaces are wet and therefore the tanks do not have to dry out before they are applied. They will assist in making both the interior and the exterior surfaces more impervious to water.

**Plastering Walls with a Special Process.** A process, known as "Ironite," developed by a mid-western firm, has proved successful in many jobs, but it is a patented process and is not a cheap method of doing work.

After the walls are properly chipped and cleaned, the "Ironite," in a powdered form, mixed with water is applied to the wall. These iron particles cover the walls and joints or rough places. They oxidize and expand, and the expansion of the particles fills the surface pores and seals them against leakage. Several coats must be applied to fill all openings thoroughly. The entire surface is then plastered with a good mortar applied in the usual manner. Cracks in concrete walls have been repaired successfully with this material.

**Alum, Soap, Oil, etc.** Soap, alum, and oil have been used in concrete to make it water-tight. The theory is that these materials are water-repellent. They have been used in different combinations and in different quantities. They have also been used as a wash for covering the surface of concrete. Heavy oils have been used with some success. Their use is questionable, and, in many cases, they are effective only for a few years. Finely ground clay has also been added to the mix. The clay will decrease the strength of the concrete and if more than a small percentage is used the amount of cement should be increased. There are many admixtures on the market for making concrete water-tight. The bases of these mate-

rials vary and the results are often uncertain. Admixtures should be carefully investigated before they are used.

**Hydrated Lime.** Hydrated lime is an admix which has been successfully used to assist in making concrete impervious. The lime assists in making the concrete flow more freely, a small reduction can be made in the amount of mixing water and the fine particles assist in filling the voids in the concrete. For a 1: 2: 4 or a 1: 2½: 3½ concrete, hydrated or well-slaked lime amounting to 6 or 7 per cent of the weight of the cement may be added to every batch of concrete. The lime should be placed in the mixer with the cement. All ingredients should be mixed thoroughly, using the minimum amount of water to secure a plastic mixture.

**Asphalt.** Asphalt is laid in thicknesses from ¼ inch to 1 inch as a waterproofing course. It usually is laid in one or more continuous sheets. It is also used for filling in contraction joints in concrete. The backs of retaining walls, of either concrete, stone, or brick, are often coated with asphalt to make them waterproof, the asphalt being applied hot with a mop. The bottoms of reservoirs have been constructed of concrete blocks 6 to 8 feet square with asphalt joints ⅜ inch to ½ inch in thickness and extending at least halfway through the joint, that is, for a block 6 inches in thickness the asphalt would extend down at least 3 inches.

Asphalt is a mineral substance composed of different hydrocarbons, which are widely scattered throughout the world. There is a great variety of forms in which it is found, ranging from volatile liquids to thick semifluids and solids. These are intermixed with different kinds of inorganic or organic matter, but are sometimes found in a free or pure state. Liquid varieties are known as *naphtha* and *petroleum;* the viscous or semifluid as *maltha* or *mineral tar;* and the solid as *asphalt* or *asphaltum.* The most noted deposit of asphalt, which is used extensively in this country for paving and roofing materials, is found in the island of Trinidad and at Bermudez, Venezuela. The bituminous limestone deposited at Seyssel and Pyrimont, France; in Val-de-Travers, Canton of Neuchatel, Switzerland; and at Ragusa, Sicily, are known as rock asphalt and are perhaps the best for waterproofing purposes.

In the construction of the filter plant at Lancaster, Pa., in 1905, a pure-water basin was constructed of reinforced concrete. The

pure-water basin is 100 feet wide by 200 feet long and 14 feet deep, with buttresses spaced 16 feet 6 inches center to center. The walls at the bottom are 15 inches thick, and 12 inches thick at the top. A wet mixture of 1 part cement, 3 parts sand and 5 parts stone was used. In constructing the floor of the pure-water basin, a thin layer of asphalt was used, as shown in Fig. 21, but no waterproofing material was used in the walls, and both were found to be water-tight. With some alterations for operating purposes, it is still in use. Also, a reinforced concrete roof has been built over it.

**Felt Laid with Asphalt.** Alternate layers of paper or felt are laid with asphalt or tar, and are frequently used to waterproof floors,

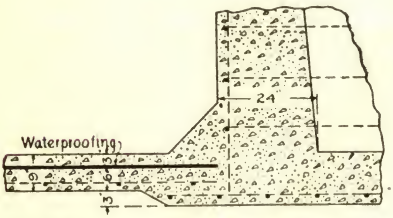



Fig. 21. Floor of Pure-Water Basin

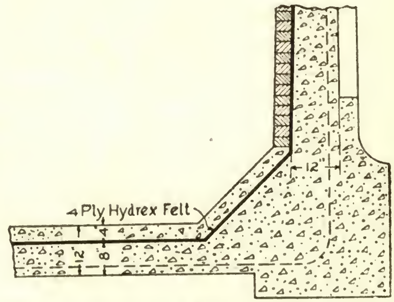



Fig. 22. Method of Waterproofing Reservoirs by Means of "Hydrex" Felt

tunnels, subways, roofs, arches, etc. These materials range from ordinary tar paper laid with coal-tar pitch or asphalt to asbestos or asphalt felt laid in coal tar or asphalt. Coal-tar products have come into very common use for this work but the coal tar should contain a large percentage of carbon to be satisfactory.

In using these materials for rendering concrete water-tight, usually a layer of concrete or brick is laid first. On this is mopped a layer of hot asphalt; felt or paper is then laid on the asphalt, the latter being lapped from 6 to 12 inches. After the first layer of felt is placed, it is mopped over with hot asphalt compound, and another layer of felt or paper is laid, the operation being repeated until the desired thickness is secured, which is usually from 2 to 10 layers—or, in other words, the waterproofing varies from 2-ply to 10-ply. A waterproofing course of this kind, or a course as described in the paragraph on asphalt waterproofing, forms a distinct joint, and the

strength in bending of the concrete on the two sides of the layer must be considered independently.

When asphalt, or asphalt laid with felt paper, is used for waterproofing the interiors of the walls of tanks, a 4-inch course of brick is required to protect and hold in place the waterproofing materials. Fig. 22 shows a wall section of a reservoir (*Engineering Record*, Sept. 21, 1907) constructed for the New York, New Haven and Hartford

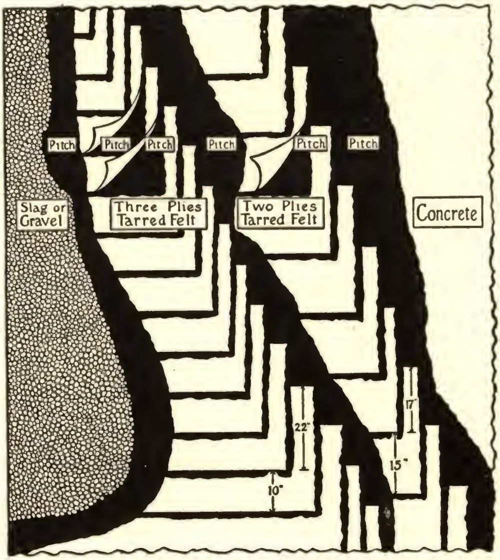



**Fig. 23. Section Showing Method of Waterproofing Concrete**

Railroad, which illustrates the methods described above. The waterproofing materials for this reservoir consist of 4-ply "Hydrex" felt, and "Hydrex" compound was used to cement the layers together.

Fig. 23 is an illustration of the method used by the Barrett Manufacturing Company in applying their 5-ply coal-tar pitch and felt roofing material. It illustrates in a general way the method used in applying waterproofing. The surfaces to be waterproofed are mopped with pitch or asphalt. While the pitch is still hot, a layer of felt is placed, which is followed with alternate layers of pitch or asphalt until the required number of layers of felt has been secured. In no place should one layer of felt be permitted to touch the layer above or below it. When the last layer of felt is laid and

thoroughly mopped with the coal-tar, something should be placed over the entire surface waterproofed to protect it from being injured. For roofing, this protection is gravel, as shown in Fig. 23. In waterproofing the back of concrete or stone arches usually a layer of brick is placed and then the joints between the bricks are filled with pitch. Bricks used in this manner also assist in holding the waterproofing in place. Five layers of felt and pitch should be a sufficient protection against a head of water of ten feet.

## PRESERVATION OF STEEL IN CONCRETE

Structural steel and reinforcing steel when properly embedded in concrete will be preserved indefinitely. The concrete must be of sufficient thickness and density to protect the steel from moisture and air. The proof of the above statement has been confirmed many times by the removal of old structures. The steel has been found in good condition where it was properly protected. Where cracks occur in the concrete or the concrete has been broken or destroyed, generally the steel was corroded. With our knowledge of cement, aggregates, methods of mixing, etc., the structures of today should have much better protection than those constructed several years ago.

In steel frame buildings, the columns should be protected by at least 2 inches of concrete. Some building codes require 3 inches. The beams should be protected by 1½ inches of concrete on the sides and bottom, and the girders by 1½ inches of concrete on the sides and 2 inches on the bottom. The top of the steel should be at least 2 inches below the top of the slab, probably 2½ inches is better. If the top of the steel is 2½ inches below the top of the slab, there is less danger of a crack over the top of the beam and it assists in placing the electric conduits.

In reinforced concrete buildings, ordinary footings of reinforcing steel should be protected by 3 inches of concrete. For very large footings, this protection should be increased to 4 inches. All reinforcing steel in the columns or pedestals below the ground should be protected by 3 to 4 inches of concrete. All columns above the ground line should be protected by 2 inches of concrete; but in light buildings, especially where the columns are to be plastered, this thickness may be reduced to 1½ inches if the fire hazard is not great. The

beams should be protected by $1\frac{1}{2}$ inches of concrete on the sides and the bottom, the girders by $1\frac{1}{2}$ inches on the sides and 2 inches on the bottom.

The reinforcing steel in retaining walls should be protected by 2 inches of concrete. Water basins, reservoirs, etc., should be protected by $2\frac{1}{2}$ inches.

When steel grillages are used for the support of a structure, at least 6 inches of concrete should be placed under the steel and at the ends and the flanges of the beams should be protected by at least 4 inches of concrete. The distance from center to center of beams should be sufficient so that the concrete can be worked between them. The base plate and the column below the ground level should be protected by 4 inches of concrete. This concrete should be of a mixture that will produce a compression strength of at least 2,000 lbs. per square inch at the age of 28 days.

## FIRE PROTECTIVE QUALITIES OF CONCRETE

**High Resisting Qualities.** The various tests which have been conducted—including the involuntary tests made as the result of fires—have shown that the fire-resisting qualities of concrete, and even resistance to a combination of fire and water, are greater than those of any other known type of building construction. Fires and experiments which test buildings of reinforced concrete have proved that where the temperature ranges from 1,400 to 1,900 degrees Fahrenheit, the surface of the concrete may be injured to a depth of $\frac{1}{2}$ to $\frac{3}{4}$ inch or even of one inch; but the body of the concrete is not affected, and the only repairs required, if any, consist of a coat of plaster.

**Thickness of Concrete Required for Fireproofing.** Actual fires and tests have shown that 2 inches of concrete will protect an I-beam with good assurance of safety. Reinforced concrete beams and girders should have a clear thickness of $1\frac{1}{2}$ inches of concrete outside the steel on the sides and 2 inches on the bottom; slabs should have at least 1 inch below the slab bars, and columns 2 inches. Structural steel columns should have at least 2 inches of concrete outside of the farthest projecting edge.

**Theory.** The theory of the fireproofing qualities of Portland cement concrete given by Mr. Spencer B. Newberry is that the

capacity of the concrete to resist fire and prevent its transference to steel is due to its *combined water and porosity.* In hardening, concrete takes up 12 to 18 per cent of the water contained in the cement. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water is given off at 500 degrees Fahrenheit, but dehydration does not take place until 900 degrees Fahrenheit is reached. The mass is kept for a long time at comparatively low temperature by the vaporization of water absorbing heat. A steel beam embedded in concrete is thus cooled by the volatilization of water in the surrounding concrete.

Resistance to the passage of heat is offered by the porosity of concrete. Air is a poor conductor, and an air space is an efficient protection against conduction. The outside of the concrete may reach a high temperature; but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is carried off by the metal as fast as it is supplied.

**Cinder vs. Stone Concrete.** Mr. Newberry says: "Porous substances, such as asbestos, mineral wool, etc., are always used as heat-insulating material. For this same reason, cinder concrete, being highly porous, is a much better nonconductor than a dense concrete made of sand and gravel or stone, and has the added advantage of being light."

# CHAPTER IV

## STEEL FOR REINFORCING CONCRETE

**Quality of Reinforcing Steel.** Four classes of steel are used for reinforcing bars: three grades of billet steel and old railroad rails. The three grades of billet steel are the structural steel grade, the intermediate, and the hard. Billet steel is made from new billets of either acid Bessemer, electric-furnace, or open-hearth steel, which excludes re-rolled material. Old-rail steel is similar in physical qualities to hard steel; the tensile strength and yield point are higher than are those of the other grades, but they are more brittle and the elongation is less than for structural steel. The structural steel grade can be used only with a working tension not to exceed 16,000 lbs. per sq. in., which may be increased to 20,000 pounds for rail steel, or the intermediate or hard grades of billet steel. The intermediate grade is in favor for high-class work, since it combines higher permissible tensile stress and greater ductility than does the hard grade, which makes it safer under vibrating loads. The old-rail steel is popular on account of its lower price. Billet steel is generally more uniform in quality, is usually a little higher priced, but is extensively used.

## TYPES OF BARS

The steel bars used in reinforcing concrete usually consist of small bars of such shape and size that they may easily be bent and placed in the concrete so as to form a monolithic structure. To distribute the stress in the concrete, and secure the necessary bond between the steel and concrete, the steel required must be supplied in comparatively small sections. All types of the regularly rolled small bars of square, round, and rectangular section, as well as some of the smaller sections of structural steel, such as angles, T-bars, and channels, and also many special rolled bars, have been used for reinforcing concrete. These bars vary in size from ¼ inch for light construction, up to 1½ inches for heavy beams,

and up to 2 inches for large columns. In Europe plain round bars have been extensively used for many years and the same is true in the United States, but not to the same extent as in Europe; that is, in America a much larger percentage of work has been done with *deformed* bars.

**Plain Bars.** With plain bars, the transmission of stresses is dependent upon the adhesion between the concrete and the steel. Square and round bars show about the same adhesive strength, but the adhesive strength of flat bars is far below that of the round and square bars. The round bars are more convenient to handle and easier obtained, and have, therefore, generally been used when plain bars were desirable.

**Structural Steel.** Small angles, T-bars, and channels have been

Fig. 24. Square Twisted Reinforcing Steel Bar
*Courtesy of Inland Steel Company*

used to a greater extent in Europe than in this country. They are principally used where riveted skeleton work is prepared for the steel reinforcement; and in this case, usually, it is desirable to have the steel work self-supporting.

**Deformed Bars.** There are many forms of reinforcing materials on the market, differing from one another in the manner of forming the irregular projections on their surface. The object of all these special forms of bars is to furnish a bond with the concrete, independent of adhesion. This bond formed between the deformed bar and the concrete is called a *mechanical bond.* Some of the most common types of bars used are the square twisted bar; the Bethlehem, the Havemeyer; the Kahn, and the Ryerson.

*Square Twisted Bar.* The twisted bar, shown in Fig. 24, was one of the first steel bars shaped to give a mechanical bond with concrete. This type of bar is a commercial square bar twisted while cold. There are two objects in twisting the bar—*first,* to give the metal a mechanical bond with the concrete; *second,* to increase the elastic limit and ultimate strength of the bar. In twisting the bars, usually one complete turn is given the bar in nine or ten diameters of the bars, with the result that the elastic limit of the bar is

increased from 40 to 50 per cent, and the ultimate strength is increased from 25 to 35 per cent. These bars can readily be bought already twisted; or, if it is desired, square bars may be bought and twisted on the site of the work. At the present time, it is not extensively used.

*Havemeyer Bar.* The Havemeyer bar, Fig. 25, was invented by Mr. J. F. Havemeyer. This has a uniform cross section throughout

**Fig. 25. Havemeyer Bar for Reinforcement of Concrete**
*Courtesy of Concrete Steel Company*

**Fig. 26. Ryerson Bar**
*Courtesy of Joseph Ryerson & Son, Inc.*

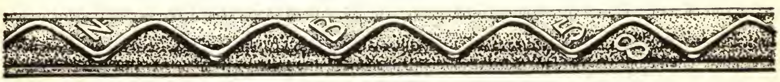


**Fig. 27. Bethlehem Bar**
*Courtesy of Bethlehem Steel Company*

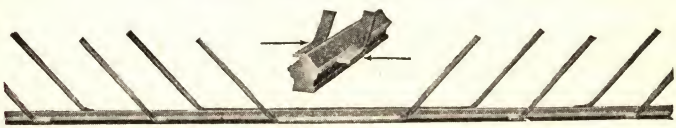


**Fig. 28. Kahn Trassed Bar for Reinforcement of Concrete**
*Courtesy of The Kahn System*

its length. The bonding of the bar to the concrete is uniform at all points, and the entire section is available for tensile strength.

*Ryerson Bar.* In Fig. 26 is shown the Ryerson bar. This bar is rolled with ribs at right angles to the axis of the bar to form a mechanical bond between the concrete and the steel.

*Bethlehem Bar.* The Bethlehem bar, Fig. 27 has been developed from the corrugated bar invented by A. L. Johnson, member of the American Society of Civil Engineers. This bar has a uniform cross section throughout its length. It is rolled with the letters N. B. (new billet) on the side of the bar, and likewise with numbers indicating the size of the bars, as shown in tne figure.

*Kahn Bar.* The Kahn bar, Fig. 28, was invented by Mr. Julius

TABLE IX

**Areas and Weights of Reinforcing Bars**

| Thickness or Diameter (inches) | Area (square inches) | Weight 1 foot long (pounds) | Circumference (round bars) (inches) |
|---|---|---|---|
| ¼ rd. | .0491 | .167 | .7854 |
| ⅜ rd. | .1104 | .376 | 1.1781 |
| ½ rd. | .1963 | .668 | 1.5708 |
| ½ sq. | .2500 | .850 | ...... |
| ⅝ rd. | .3068 | 1.043 | 1.9635 |
| ¾ rd. | .4418 | 1.502 | 2.3562 |
| ⅞ rd. | .6013 | 2.044 | 2.7489 |
| 1 rd. | .7854 | 2.670 | 3.1416 |
| 1 sq. | 1.0000 | 3.400 | ...... |
| 1⅛ sq. | 1.2656 | 4.303 | ...... |
| 1¼ sq. | 1.5625 | 5.313 | ...... |

Kahn, Member, American Society of Civil Engineers. This bar is designed with the assumption that the shear members should be rigidly connected to the horizontal members. The bar is rolled with a cross section as shown in the figure. The thin edges are cut and turned up and form the shear members. These bars are manufactured in several sizes. A rib bar is also rolled by this company.

**Standard Sizes.** In September, 1924, a Joint Conference of Representatives of Manufacturers, Distributors, and Users of Concrete Reinforcement Bars was held in Washington, when they unanimously adopted the areas and sizes of bars as shown in Table IX to become effective as applying to new production January 1, 1925; every effort to be made to clear current orders of eliminated sizes before March 1, 1925. This action cuts the number of standard sizes to eleven and eliminates many sizes which were formerly quite commonly used. The gradation, in percentage of increase from one size to the next larger, is so nearly uniform that there is no material disadvantage, while the conservation economy resulting from the elimination of needlessly small variations in sizes is great. All designs in this course are based on the eleven sizes given in Table IX.

The Concrete Reinforcing Steel Institute has requested all manufacturers of new billet reinforcing bars to mark each bar with the letters N. B. so that they can be distinguished readily as new billet bars.

### TABLE X
### Standard Sizes of Expanded Metal

| Mesh in Inches | Gage No. | Weight in Lb. Per Sq. Ft. | Sectional Area 1 Foot Wide in Sq. In. |
|---|---|---|---|
| 3 | 16 | .30 | .082 |
| 3 | 10 | .625 | .177 |
| 6 | 4 | .86 | .243 |

**Expanded Metal.** Expanded metal, Fig. 29, is made from plain sheets of steel, slit in regular lines and opened into meshes of any desired size or section of strand. It is commercially designated by giving the gage of the steel and the amount of displacement between the junctions of the meshes. The most common manufactured sizes are given in Table X.

### TABLE XI
### Standard Sizes of Electrically Welded Mesh Fabric

| Spacings of Wires in Inches | | Gage Number | | Sect. Areas, Sq. In. per Ft. | | Weight per 100 Sq. Ft. |
|---|---|---|---|---|---|---|
| Longit. | Trans. | Longit. | Trans. | Longit. | Trans. | |
| 2 | 16 | 3 | 8 | .280 | .015 | 103.6 |
| 2 | 16 | 4 | 9 | .239 | .013 | 88.5 |
| 2 | 16 | 5 | 10 | .202 | .011 | 74.6 |
| 2 | 16 | 6 | 10 | .174 | .011 | 64.7 |
| 3 | 16 | 3 | 8 | .187 | .015 | 72.0 |
| 3 | 16 | 4 | 9 | .159 | .013 | 61.4 |
| 3 | 16 | 5 | 10 | .135 | .011 | 51.8 |
| 3 | 16 | 6 | 10 | .116 | .011 | 45.1 |
| 4 | 16 | 3 | 8 | .140 | .015 | 56.1 |
| 4 | 16 | 4 | 9 | .120 | .013 | 47.9 |
| 4 | 16 | 5 | 10 | .101 | .011 | 40.4 |
| 4 | 16 | 6 | 10 | .087 | .011 | 35.2 |
| 4 | 16 | 7 | 11 | .074 | .009 | 29.7 |
| 6 | 12 | 2 | 2 | .108 | .054 | 59.4 |
| 6 | 12 | 3 | 3 | .093 | .047 | 51.2 |
| 6 | 12 | 4 | 4 | .080 | .040 | 43.8 |
| 6 | 6 | 4 | 4 | .080 | .080 | 57.8 |
| 6 | 6 | 6 | 6 | .058 | .058 | 42.0 |
| 6 | 6 | 8 | 8 | .041 | .041 | 29.9 |
| 4 | 4 | 6 | 6 | .087 | .087 | 61.9 |
| 4 | 4 | 8 | 8 | .062 | .062 | 44.1 |
| 2 | 2 | 10 | 10 | .086 | .086 | 60.3 |
| 2 | 2 | 12 | 12 | .052 | .052 | 36.8 |

**Wire Mesh.** Wire mesh is often used for reinforcing slabs in a steel frame building. The mesh is placed easily and quickly. There are two general types of mesh, the welded mesh and the triangular mesh. A few of the standard sizes of the electrically welded mesh are shown in Table XI.

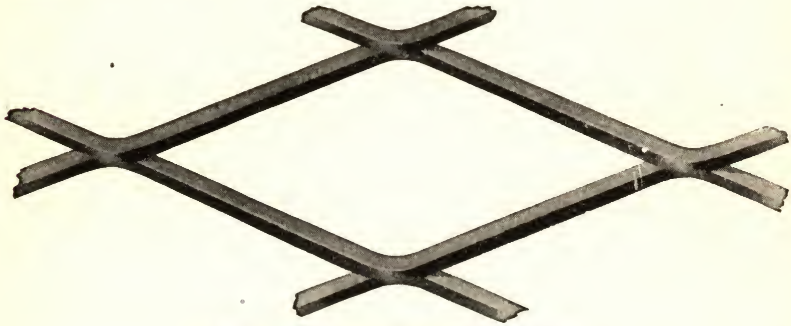


Fig. 29. Example of Expanded Metal Fabric
*Courtesy of Northwestern Expanded Metal Company*

## SPECIFICATIONS FOR REINFORCING BARS

### *BILLET-STEEL

**Process of Manufacture.** Steel may be made by the open-hearth, electric-furnace or acid-Bessemer process.

Bars shall be rolled from new billets properly identified.

**Physical Properties.** The physical properties of reinforcing bars shall conform to the limits as given in Table XII.

**Chemical Determinations.** In order to determine if the material conforms to the prescribed chemical limitations, analysis shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt or blow of steel, and a certified copy of such analysis shall be furnished to the engineer or his inspector.

**Yield Point.** For the purpose of these specifications, the yield point shall be determined by careful observation of the drop of the testing machine, or by other equally accurate methods.

**Form of Specimens.** (a) Tensile and bending test specimens of cold-twisted bars shall be cut from the bars after twisting, and shall be tested in full size without further treatment, unless otherwise

*In all essential points, these agree with the specifications of the American Society for Testing Materials, but the wording is different.

## TABLE XII

### Physical Properties of Billet-Steel Reinforcing Bars

| Properties Considered | Plain Bars | | | Deformed Bars | | |
|---|---|---|---|---|---|---|
| | Structural Steel Grade | Intermediate Grade | Hard Grade | Structural Steel Grade | Intermediate Grade | Hard Grade |
| Tensile strength in pounds per square inch..... | 55,000 to 70,000 | 70,000 to 85,000 | 80,000 min. | 55,000 to 70,000 | 70,000 to 85,000 | 80,000 min. |
| Yield point, minimum, in pounds per square inch. | 33,000 | 40,000 | 50,000 | 33,000 | 40,000 | 50,000 |
| Elongation, in 8 in., minimum, percentage*..... | $\frac{1,400,000}{\text{Ten. str.}}$ | $\frac{1,300,000}{\text{Ten. str.}}$ | $\frac{1,200,000}{\text{Ten. str.}}$ | $\frac{1,250,000}{\text{Ten. str.}}$ | $\frac{1,125,000}{\text{Ten. str.}}$ | $\frac{1,000,000}{\text{Ten. str.}}$ |
| Cold bend without fracture: thickness or diameter less than ¾"........ | 180°, $d=1t$ | 180°, $d=2t$ | 180°, $d=3t$ | 180°, $d=1t$ | 180°, $d=3t$ | 180°, $d=4t$ |
| diameter ¾" or over............ | 180°, $d=1t$ | 90°, $d=2t$ | 90°, $d=3t$ | 180°, $d=2t$ | 90°, $d=3t$ | 90°, $d=4t$ |

*Modifications of the elongation requirement are shown below.
Cold bending test: $d$=diameter of pin about which specimen is bent;
$t$=thickness or diameter of the specimen.
Cold twisted bars must have a minimum yield point of 55,000 lbs. per sq. in.; for the bending test, bars under ¾", $d=2t$; bars ¾" or over, $d=3t$.

specified as in (c), in which case the conditions therein stipulated shall govern.

(b) Tensile and bending test specimens may be cut from the bars as rolled, but tensile and bending test specimens of deformed bars may be planed or turned for a length of at least 9 inches, if deemed necessary by the manufacturer in order to obtain uniform cross section.

(c) If it is desired that the testing and acceptance for cold-twisted bars be made upon the rolled bars before being twisted, the bars shall meet the requirements of the structural steel grade for plain bars given in this specification.

**Standard Sizes.** In September, 1924, a Joint Conference of Representatives of Manufacturers, Distributors, and Users of Concrete Reinforcement Bars was held in Washington, when they unanimously adopted the areas and sizes ot bars as shown in Table IX and all designs in this text are based on the eleven sizes given.

The specification regarding chemical composition states that the phosphorus in acid-Bessemer steel shall not exceed 0.10 per cent and in open-hearth or electric-furnace steel shall not exceed 0.05 per cent for basic and 0.08 per cent for acid steel.

**Number of Tests.** At least one tensile test and one bending test shall be made from each melt of open-hearth or electric-furnace steel and each melt of 10 tons of Bessemer steel rolled. In case bars differing ⅜ inch and more in diameter or thickness are rolled from one melt or blow, a test shall be made from the thickest and thinnest material rolled. Should either of these test specimens develop flaws, or should the tensile test specimen break outside of the middle third of its gaged length, it may be discarded and another test specimen substituted therefor. In case a tensile test specimen does not meet the specifications an additional test may be made.

**Modification in Elongation for Thin and Thick Material.** For bars less than $\frac{7}{16}$ inch and more than ¾ inch nominal diameter or thickness, the following modifications shall be made in the requirements for elongation:

(a) For each increase of $\frac{1}{32}$ inch in diameter or thickness above ¾ inch, a deduction of 0.25 per cent shall be made from the specified percentage of elongation.

(b) For each decrease of $\frac{1}{32}$ inch in diameter or thickness below $\frac{7}{16}$ inch, a deduction of 0.5 per cent shall be made from the specified percentage of elongation.

(c) The above modifications in elongation shall not apply to cold-twisted bars.

**Number of Twists.** Cold-twisted bars shall be twisted cold with one complete twist in a length equal to not more than 12 times the thickness of the bar.

**Finish.** Material must be free from injurious seams, flaws, or cracks, and have a workmanlike finish.

**Variation in Weight.** Bars for reinforcement are subject to rejection if the actual weight of any lot varies more than 5 per cent over or under the theoretical weight of the lot.

## RAIL-STEEL

**Process of Manufacture.** The bars are rolled from standard section T-rails, as plain, deformed, or hot-twisted. The hot-twisted

### TABLE XIII

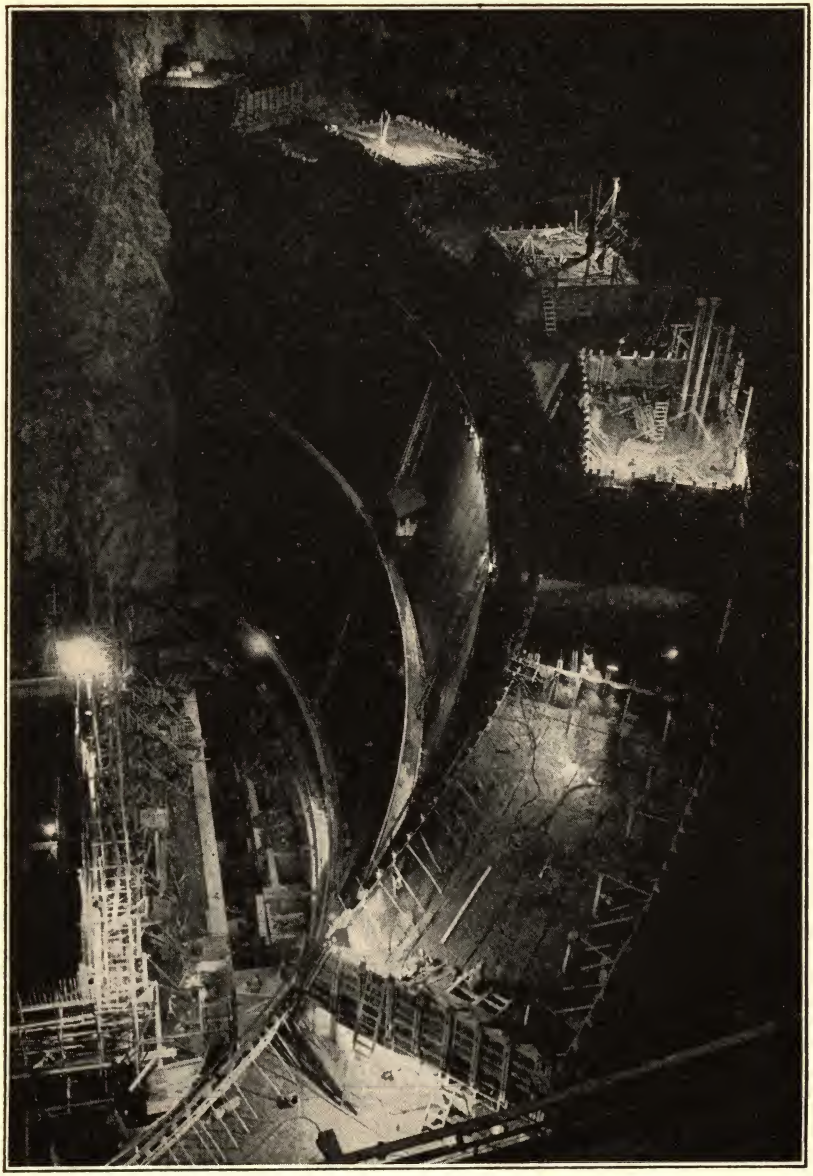
### Physical Properties of Rail-Steel Reinforcement Bars

| Properties Considered | Plain Bars | Deformed and Hot-Twisted Bars |
|---|---|---|
| Tensile strength, in pounds per sq. in. | 80,000 | 80,000 |
| Yield point, in pounds per sq. in. | 50,000 | 50,000 |
| Elongation in 8 inches, percentage* | $\frac{1,200,000}{\text{Tens. str.}}$ | $\frac{1,000,000}{\text{Tens. str.}}$ |
| Cold bend without fracture: thickness or diameter less than ¾″ | 180°, $d=3t$ | 180°, $d=4t$ |
| Cold bend: diameter ¾″ or over | 90°, $d=3t$ | 90°, $d=4t$ |

*Modifications of the elongation requirement are the same as those given for billet-steel bars.
Cold bending test: $d$=diameter of pin about which specimen is bent.
$t$=thickness or diameter of the specimen.
The specimen shall bend cold around a pin without cracking on the outside.
The other specifications as previously given for billet-steel bars also apply to rail-steel bars.

bars shall have one complete twist in a length not over 12 times the thickness of the bar.

**Physical Properties.** The bars shall conform to the requirements given in Table XIII.



VIEW TAKEN AT NIGHT DURING CONSTRUCTION OF SEMINOE DAM AND POWER PLANT FROM LEFT CABLEWAY BENCH, NORTH PLATTE RIVER, WYOMING

# CHAPTER V

## CONCRETE CONSTRUCTION

Concrete is extensively used for constructing the many different types of foundations, retaining walls, dams, culverts, etc. The ingredients of which concrete is made, the proportioning and the methods of mixing these materials, etc., have been discussed. Various details of the use of concrete in the construction of foundations, etc., are discussed during the treatment of the several kinds of work.

**Rubble Concrete.** Rubble concrete includes any class of concrete in which large stones are placed. The chief use of this concrete is in constructing dams, lock walls, breakwaters, retaining walls, and bridge piers.

The cost of rubble concrete in large masses should be less than that of ordinary concrete, as the expense of crushing the stone used as rubble is saved, and each large stone replaces a portion of cement and aggregate; therefore, this portion of cement is saved, as well as the labor of mixing it. The weight of a cubic foot of stone is greater than that of an equal amount of ordinary concrete, because of the pores in the concrete; the rubble concrete is therefore heavier, which increases its value for certain classes of work. In comparing rubble concrete with rubble masonry, the former is usually found cheaper because it requires very little skilled labor. For walls 3 or 3½ feet thick, the rubble masonry will usually be cheaper, owing to the saving in forms.

*Proportion and Size of Stone.* Usually the proportion of rubble stone is expressed in percentage of the finished work. This percentage varies from 20 to 65 per cent. The percentage depends largely on the size of the stone used, as there must be nearly as much space left between small stones as between large ones. The percentage therefore increases with the size of the stones. When "one-man" or "two-man" rubble stone is used, about 20 per cent to 25 per cent of the finished work is composed of these stones. When the stones are large enough to be handled with a derrick, the pro-

portion is increased to about 33 per cent; and to 50 per cent, when the rubble stones average from 1 to $2\frac{1}{4}$ cubic yards each.

The distance between the stones may vary from 3 inches to 15 or 18 inches. With a wet mixture of concrete, which is generally used, the stones can be placed much closer than if a dry mixture is used. With the latter mixture, the space must be sufficient to allow the concrete to be thoroughly rammed into all of the crevices. Specifications often state that no rubble stone shall be placed nearer the surface of the concrete than 6 to 12 inches.

*Rubble Masonry Faces.* The faces of dams are very often built of rubble, ashlar, or cut stone, and the filling between the faces made of rubble concrete. For this style of construction, no forms are required. For rubble concrete, when the faces are not constructed of stone, wooden forms are constructed as for ordinary concrete.

*Comparison of Quantities of Materials.* The mixture of concrete used for this class of work is often 1 part Portland cement, 2 parts sand, and 4 parts stone. The quantities of materials required for one yard of concrete, according to Table VII, are 1.48 barrels of cement, 0.45 cubic yard of sand, and 0.90 cubic yard of stone. If rubble concrete is used, and if the rubble stone laid averages 0.40 cubic yard for each yard of concrete, then 40 per cent of the cubic contents is rubble, and each of the other materials may be reduced 40 per cent. Reducing these quantities gives $1.48\times0.60=0.89$ barrel of cement; $0.45\times0.60=0.27$ cubic yard of sand; and $0.90\times0.60=0.54$ cubic yard of stone, per cubic yard of rubble concrete.

The construction of a dam on the Quinebaug River is a good example of rubble concrete. The height of the dam varies from 30 to 45 feet above bed rock. The materials composing the concrete consist of bank sand and gravel excavated from the bars in the bed of the river. The rock and boulders were taken from the site of the dam, and were of varying sizes. Stones containing 2 to $2\frac{1}{2}$ cubic yards were used in the bottom of the dam, but in the upper part of the dam smaller stones were used. The total amount of concrete used in the dam was about 12,000 cubic yards. There was $1\frac{1}{2}$ cubic yards of concrete for each barrel of cement used. The concrete was mixed wet, and the large stones were so placed that no voids or hollows would exist in the finished work.

**Depositing Concrete Under Water.** In depositing concrete under

water, some means must be taken to prevent the separation of the materials while passing through the water. The three principal methods are by means of closed buckets; by means of cloth or paper bags; and by means of tubes.

*Buckets.* For depositing concrete by the first method, special buckets are made with a closed top and hinged bottom. Concrete deposited under water must be disturbed as little as possible and, in tipping a bucket, the material is apt to be disturbed. Several different types of buckets with hinged bottoms have been devised to open automatically when the place for depositing the concrete has been reached. In one type, the latches which fasten the trap-doors are released by the slackening of the rope when the bucket reaches the bottom, and the doors are open as soon as the bucket begins to ascend. In another type, in which the handle extends down the sides of the bucket to the bottom, the doors are opened by the handles sliding down when the bucket reaches the bottom. The doors are hinged to the sides of the bucket and, when opened, permit the concrete to be deposited in one mass. In depositing concrete by this means, it is found rather difficult to place the layers uniformly and to prevent the formation of mounds.

*Bags.* This method of depositing concrete under water is by means of open-woven bags or paper bags, two-thirds to three-quarters filled. The bags are sunk in the water and placed in courses—if possible, header and stretcher system—arranging each course as laid. The bagging is close enough to keep the cement from washing out and, at the same time, open enough to allow the whole to unite into a compact mass. The fact that the bags are crushed into irregular shapes which fit into each other tends to lock them together in a way that makes even an imperfect joint very effective. When the concrete is deposited in paper bags, the water quickly soaks the paper; but the paper retains its strength long enough for the concrete to be deposited properly.

*Tubes.* The third method of depositing concrete under water is by means of long tubes, 4 to 14 inches in diameter. The tubes extend from the surface of the water to the place where the concrete is to be deposited. If the tube is small, 4 to 6 inches in diameter, a cap is placed over the bottom, the tube filled with concrete and lowered to the bottom. The cap is then withdrawn, and as fast as

the concrete drops out of the bottom, more is put in at the top of the tube, and there is thus a continuous stream of concrete deposited.

When a large tube is used to deposit concrete in this manner, it will be too heavy to handle conveniently if filled before being lowered. The foot of the tube is lowered to the bottom, and the water rises into the chute to the same level as that outside; and into this water the concrete must be dumped until the water is wholly replaced or absorbed by the concrete. This has a tendency to separate the cement from the sand and gravel, and will take a yard or more of concrete to displace the water in the chute. There is a danger that this amount of badly washed concrete will be deposited whenever it is necessary to charge the chute. This danger occurs not only when the charge is accidentally lost, but whenever the work is begun in the morning, or at any other time. Whenever the work is stopped, the charge must be allowed to run out, or it would set in the tube. The tubes are usually charged by means of wheelbarrows, and a continuous flow of concrete must be maintained. When the chute has been filled, it is raised slowly from the bottom, allowing a part of the concrete to run out in a conical heap at the foot.

This method has also been used for grouting stone. In this case, a 2-inch pipe, perforated at the bottom, is used. The grout, on account of its great specific gravity, is sufficient to replace the water in the interstices between the stones, and firmly cement them into a mass of concrete. A mixture of one part cement and one part sand is the leanest mixture that can be used for this purpose, as there is a great tendency for the cement and sand to separate.

## FOUNDATIONS

**Importance of Foundations.** It would be impossible to overemphasize the importance of foundations, because the very fact that the foundations are underground and out of sight detracts from the consideration that many will give to the subject. It is probably true that a yielding of the subsoil is responsible for a very large proportion of the structural failures which have occurred. It is also true that many failures of masonry, especially those of arches, are considered as failures of the superstructure, because of breaks occurring in the masonry of the superstructure, which have really been due, however, to a settlement of the foundations, resulting in

unexpected stresses in the superstructure. It is likewise true that the design of foundations is one which calls for the exercise of experience and broad judgment, to be able to interpret correctly such indications as are obtainable as to the real character of the subsoil and its probable resistance to concentrated pressure.

**Character of Soil.** The character of soil on which it may be desired to place a structure varies all the way from the most solid rock to that of semi-fluid soils whose density is but little greater than that of water. The gradation between these extremes is so uniform that it is practically impossible to draw a definite line between any two grades. It is convenient, however, to group subsoils into three classes, the classification being based on the method used in making the foundation. These three classes of subsoils are: firm; compressible; and semi-fluid.

*Firm Subsoils.* These comprise all soils which are so firm, at least at some reasonably convenient depth, that no treatment of the subsoil, or any other special method, needs to be adopted to obtain a sufficiently firm foundation. This, of course, practically means that the soil is so firm that it can safely withstand the desired unit pressure. It also means that a soil which might be classed as firm soil for a light building should be classed as compressible soil for a much heavier building. It frequently happens that the top layers must be removed because the surface rock has disintegrated. Also, the surface must be leveled. Nothing further needs to be done to a grade of this kind.

*Compressible Subsoils.* These include soils which might be considered as firm soils for light buildings, such as dwelling houses, but which could not withstand the concentrated pressure that would be produced, for example, by the piers or abutments of a bridge. Such soils may be made sufficiently firm by methods described later.

*Semi-Fluid Subsoils.* These are soils such as are frequently found on the banks or in the beds of rivers. They are so soft that they cannot sustain, without settlement, even the load of a house, to say nothing of a heavier structure; nor can they be materially improved by any reasonable method of compression. The only possible method of placing a heavy structure in such a locality consists in sinking some sort of a foundation through such soft soil until it reaches and is supported by a firm soil or by rock, which may

be 50 or even 100 feet below the surface. The general methods of accomplishing these results will be detailed in the following pages.

**Examination of Soil with Auger.** The first step is to excavate the surface soil to the depth at which it would be convenient to place the foundation and at which the soil appears, from mere inspection, to be sufficiently firm for the purpose. An examination of the trenches or foundation pits with a post auger or steel bar will generally be sufficient to determine the nature of the soil for any ordinary building. The depth to which such an examination can be made with a post auger or steel bar will depend on the nature of the soil. In ordinary soils there will not be much difficulty in extending such an examination 3 to 6 feet below the bottom of the foundation pits. In common soils or clay, boring 40 feet deep, or even deeper, can readily be made with a common wood auger, turned by men. From the samples brought up by the auger, the nature of the soil can be determined; and some knowledge regarding the compactness of the soil can be determined in this manner.

**Testing Compressive Value of Soil.** In order to test a soil to find its compressive value, the bottom of the pit should be leveled for a considerable area, and stakes should be driven at short intervals in each direction. The elevations of the tops of all the stakes should be very accurately taken with a spirit level. For convenience, all stakes should be driven to the same level. A mast whose base has an area one foot square can support a platform which may be loaded with several tons of building material, such as stone, brick, steel, etc. This load can be balanced with sufficient closeness so that some very light guys will maintain the unstable equilibrium of the platform. As the load on the platform is greatly increased, at some stage it will be noted that the mast and platform have begun to sink slightly, and also that the soil in a circle around the base of the mast has begun to rise. This is indicated by the rising of the tops of the stakes. Even a very ordinary soil may require a load of five or six tons on a square foot before any yielding will be observable. One advantage of this method lies in the fact that the larger the area of the foundation, the greater will be the load per square foot which may be safely carried, and that the uncertainty of the result is on the safe side. A soil which might yield under a load concentrated on a mast one foot square would probably be safe under that same unit

load on a continuous footing which was perhaps three feet wide; and if, in addition, a factor of safety of three or four was used, there would probably be no question as to the safety. Such a test need be applied only to an earthy soil. It would be practically impossible to produce a yielding by such a method on any kind of rock or even on a compacted gravel.

**Bearing Power of Ordinary Soils.** A distinction must be maintained between the crushing strength of a cube of rock or soil, and the bearing power of that soil when it lies as a mass of indefinite extent under some structure. A soil can fail only by being actually displaced by the load above it, or because it has been undermined, perhaps by a stream of water. A sample of rock which might crush with comparative ease when tested as a six-inch cube in a testing machine, will, as part of a mass of rock, probably withstand a great concentrated load, due to the lateral support it receives from the surrounding stone. Even gravel, which would have absolutely no strength if an attempt were made to place a cube of it in a testing machine, will support several tons when lying in a pit where it is confined laterally in all directions.

*Rock.* The ultimate crushing strength of stone varies greatly. The crushing strength is usually determined by making tests on small cubes. Tests made on prisms of a less height than width show a much greater strength than tests made on cubes of the same material, which shows that the bearing strength of rock on which foundations are built is much greater than the cubes of this stone.

*Sand and Gravel.* Sand and gravel are often found together. When compact and of sufficient thickness, they make a firm foundation. Dry sand or set sand, when prevented from spreading laterally, forms one of the best beds for foundations; but it must be well protected from running water, as it is easily moved by scouring. Clean, dry sand will safely support a load of 3,000 to 8,000 pounds per square foot; and when compact and well cemented, from 8,000 to 10,000 pounds per square foot. Ordinary gravel, well bedded, will safely bear a load of 5,000 to 8,000 pounds per square foot; and when well cemented, it will bear from 8,000 to 10,000 pounds per square foot.

*Clay.* There is great variation in clay soils, ranging from a very soft mass which will squeeze out in all directions when a very

small pressure is applied, to shale or slate which will support a very heavy load. As the bearing capacity of ordinary clay is largely dependent upon its dryness, it is very important that a clay soil should be well drained, and that a foundation laid on such a soil should be at sufficient depth to be unaffected by the weather. If the clay cannot be easily drained, means should be taken to prevent the penetration of water. When the strata are not horizontal, great care must be taken to prevent the flow of the soil under pressure. When gravel or coarse sand is mixed with the clay, the bearing capacity of the soil is greatly increased.

The bearing capacity of a soft clay is from 1,000 or less to 3,000 pounds per square foot; of a thick bed of medium dry clay, 4,000 to 6,000 pounds per square foot, and for a thick bed of dry clay, 7,000 to 9,000 pounds per square foot.

*Soft or Semiliquid Soils.* The soils of this class include mud, silt, quicksand, etc. Such soils should be removed so that a more compact material can be had. Sometimes it is practical to make a widespread reinforced concrete footing to support a light building, but generally it is necessary to use piling to overcome the difficulty. Soils that will apparently carry a safe load of 1,000 pounds per square foot will often settle under a constant load. Sometimes a soil can be drained with deep sewer lines and the soil will harden when the water is removed. Usually it will be more satisfactory to support the structure on piles than to attempt to drain the subsoil or to improve it in other ways.

**Preparing the Bed on Rock.** The preparation of a rock bed on which a foundation is to be placed is a simple matter compared with that required for some soils on which foundations are placed. The bed rock is prepared by cutting away the loose and decayed portions of the rock and making the plane on which the foundation is placed perpendicular to the direction of the pressure. If the rock bed is an inclined plane, a series of steps can be made for the support of the foundation. Any fissures in the rock should be filled with concrete.

**Preparing the Bed on Firm Earth.** Under this heading is included hard clay, gravel, and clean, dry sand. The bed is prepared by digging a trench deep enough so that the bottom of the foundation is below the frost line, which is usually 3 to 6 feet below the surface.

Some provision, similar to that shown in Fig. 31, should be made for drainage.

Care should be taken to proportion the load per unit of area so that the settlement of the foundation will be uniform.

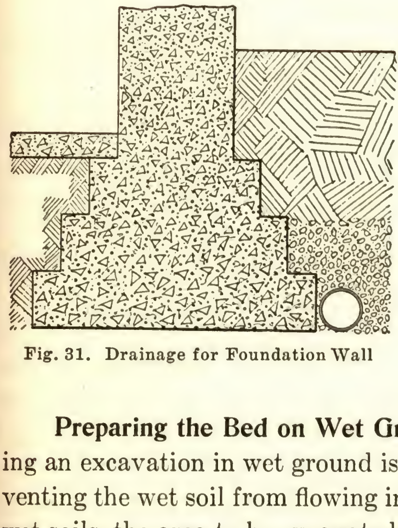


Fig. 31. Drainage for Foundation Wall

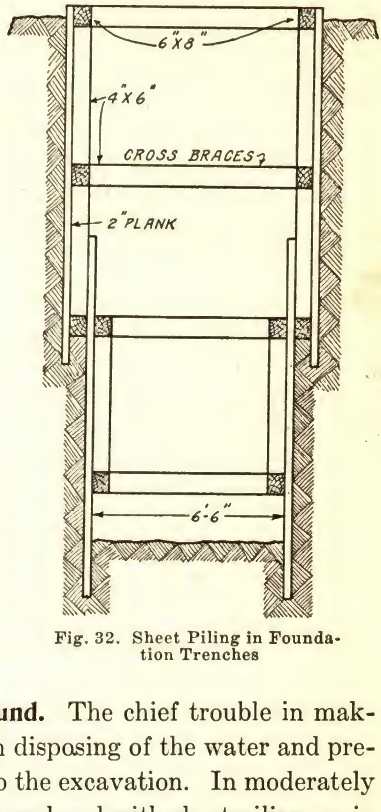



Fig. 32. Sheet Piling in Foundation Trenches

**Preparing the Bed on Wet Ground.** The chief trouble in making an excavation in wet ground is in disposing of the water and preventing the wet soil from flowing into the excavation. In moderately wet soils, the area to be excavated is enclosed with sheet piling, as in Fig. 32. This piling usually consists of ordinary plank, 2 inches thick and 6 to 10 inches wide, and is often driven by hand; or it may be driven by methods that will be described later. The piling is driven in close contact, and in very wet soil it is necessary to drive a double row of the sheeting. To prevent the sheeting from being forced inwards, cross braces are used between the longitudinal timbers. When one length of sheeting is not long enough, an additional length can be placed inside. A more extended discussion of pile driving will be given in the treatment of the subject "Piles."

The water can sometimes be bailed out, but it is generally necessary to use a hand or steam pump to free the excavation of water. Quicksand and very soft mud are often pumped out along with the water by a centrifugal or mud pump.

Sometimes, areas are excavated by draining the water into a hole the bottom of which is always kept lower than the general level of the bottom of the excavation. A pump may be used to dispose of the water drained into the hole or holes.

When a very soft soil extends to a depth of several feet, piles are usually driven at uniform distances over the area and a grillage is constructed on top of the piles. This method of constructing a foundation is discussed in the section on "Piles."

## FOOTINGS

**Requirements of Footing Course.** The three requirements of a footing course are:

(1) That the area shall be such that the total load divided by the area shall not be greater than the allowable unit pressure on the subsoil.

(2) That the line of pressure of the wall, or pier, shall be directly over the center of gravity—and hence the center of upward pressure—of the base of the footings.

(3) That the footing shall have sufficient structural strength so that it can distribute the load uniformly over the subsoil.

When it has been determined with sufficient accuracy how much pressure per square foot may be allowed on the subsoil and when the total load of the structure has been computed, it is a very simple matter to compute the width of continuous footings or the area of column footings.

The second requirement is easily fulfilled when it is possible to spread the footings in all directions as desired. A common exception occurs when putting up a building which entirely covers the width of the lot. The walls are on the building line; the footings can expand inward only. The lines of pressure do not coincide, as shown in Fig. 33. A construction such as shown in Fig. 33 will generally result in cracks in the building, unless some special device is adopted to prevent them. One general method is to introduce a tie of sufficient strength from *a* to *b*, Fig. 33. The other general method is to introduce cantilever beams under the basement, which either extend clear across the building or else carry the load of

interior columns so that the center of gravity of the combined loads will coincide with the central pressure line of the upward pressure of the footings.

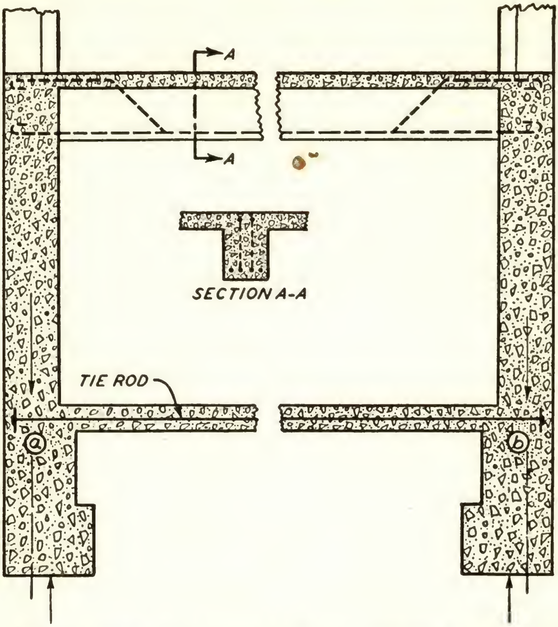



**Fig. 33. Construction where Lines of Downward and Upward Pressure Do Not Coincide**

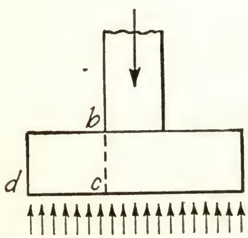



**Fig. 34. Transverse Stresses in Footing Determining Its Thickness**

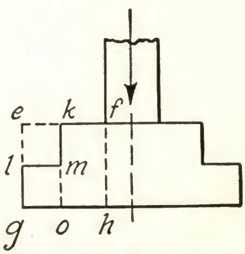



**Fig. 35. Saving of Material in Very Thick Footing**

The third requirement practically means that the thickness of the footing (*bc*, Fig. 34) shall be great enough so that the footing can resist the transverse stresses caused by the pressure of the subsoil on the area between *c* and *d*. When the thickness must be made very great, such as *fh*, Fig. 35, on account of the wide offset, *gh*, material

TABLE XIV

**Ratio of Offset to Thickness for Footings of Various Kinds of Masonry**

| Kind of Masonry | Modulus of Rupture (Minimum and Maximum Values) | Assumed Safe Value ($R$) | Average | Pressure on Bottom of Footing (Tons per Square Foot) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
| 1:1½:3 | 500—580 | 540 | 90 | 2.08 | 1.47 | 1.20 | 1.04 | .93 | .85 | .79 |
| 1:2½:3½ | 415—485 | 450 | 75 | 1.90 | 1.34 | 1.10 | 0.95 | .85 | .78 | .72 |
| 1:2:4 | 345—400 | 375 | 65 | 1.77 | 1.25 | 1.02 | 0.88 | .79 | .72 | .67 |
| 1:3:5 | 275—325 | 300 | 50 | 1.55 | 1.10 | 0.90 | 0.78 | .69 | .63 | .59 |
| 1:3:6 | 215—245 | 230 | 40 | 1.39 | 0.98 | 0.80 | 0.69 | .62 | .57 | .53 |

may be saved by cutting out the rectangle *ekml*. The thickness *mo* is computed for the offset *go*, just as in the first case; while the thickness *km* of the second layer may be computed from the offset *kf*. Where the footings are made of stone or of plain concrete, whose transverse strength is always low, the offsets are necessarily small; but when using timber, reinforced concrete, or steel I-beams, the offsets may be very wide in comparison with the depth of the footing.

**Calculation of Footings.** The method of calculation is to consider the offset of the footing as an inverted cantilever which is loaded with the calculated upward pressure of the subsoil against the footing. If Fig. 34 is turned upside down, the resemblance to the ordinary loaded cantilever will be more readily apparent. Considering a unit length $l$ of the wall and the amount of the offset $o$ equal to $dc$ in Fig. 34, and calling $P$ the unit pressure from the subsoil, we have $Pol$ as the pressure on that area, and its lever arm about the point $c$ is $\frac{1}{2}o$. Therefore, its moment equals $\frac{1}{2}Po^2l$. If $t$ represents the thickness $bc$ of the footing, the moment of resistance of that section equals $\frac{1}{6}Rlt^3$, in which $R$ equals the unit compression (or unit tension) in the section. We therefore have the equation

$$\frac{1}{2}Po^2l=\frac{1}{6}Rlt^2 \tag{1}$$

By transposition

$$\frac{o^2}{t^2}=\frac{R}{3P}; \text{ or } \frac{o}{t}=\sqrt{\frac{R}{3P}} \tag{2}$$

The fraction $\frac{o}{t}$ is the ratio of the offset to its thickness. The solution of the above equation, using what are considered to be conservatively

safe values for $R$ for various grades of plain concrete, is given in Table XIV.

**Example.** The load on a wall has been computed as 19,000 pounds per running foot of the wall, which has a thickness of 18 inches just above the footing. What must be the breadth and thickness of 1:2:4 concrete slabs which may be used as a footing on soil which is estimated to bear safely a load of 2.0 tons per square foot?

*Solution.* Dividing the computed load (19,000) by the allowable unit pressure (2.0 tons equals 4,000 pounds), we have 4.75 feet as the required width of the footing.

$\frac{1}{2}$ (4.75—1.5)=1.625 feet, the breadth of the offset $o$

From Table XIV we find that for a subsoil loading of 2.0 tons per square foot the offset for 1:2:4 concrete may be 0.88 times its thickness. Therefore, $\frac{1.625}{0.88}$=1.84 feet=22.0 inches, is the required thickness.

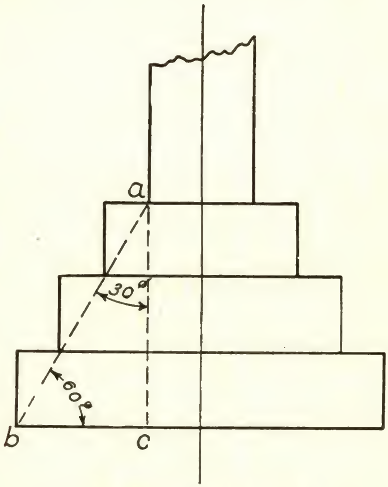



Fig. 36. Type of Footing

Instead of figuring the dimensions of a footing as just described, they may be designed by carefully drawing a few lines. After finding the size of a footing under a pier or a column, or the width of a footing under a wall, the projections and depths of the masonry for the footing is found as shown in Fig. 36. A line is drawn from the point $a$, at an angle of 30 degrees from the vertical, and extending to the outer edge of the footing. The point $a$ may be the edge of a wall, column, or column base.

The height $ac$ is then divided into any number of steps desired

and the lines of the footing drawn. The dimensions can be either figured or scaled if the drawing has been carefully made.

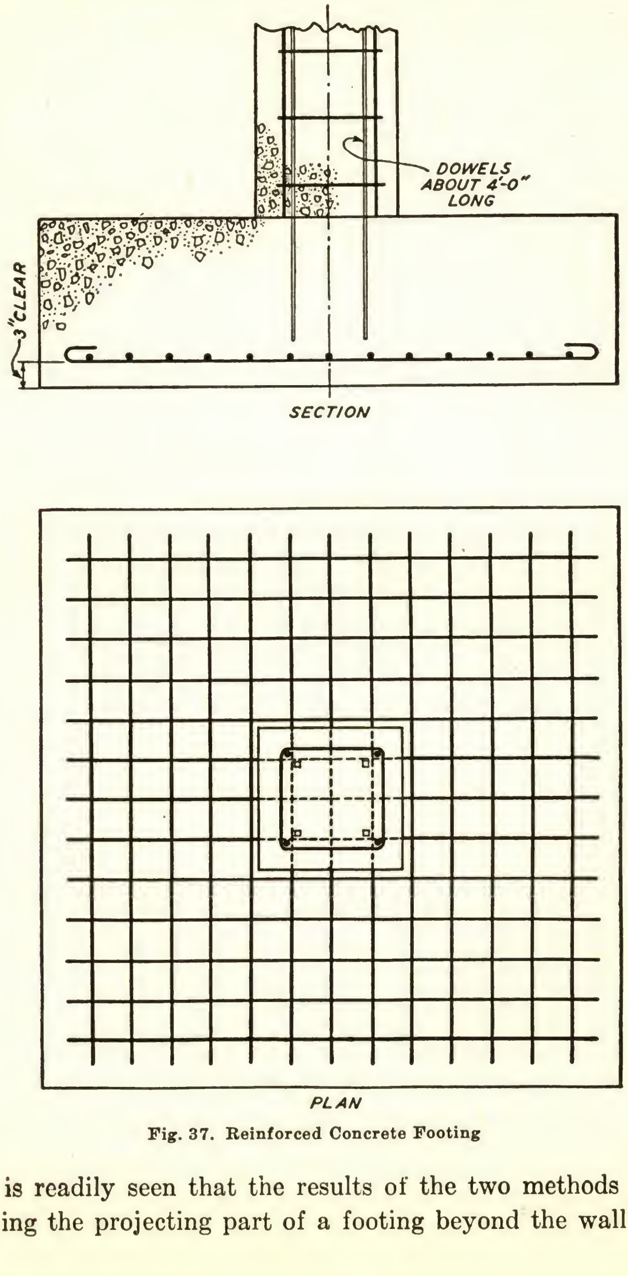



Fig. 37. Reinforced Concrete Footing

It is readily seen that the results of the two methods of proportioning the projecting part of a footing beyond the wall do not

agree. The first method permits tension in the bottom of the concrete and the graphical method keeps it in compression. The latter

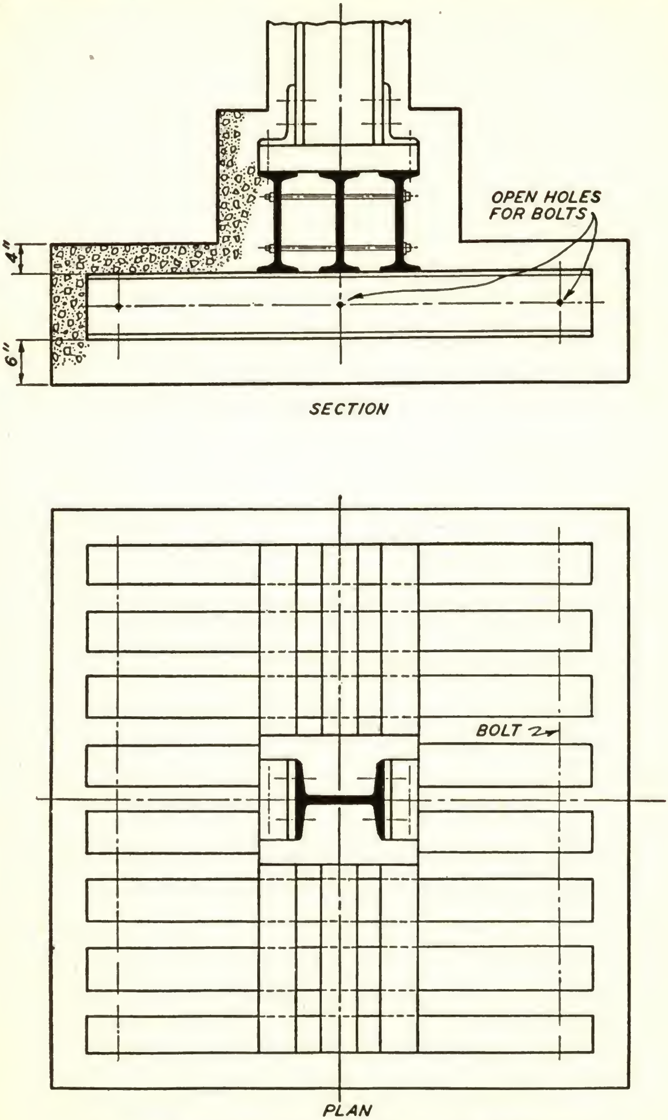



Fig. 38. Structural Steel Footing Protected with Concrete

method is the one to be used for general work. The offset can be quickly calculated after selecting a vertical height and multiplying

it by the tangent of 30°. That is, if a vertical height of 12 inches is selected, then 12 times tangent 30° (0.57735) equals 6.93 inches, which is the offset.

This type of footing is used only for plain concrete, stone or brick. Reinforced concrete and structural steel are used when wide projections are necessary.

**Wall Footings.** Where wide projections are required on each side of the wall, reinforced concrete slabs or beams or structural steel beams are used. This is possible on account of their great transverse strength. When structural steel beams are used, they must be encased in concrete.

**Column Footings.** In Fig. 37 is shown a reinforced concrete footing for a column. In Fig. 38 a similar footing is shown in which steel beams are used and protected with concrete. For both wall footings and column footings, reinforced concrete is usually more economical than the use of structural steel, and so this type of footing is generally used. The reinforcing steel can be secured from stock and construction work can be started more promptly than if structural steel is required. It also requires nearly as much concrete and form work for the steel footings as for the reinforced concrete footings. The bars weigh much less than the structural steel, but the price per pound is usually greater.

## PILE FOUNDATIONS

**Piles.** When a building is constructed on a soft or unreliable ground, and spread foundations would not be sufficient to support the load, or if a very large load must be supported on a small area, then some form or pile foundation is usually used.

Formerly the term pile was understood to be a stick of timber driven in the ground to support a structure, but about 1900, concrete piling came into use and since that time their use has gradually increased and they are now extensively employed.

Concrete piles have several advantages over wood piles in many jobs, although there are some jobs where long piles are required and for such use the wood pile has the advantage in being lighter to handle. Steel piling is now being used and will encroach on the wood pile, rather than the concrete pile, where long lengths are required. For the steel piling, 8″-**H**, 10″-**H**, 12″-**H**, etc., column

sections are used. The **H** columns can be secured in long lengths and they can easily be spliced in a satisfactory manner.

Steel and reinforced concrete sheet piling have been used extensively where wood sheet piling was formerly used.

**Wood Bearing Piles.** A wood pile is cut from the body of a tree. It may vary in length from 16 to 80 feet or more. These sticks of timber must be fairly straight. The diameter of the pile under the

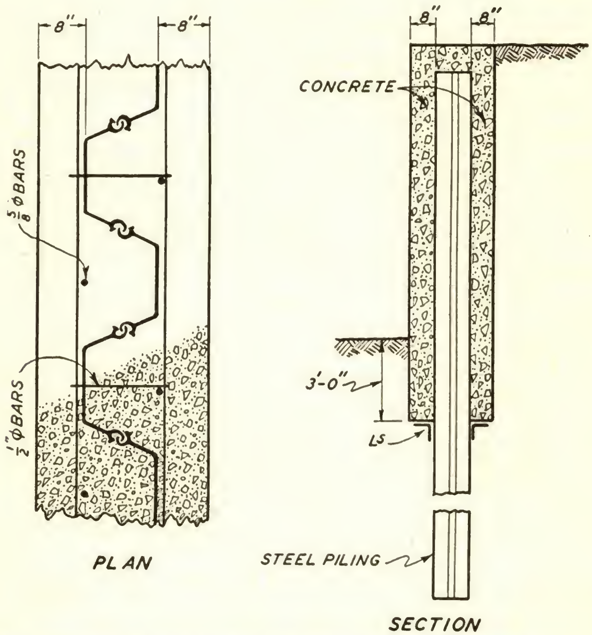



Fig. 39. Retaining Wall of Arch Web Steel Sheet Piling Protected by Concrete

bark at the small end is seldom less than 6 inches. They should be cut from close-grained trees and should be free from defects such as ring shakes, unsound knots, decay, or other defects that will materially reduce their strength. In the firmer soils or where long piles are required, yellow pine, fir and oak are usually used. For short piles and soft soils, white pine, spruce, or hemlock may be used.

Wood piles, when used for foundations in permanent work, must always be under water so that they will not rot. The safe load for fair-sized wood piles is about 15 tons. The load will vary with the conditions, and tests should be made. The usual spacing of piles

is 2′6″ on centers, and this should be a minimum spacing for piles.

**Wood Sheet Piling.** Wood sheet piling is often used in supporting earth around deep excavations. This plank may be of single, double or triple thickness. It is used in cofferdams where water as well as earth must be kept out of the excavation. It is driven in close contact to prevent leakage. Sheet piling is kept in place

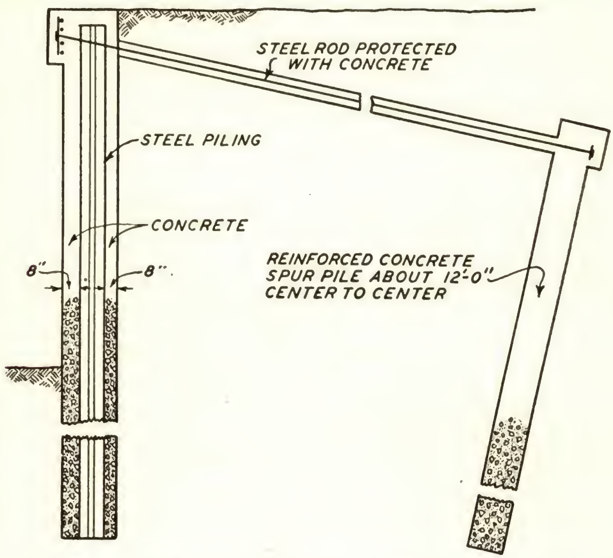



Fig. 40. Retaining Wall of Arch Web Steel Sheet Piling Tied Back at Top

by longitudinal stringers on both sides of the sheet piling, the stringers being supported by vertical timbers or piles.

**Steel Sheet Piling.** Steel sheet piling is often used where formerly wood sheet piling was used. Many types of steel piling have arched webs as shown in Fig. 39. Sheet piling is generally more flexible and easier to make watertight than wood piling. It can be pulled out, after being used, and reused many times.

The arch-web piling covered with concrete is used in constructing retaining walls, specially along a water front. In this case the steel piling is part of the permanent construction. This piling is usually driven, assisted by water jets. The concrete on the top of the wall should have a minimum thickness of at least 8 inches above the top of the piling. Where constructed along a water front, the concrete should extend down to the low-water level or lower. The

concrete should be reinforced to hold it in place and the piling should be punched in the web so that tie bars can pass through the piling and hold the longitudinal and vertical bars in place. Fig. 39 shows a wall constructed as described above.

Fig. 40 shows a similar wall with the top tied back to a pile driven several feet in the rear. This type of wall is more economical than the type shown in Fig. 39. The concrete for such work should be of a dense mixture, the minimum mix being 1 part Portland cement, 2½ parts sand, and 3½ parts stone.

**Reinforced Concrete Sheet Piling.** Precast reinforced concrete sheet piling has been used to some extent over a period of several

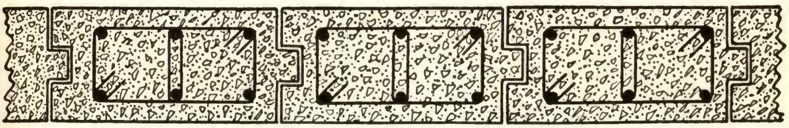


Fig. 41. Reinforced Concrete Sheet Piling

years. It must be reinforced for the stresses to which it will be subjected and also for handling. A rich mixture of concrete must be used. In Fig. 41 is shown, in section, the tongue and groove type. This piling should be driven by a combination of jetting and hammering.

A good wall can be constructed by connecting the top of the piling to a beam and tying the beam back to a spur pile as shown in Fig. 40. It may be used also without spur piles, but stronger and longer piling will be required due to the nature of the stresses.

**Concrete and Reinforced Concrete Piles.** Concrete and reinforced concrete piles have certain advantages over wood piles. They are more permanent, since they are not subject to decay or the attack of wood borers. They are often more economical because of the greater carrying capacity, which means a reduction in the size of the footings due to the reduction in the number of piles. Generally the amount of pumping, sheet piling, and excavation is reduced when reinforced concrete piles are used. The top of the pile may project above the ground and make a column for supporting the superstructure. The load that can be placed on a concrete pile will depend on the type of pile, size, length, the ground in which it is placed,

etc.; but 30 tons is probably an average load for a fair-sized concrete pile.

Concrete and reinforced concrete piles may be classified under

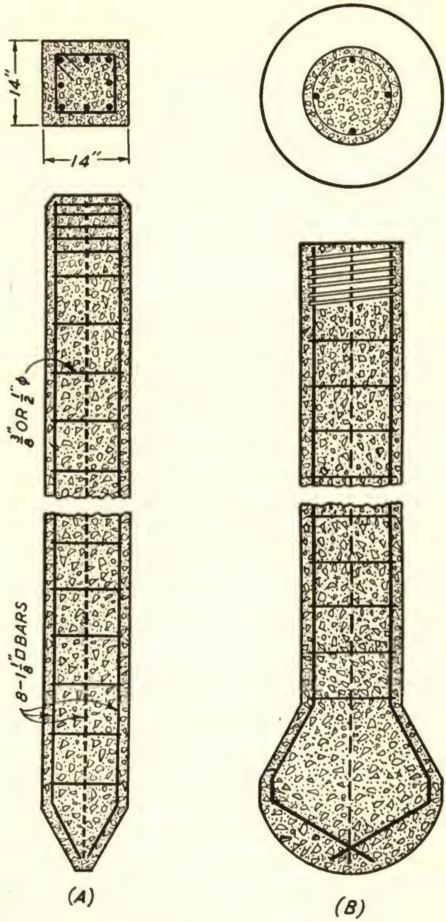



Fig. 42. Reinforced Concrete Piles

two headings: (a) those that are formed, cured, and driven or jetted into place as shown in Fig. 42 at (*A*); (b) those that are formed in place, with or without the use of steel forms, as shown in Fig. 42 at (*B*). The latter are known as cast-in-place piles. They may be of plain or reinforced concrete.

Precast piles must be reinforced in order to be handled. The

amount of steel used in reinforcing these piles depends largely on the manner in which they are to be handled, and on the number and location of points where the cables will be attached to the piles for handling. Where the piles project several feet above the ground level when placed in the structure, they must be designed as a column.

Reinforced concrete columns can be made in any thickness and length if equipment for handling them can be secured. Piles of this type have been cast 24×24 inches and over 100 feet long for use in railroad trestles. A rich, dense concrete must be used, especially where one end projects above the ground line. A mix composed of 1 part cement, 2½ parts sand and 3½ parts stone may be used; or a better mixture consists of 1 part cement, 1¾ parts sand and 3 parts stone.

These pilings may be driven much the same as a wood pile, but the head must be well protected. They should be level on top with the corners chamfered. Where conditions permit the use of a water jet, it will assist the driving materially, save the head of the pile, and reduce the amount of driving usually required. When well designed and driven into hard material, a pile should support a load of 30 tons. Concrete piles can be driven into soils that are too hard for ordinary wood piles.

In Fig. 43 is shown a structure for supporting a highway over swamp land, or a stream, where piling is necessary. The column bents should be spaced 16 to 20 feet apart. Each column should be supported by two piles. In such a structure the load on the piling is not great, but it is necessary to use two piles under each column to balance the load. The piles are capped with reinforced concrete and a tie beam is placed between the footings. This beam will take the thrust caused by the inclined columns and also will assist in stabilizing the small foundations built on top of the piles. The tops of the columns are connected by a girder which supports the floor beams and slabs. If it is necessary to use long columns, 16 feet or over, they should be braced.

**Steel Shell Concrete Piles.** Steel shell concrete piles have been used to advantage in many places. This is especially true in cities where tall buildings are constructed and room for piles of other types is not available. The bottom of these piles must rest on rock or some other very firm material. This type of piling was used for support-

ing a sixteen story building at Fourteenth Street and Fifth Avenue, New York City, where loads on the wall columns were about 500 tons and a small group of the steel piles transferred the load to solid rock about 15 feet below. The piles were 5 in number for each column and were driven close to an existing wall without damaging

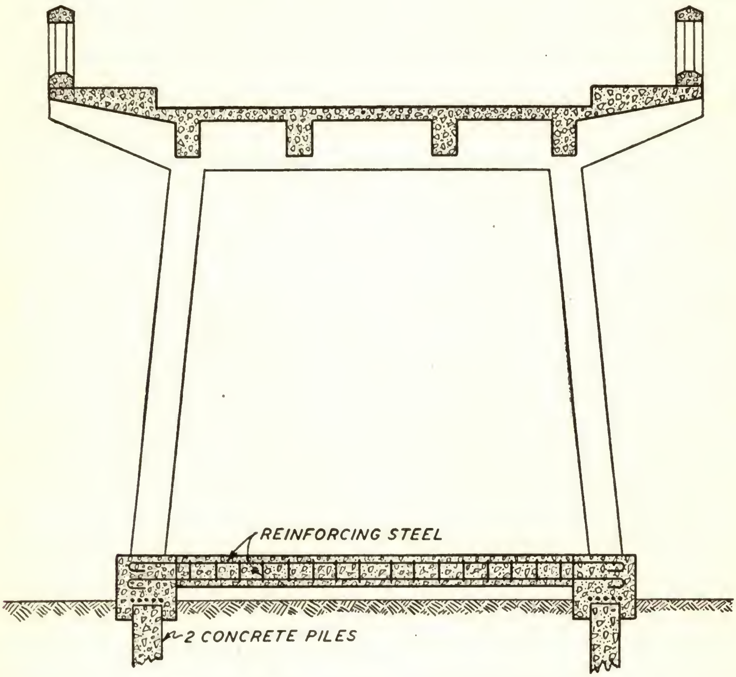



Fig. 43. Highway Bridge Supported by Piles

it. The outer shell of the steel pipe was $\frac{3}{8}$ inch in thickness and 12 inches inside diameter, and was filled with concrete reinforced with 4 vertical bars 2 inches in diameter. This gave a total cross-sectional area of 27.2 square inches of steel with an allowable load of 16,000 pounds per square inch, and 100 square inches of concrete on which a load of 500 pounds per square inch was allowed by the building code of New York. The tubes and bars had an even bearing on the bed rock. The tubes were sunk by the use of a special air hammer and an inside hydraulic jet.

The building laws of Philadelphia, Pa., are more conservative for the loading of this type of piles. They require the steel tube

to be at least 9 inches in diameter with a thickness of at least $\frac{5}{16}$ inch. The allowable load is 1,000 pounds per square inch on the net section of the concrete. No allowance is made for the metal in the pile. Reinforcing steel is not required.

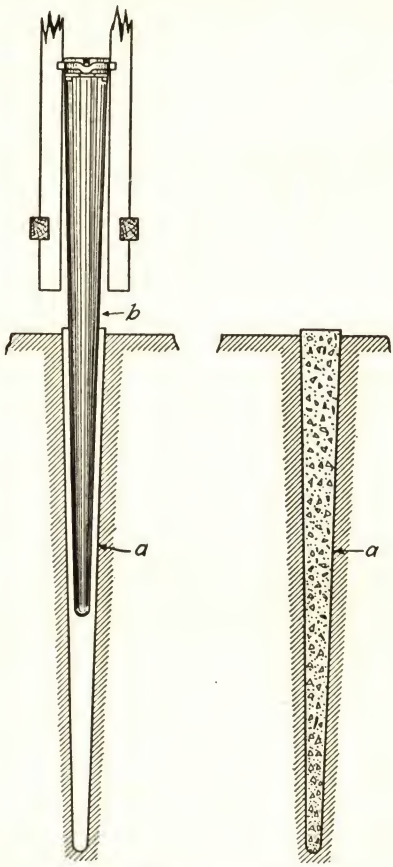



Fig. 44. Raymond Concrete Pile

Often these piles are driven by steam or air hammers. The material inside of the steel pipe may be driven out by compressed air, blowpipes or water jets. Such a pile, when well placed on rock, will usually support 75 to 90 tons per pile. A very rich mixture of concrete is used, that is 1:1:2 or 1:1½:2½. These piles will carry a greater load than any other type described in this text except for very large caisson piles.

*Raymond Concrete Pile.* The Raymond concrete pile, Fig. 44, is constructed in place. A collapsible steel pile core is encased in a thin, closely-fitting, sheet-steel shell. The core and shell are driven to the required depth by means of a pile driver. The core is so constructed that when the driving is finished, it is collapsed and with-

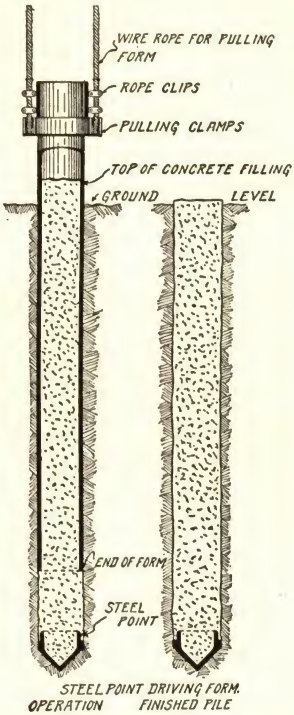



**Fig. 45. Standard Simplex Concrete Piles**

drawn, leaving the shell in the ground, which acts as a mold for the concrete. When the core is withdrawn, the shell is filled with concrete, which is tamped during the filling process. These piles are usually 18 to 20 inches in diameter at the top and 8 inches at the point. When it is desirable, the pile can be made larger at the small end. The sheet steel used for these piles is usually No. 24 gage. When it is desirable to reinforce these piles, the bars are inserted in the shell after the core has been withdrawn and before the concrete is placed.

*Simplex Concrete Pile.* The different methods for producing

the Simplex pile cover the two general classifications of concrete piles —namely, those molded in place, and those molded above ground and driven with a pile driver. Fig. 45 shows the standard methods of producing the Simplex pile; a steel point is shown which has been driven and imbedded in the ground, the concrete deposited, and the form partially and entirely withdrawn.

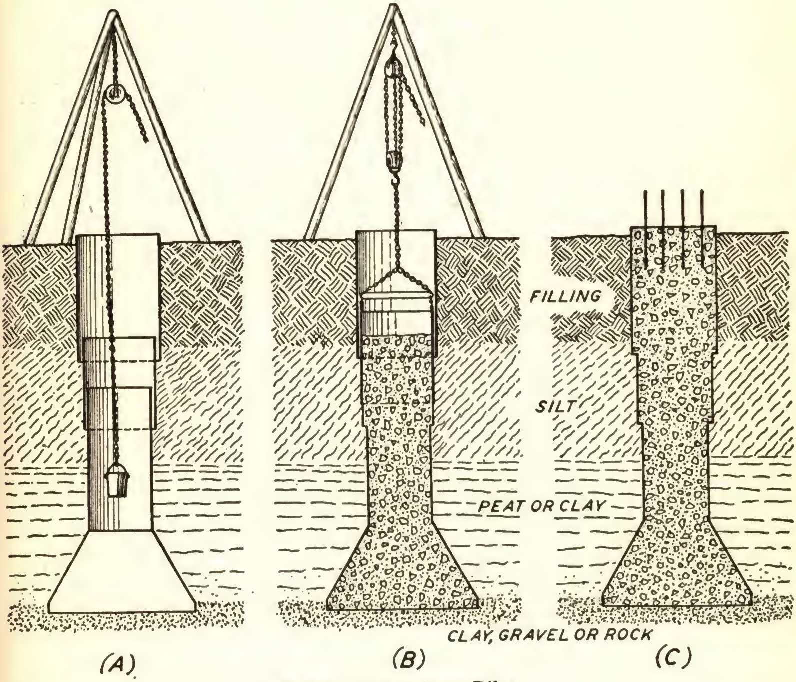



Fig. 46. Gow Caisson Piles

The concrete is deposited in the steel form by means of drop-bottom buckets, holding about 600 pounds for each charge. This method of dropping concrete in mass is used to prevent voids.

*Gow Caisson Pile.* The Gow system of piling consists of constructing a single cylindrical shaft of concrete of sufficient size to carry the entire weight of a building column through an unsatisfactory soil to one that will safely support the load when properly distributed. The supporting material for these piles should be rock, hard shale, cemented gravel, or other material of a very compact nature. In constructing these piles, a cylindrical excavation is made

to a depth of 6 to 8 feet, depending on the nature of the soil penetrated, and then a steel cylinder is placed in the hole. The excavation is continued until the depth of the hole is as great or greater than the length of the steel form. A second form is placed inside of the first form and the excavation continued. This operation is repeated until the satisfactory bearing is found. To secure a greater bearing area, the shaft is "belled out" as shown in Fig. 46 at (*A*). Fig. 46 at (*B*) shows the concrete being placed and the cylinders being removed. The cylinder is not withdrawn before the concrete is placed. In Fig. 46 at (*C*), the finished pile is shown with steel dowels in place for anchoring the column above. These piles usually are not made less than 3 feet in diameter at the lower end, as men cannot conveniently work in a smaller space.

For example, suppose a load of 750,000 pounds is to be supported by one of these piles. If a concrete with a working stress of 500 pounds were used, the diameter of the smallest cylinder would be computed from

$$\pi r^2 = \frac{750{,}000}{500}$$
$$r^2 = 477.4$$
$$r = 21.87$$

Therefore, the diameter would be $21.9 \times 2 = 43.8$ inches. The diameter of the cylinder in this case would be 44 inches.

If the bearing material were a soft rock that would support a load of 15 tons per square foot, then the diameter of the bell would be

$$\pi r^2 = \frac{750{,}000}{30{,}000}$$
$$r^2 = 7.95$$
$$r = 2.82$$

Therefore, the diameter of the bell at the bottom would be $2.82 \times 2 = 5.64$ feet, or, 5 feet 8 inches.

Fig. 47 shows a pedestal pile used by the MacArthur Concrete Pile and Foundation Company. A core and cylindrical casing are first driven to the required depth as shown in Fig. 47 at (*A*). The core is then removed and a charge of concrete dumped in the casing, Fig. 47 at (*B*). The core is replaced and used as a rammer to compress the concrete into the soil, Fig. 47 at (*C*). The process is repeated

until a base about 3 feet in diameter is formed as indicated in Fig. 47 at (*D*). The cylindrical casing is then filled and the casing is withdrawn. The concrete column is 17 inches in diameter. This pile has the advantage of having a direct bearing of a few square feet at the bottom as well as the frictional resistance of its surface.

**Carrying Capacity of Piles.** The carrying capacity of a pile depends largely on the soil it is driven into and on the length and

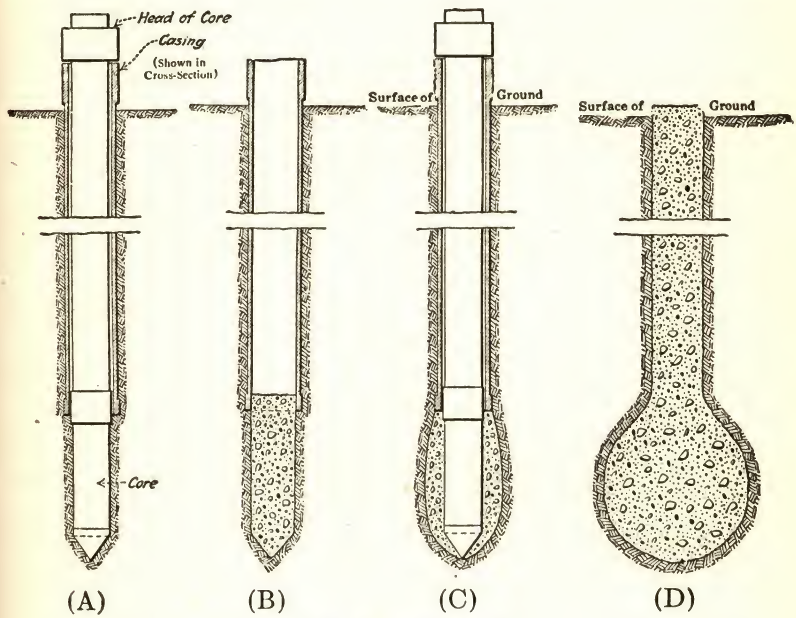



Fig. 47. Pedestal Pile

size of the pile. A pile driven into and supported only by soft material supports the load by the friction between the pile and the earth around it and by the compressed material below the point of the pile. The pile which is driven into a hard material or that is supported by rock is a great deal more reliable than one driven into mud.

There have been several formulas advanced for the driving of piles where the penetration during the last blows is measured and used to determine the value of the carrying capacity of the pile. Probably the best-known formula for this work is the "Engineer-

ing News Formula." The formula for the drop-hammer method of driving is:

$$R = \frac{2Wh}{S+1.0} \tag{3}$$

For the single-acting steam hammer

$$R = \frac{2Wh}{S+0.1} \tag{4}$$

In the above formulas, $R$ equals the resistance of the pile, $S$ is the penetration of the pile during the last blows, in inches; $W$ is the weight of the hammer in pounds; and $h$ is the fall of the hammer in feet.

The steam-hammer driving pile is preferable to the drop hammer. In using the steam hammer, the blows are delivered rapidly and the soil does not have a chance to settle between the successive blows, while with the drop hammer the blows are much less rapid, which gives the soil a chance to settle around the pile. This is taken into account in the 2 formulas just given. The formulas were originally written for wood piles, but for several years have been used for concrete and reinforced piles.

The results of piles driven by these formulas have not always been successful, as in many cases excessive settlement has occurred. Probably this was in part due to the lack of knowledge concerning the soil below the bottom of the pile. Also, when the piles are being driven, the moisture in the earth is driven away from the piles, and, when it returns, the friction between the piles and soil is reduced, due to additional moisture.

A thorough examination should be made of the soil where a building is to be constructed. This examination will consist of deep borings, test pits, driving sample piling (if a piling foundation is being considered) loading the piles, and redriving after the load is removed. The test load should remain on the piling for at least 3 weeks and it must be taken into consideration that a single pile will carry a greater load than an individual pile in a group, due to the soil being disturbed by driving piles close together. With the information obtained from driving the test piles, an experienced engineer can determine if additional driving, compared with the "Engineering News Formula" is required. Where firm soil will be

encountered, specifications often state that the piles shall be driven to a penetration of ¼ inch per blow or to refusal.

**Example 1.** The mean penetration of a pile during the last five blows was ½ inch, driven by a 2,000-pound steam hammer dropping 3 feet. What is the safe load?

*Solution.* $$R=\frac{2Wh}{S+0.1}=\frac{2\times2000\times3}{\frac{1}{2}+0.1}=20{,}000 \text{ pounds}$$

**Example 2.** Piles are required with a resistance of 30,000 pounds each. The equipment consists of a drop hammer weighing 3,000 pounds, with a fall of 20 feet. What should be the average of the last five blows?

*Solution.* $$30{,}000=\frac{2Wh}{S+1}=\frac{2\times3000\times20}{S+1}=\frac{120000}{S+1}$$

$$S=\frac{120000}{30000}-1=3 \text{ inches}$$

**Water-Jetted Piles.** It is difficult to drive piles into sand with a drop hammer or a steam-hammer pile driver; therefore, the jetting method is generally used. To jet a pile, a good water pressure is required and it can be secured by the use of pumps. Attached to the pump is a strong hose about 2 inches in diameter with a nozzle attached to one end, the nozzle being about ¾ inch in diameter or less. Where piles 20 feet in length or less are to be jetted, the hose can be fastened to a stick of wood of the required length, the pump started, and a hole bored straight down into the ground to the depth required for the piling. When the correct depth is reached, the hose is withdrawn and the pile is placed in the hole. After jetting, the top of the piles may be 4 to 5 feet or more below the surface of the ground, depending on the required depth of the footing. When the jetting of the piles for a footing has been completed, the excavation is made down to a point about 8 inches below the top of the piles. Piles are then tightened up, preferably with a steam hammer. After this operation has been completed, the footing is concreted.

Often jetting is used in combination with driving, especially where large reinforced concrete piles are required. In such cases, two jets are usually required to keep the piles vertical. The jets are on opposite sides of the pile and are secured to it. The use of jets will decrease the amount of hammering required, which will generally save the head of the pile from being badly battered as

might be the case without the use of the water. Sometimes precast reinforced concrete piling is made with a hole through the entire length of the pile for jetting purposes.

**Advantages of Concrete and Reinforced Concrete Piles.** A group of concrete or reinforced concrete piles must be capped the same as a group of wood piles, but as concrete piles generally have the

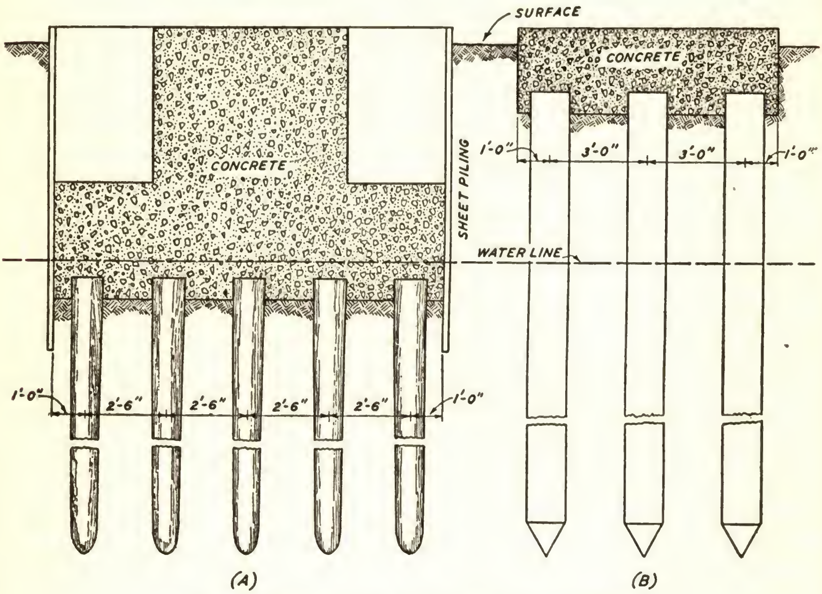



Fig. 48. Comparison of Wood and Concrete Piles

advantage of being larger, they can be driven into firmer soil, and so will carry much greater loads. They also have the advantage, compared with wood piles, of being equally durable in wet or dry soil. Wood piles, to be permanent, must be cut off below the water line so that they will always be protected from the air.

In Fig. 48 at (*A*) is shown the cross section of a foundation supported by wood piles, and in Fig. 48 at (*B*) is shown the cross section of a foundation supported by concrete piles. Both of these foundations are capped with reinforced concrete. In one case, 5 wood piles are used; in the other case, 3 concrete piles. With the same spacing in the longitudinal direction, the two foundations will support equal loads. Besides the saving in the number of piles, a saving is made in the amount of excavation and in the amount of

concrete required for the cap. With the reinforced concrete footing the sheet piling is not required, nor do workmen have to contend with the water. If firm soil is found in driving, the additional length required for the reinforced concrete piles would be equal to the difference of the elevations of the two types of foundations. For example, if the bottom of the cap for the reinforced concrete piles is at an elevation 5 feet higher than the bottom of the cap for the wood piles, then the concrete piles must be 5 feet longer than the wood piles.

**Cost.** In comparing the cost of the several types of piles there are several items to be taken into consideration. Some of these items are: amount of excavation, pumping, foundations, length of piles required, material into which they are to be driven, load to be supported, and the general purpose of the pile. There are so many varying factors to be considered in the cost of pile driving that it is impossible to give actual figures of the cost of pile driving in this text; but the figures below may be of assistance in comparing the different types of piling.

The cost of wood piles varies with their size, length and section of the country where wanted. In lengths of 30 to 50 feet with 12-inch butts and 6-inch small ends they will cost 20 to 30 cents per foot, to which must be added handling, driving, cutting off, etc. The driving cost will vary from 8 cents to 20 or 30 cents per foot, and the driving of many piles has cost more than 50 cents per foot. The actual length of the stick of timber after the cut-off has been made is the actual length of the pile.

The cost of precast piles varies with location, size and length, amount of reinforcing steel required and the amount of handling. Piles 14×14 inches and 30 to 35 feet long often cost about $1.40 to $2.00 per foot.

The cost of cast-in-place piles varies with conditions as named; many jobs have been driven at a cost of $1.50 to $2.00 per foot.

**Pile Caps.** In driving piles, especially with the drop hammer, the top of a wood or concrete pile may be badly battered unless well protected. The battered end of the pile must be cut off. To protect the head of the pile from injury, a special cushion should be used. Such a cushion for driving reinforced concrete pile is shown in Fig. 49A. The top of each pile should be chamfered and made per-

fectly horizontal so that the blows will be distributed over the entire head of the pile.

**Cutting off Heads of Piles.** The heads of all piles for an individual footing should be practically the same height. In driving wood piles an allowance usually is made in the length of the pile for this cut-off. Special saws are made for cutting wood and steel piles at

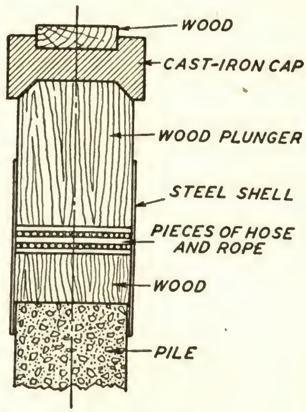



Fig. 49A. Cushion Head for Driving Reinforced Concrete Piles

the desired level. For reinforced concrete piles, pneumatic cutters and drills are best for the concrete and an acetylene torch is best for the steel bars. An advantage of cast-in-place concrete piles is that they can be concreted to the point to suit the depth of the footing.

**Foundations.** Where several piles are driven to support a pier, column, or wall, a foundation must be constructed over the top of the piles. This cap must have sufficient strength to distribute the load uniformly over the piles. Reinforced concrete is usually used for this purpose. In Fig. 48 at (*A*) is shown a concrete footing for five wood piles, and in Fig. 48 at (*B*) is shown a similar footing connecting three concrete or reinforced-concrete piles. The top of the wood piles must be cut off at the same elevation and below the water line. If the tops of the piles are below water level or several feet below the surface, a coffer-dam is built with sheet piling. The tops of the piles may be cut off below the water level with a special saw, but

the water should be pumped out before the concrete is placed. The concrete piles must be longer than the wood piles, if driven to the same depth, and the top of the footing is at the same elevation. The additional cost for extra length of concrete piles is much less than the extra items necessary for the wood piles.

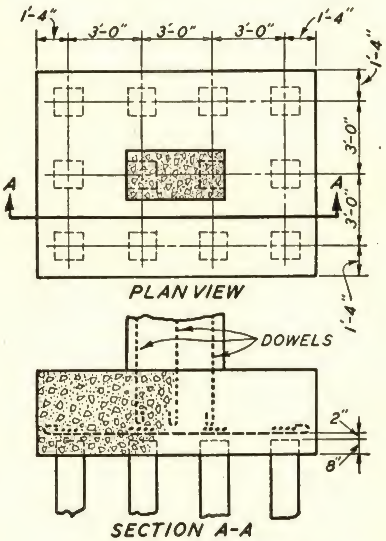



Fig. 49B. Piling Foundation

Fig. 49B shows a foundation composed of 12 precast reinforced concrete piles. These piles are placed 3 feet on centers, which is as close as concrete or long piles should be spaced. Piles are often driven 2 feet 6 inches on centers; this centering should be used only for small or short piles. To distribute the load uniformly over the tops of the piles, a reinforced concrete pad of sufficient strength for the purpose is used. The bars may be placed over the piles in straight lines or some of them may be placed diagonally through the footing from corner to corner. Piles are shown to project 8 inches into the footing and the dowels should project into the column or pier 50 diameters of the bar.

TRACK ELEVATION AT CHICAGO, ILL., FOR THE CHICAGO, ROCK ISLAND & PACIFIC RAILWAY. 1300 RAYMOND CONCRETE PILES WERE USED

*Courtesy of Raymond Concrete Pile Co.*

# CHAPTER VI

## RETAINING WALLS

A retaining wall is a wall built to sustain the lateral pressure of earth. The pressure that will be exerted on the wall will depend on the kind of material to be supported, the manner of placing it, and the amount of moisture that it contains. Earth and most other granular masses possess some frictional stability. Loose soil or a hydraulic pressure will exert a full pressure; but a compacted earth, such as clay, may exert only a small pressure due to the cohesion in the materials. This cohesion cannot be depended upon to relieve the pressure against a wall, for the cohesion may be destroyed by vibration due to moving loads or to saturation. In designing a wall the pressure due to a granular mass or a semifluid without cohesion must always be considered. The material forming the bank in the rear of the wall, on which the fill will be placed, should be stepped as shown in Fig. 53 to reduce the initial pressure against the new wall and to aid in bonding the old and new materials.

**Causes of Failure of Walls.** There are three ways in which a masonry wall may fail: (1) by sliding along a horizontal plane; (2) by overturning or rotating; (3) by failure of the ground under its toe. These are the three points that must be considered in order to design a wall that will be successful in resisting an embankment. A wall, therefore, must be of sufficient size and weight to prevent the occurrence of sliding, rotation, or crushing.

*Stability of Wall Against Sliding.* Stability against sliding is secured by making the structure of sufficient weight so that there will be no danger of a movement at the base. In Fig. 50 let $E$ be the horizontal pressure and $W$ the weight of all materials above the joint. A movement will occur when $E$ exceeds $fW$, where $f$ is the coefficient of friction. Let $n$ be a number greater than unity, the factor of safety, then in order that there be no movement $n$ must be sufficiently large so that $nE$ equals $fW$. A common value for $n$ is 2, but sometimes it is taken as low as $1\frac{1}{2}$. Substituting 2 for $n$,

$$2E = fW$$

$$W = \frac{2E}{f} \tag{5}$$

Average values of the coefficients of friction of masonry on masonry is 0.65; for masonry on dry clay, 0.50; for masonry on wet clay, 0.33; masonry on gravel, 0.60; masonry on wood, 0.50.

*Stability Against Rotation.* The stability against rotation of a wall is secured by making the wall of such dimension and weight that the resultant $R$ of the external forces will pass through the base and well within the base, as shown in Fig. 50. Generally in designing, the resultant is made to come within or at the edge of

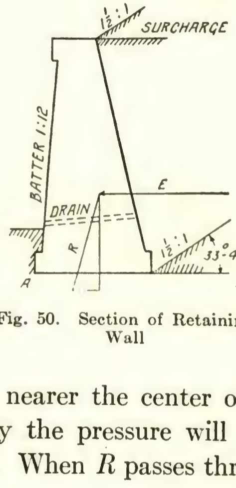



Fig. 50. Section of Retaining Wall

the middle third. The nearer the center of the base the resultant comes, the more evenly the pressure will be distributed over the foundation for the wall. When $R$ passes through $A$ the wall will fail by rotation.

*Stability Against Crushing.* The compressive unit stresses in walls must not be greater than the unit stresses permitted for safe working loads of masonry when the wall is built on a stone foundation; but when it is built on clay, sand, or gravel the allowable pressure for such foundations must not be exceeded.

**Foundations.** The foundations for a retaining wall must be below the frost line, which is about three feet below the surface in a temperate climate, and deeper in a cold climate. The foundation should be of such a character that it will safely support the wall. If necessary, the soil should be tested to determine if it will safely support the wall.

The foundation should always be well drained. Many failures of walls have occurred owing to the lack of drainage. Water behind a wall greatly increases the stresses in the wall. When water freezes behind a wall it usually causes it to bulge out, which is the first step in the failure of the wall. On a clay foundation the friction is greatly reduced by the clay becoming thoroughly soaked with water. On page 116 it is shown that the difference of the coefficients of friction of masonry on dry clay and wet clay is 0.17. There are different ways of draining a fill behind a retaining wall. The method shown in Fig. 31 for drainage often can be used. Pipes two to four inches in diameter are often built in the wall, as shown in Fig. 50.

**Design of Wall.** In designing a retaining wall the dimensions of the section of a wall are generally assumed and then the section investigated graphically to see if it is right for the conditions assumed. There are theoretical formulas for designing walls which will be given. In designing a wall, the student is advised to first make the section according to the formulas and investigate it graphically.

**Fill behind Wall.** The fills behind the walls are sometimes made horizontal with the top of the wall; at other times the fill is sloped back from the top of the wall, as shown in Fig. 50. When there is a slope to be supported, the wall is said to be surcharged, and the load to be supported is greater than for a horizontal fill.

**Face of Wall.** The front wall or face of a retaining wall may be built vertically or with a batter, but the back is sloped or stepped. When a wall is built with a batter to the front face, it usually has a slope of about one inch horizontally per foot of height.

**Thickness of Wall.** The theory and design of retaining walls is given in this text, but small walls on good foundations may be proportioned by making the thickness of the wall 0.40 to 0.50 of the height. For example, using the factor of 0.45, the thickness of a wall 10 feet high would be 4½ feet ($10 \times 0.45 = 4.5$) or if it were 6 feet high ($6 \times 0.45 = 2.7$) the thickness would be 2.7 feet.

Walls occupying prominent positions or walls of great height should be thoroughly studied and carefully designed. A retaining wall should never be less than 2½ to 3 feet in thickness at the top unless it is a very small wall; in that case, probably 18 inches would be sufficient.

**Concrete Details.** The concrete mix for walls should not be

leaner than 1 part Portland cement, 2½ parts sand and 3½ parts stone. Joints in plain concrete walls should be spaced not over 30 to 35 feet on centers. The tongue and groove should be formed in the end sections and waterproofing placed over the joints in the rear of the wall. The back of the wall should be stepped rather than sloped. The face of the wall can be finished by one of the methods mentioned elsewhere in the text.

**Value of Study of Existing Walls.** When designing a retaining wall, all existing walls in that vicinity should be examined to determine their dimensions and to discover if they have been successfully designed. Often, existing walls will give more information to an engineer than he will obtain by a theoretical or graphical study.

**Pressure behind Wall.** The development of the formulas for finding the pressure behind a wall is a long, complicated theory, and the demonstration will not be given here. The formulas given are those usually found in textbooks. They are based on the Rankine theory, which considers that the earth is a granular mass with an assumed angle of repose of 1.5 to 1, which in degrees is 33° 42′. In applying this method it is immaterial whether the forces representing the earth pressure are considered as acting directly upon the back of the wall, or are considered as acting on a vertical plane passing through the extreme back of the footing. In the latter case, the force representing the lateral earth pressure must be combined with (1) the vertical force representing the weight of the earth prism between the back of the wall and the vertical plane considered; and (2) combined with the vertical force representing the weight of the wall itself.

In the formulas for determining pressures behind a wall let $E$ equal total pressure against rear face of wall on a unit length of wall; $W$ equal weight of a unit volume of the earth; $h$ equal height of wall; and $\phi$ equal angle of repose.

*When the upper surface of the earth is horizontal,* the equation is

$$E=\tan^2\left(45°-\frac{\phi}{2}\right)\frac{Wh^2}{2} \qquad (6)$$

Since the angle of repose for the earth behind the wall has been taken as 33° 42′, Equation (6) may be reduced to the following form by substituting the value of the tangent of the angle in the equation

$$E = .286\frac{Wh^2}{2} \tag{6a}$$

*When a wall must sustain a surcharge at the slope of* 1.5 *to* 1, the equation is

$$E = \tfrac{1}{2} \cos \phi \ Wh^2 \tag{6b}$$

or

$$E = .833\frac{Wh^2}{2} \tag{6c}$$

The force $E$ is applied at one-third the height of the wall, measured from the bottom, but for surcharged wall it is applied at one-third of the height of a plane that passes just behind the wall. This is clearly shown in the different figures illustrating retaining walls.

The direction of the center of pressure $E$ is assumed as being parallel to the top of the earth back of the wall. The slope of the surcharge is generally made 1.5 horizontal to 1 vertical.

**Example.** What is the pressure per foot of length of a wall 18 feet high, earth weighing 100 pounds per cubic foot, if the fill is level with the top of the wall?

*Solution.* Substituting in Equation (6a),

$$E = .286\frac{Wh^2}{2}$$

$$= .286\frac{100 \times 18^2}{2}$$

$$= 4{,}633 \text{ pounds}$$

To illustrate the other loads and stresses and their points of application, a description of Fig. 51 will be given, followed by an example worked out in detail.

$E$ represents the pressure from fill behind the wall to be supported.

$h$ represents the height of the wall.

$B$ represents the width of the base.

$Q$ represents the distance from the toe to the point where the resultant force $R$ cuts the base.

$W_c$ represents the weight of the section of concrete wall one foot long; the line of action passing through the center of gravity of the wall.

$W_e$ represents the weight of earth, per lineal foot, resting on the base of the wall; the line of action passing through the center of gravity of the earth.

The arrow marked *C. of G. Concrete* locates the center of gravity of all concrete.

The arrow marked *C. of G. of Earth* locates the center of gravity of earth or fill above the base of the wall.

The arrow marked *C. of G., C. and E.* locates the position of the combined center of gravity of the concrete and earth.

$P$ (line $a$ to $b$) represents the weight per lineal foot of the wall, base of the wall, and the earth above the base of the wall, passing through the center of gravity of the wall and fill.

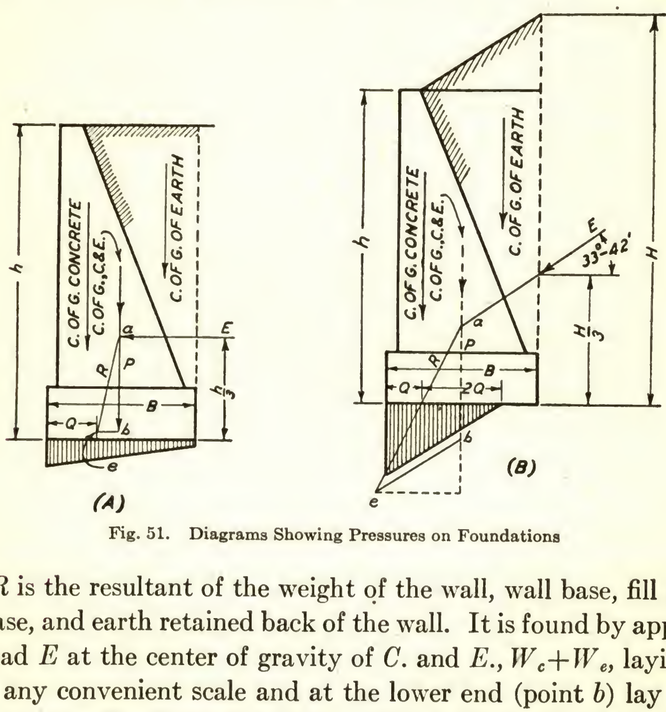



Fig. 51. Diagrams Showing Pressures on Foundations

$R$ is the resultant of the weight of the wall, wall base, fill above the base, and earth retained back of the wall. It is found by applying the load $E$ at the center of gravity of $C$. and $E$., $W_c+W_e$, laying off $P$, at any convenient scale and at the lower end (point $b$) lay off $be$ equal to force $E$. Draw the line $ae$. This is the resultant force, and the direction of it, for the wall, fill and the retained material. The stress in $ae$ may be found by scaling, or may be accurately obtained by mathematics; that is, $R=\sqrt{ab^2+be^2}$, provided it is not a surcharged wall.

**Pressure on Foundation.** The formulas for determining the pressure on the material on which the wall is constructed, recommended to the American Railway Engineering Association by a

committee appointed by that Society to investigate the subject of retaining walls, are as follows, see Fig. 51:

When $Q$ is equal to or greater than $\frac{B}{3}$ (See Fig. 51 at $A$.)

$$\text{Pressure at the toe} = (4B - 6Q)\frac{P}{B^2} \quad \textbf{(6d)}$$

$$\text{Pressure at the heel} = (6Q - 2B)\frac{P}{B^2} \quad \textbf{(6e)}$$

When $Q$ is less than $\frac{B}{3}$ (See Fig. 51 at $B$.)

$$\text{Pressure at the toe} = \frac{2P}{3Q} \quad \textbf{(6f)}$$

**Example.** Design a retaining wall to support an embankment 20 feet high, the top of the fill being level with the top of the wall; the face of the wall to be vertical, the back to slope.

*Solution.* Draw an outline of the proposed section, Fig. 52, and then investigate the section to see if it has sufficient strength to support the embankment. Make the base .50 of the height of the wall.

Width of base = 20 feet × .50 = 10 feet

Assume the width at the top as 3 feet, and find the pressure $E$ at the back by substituting in Equation (**6a**), and apply that pressure at $\frac{h}{3}$.

$$E = .286\frac{Wh^2}{2}$$

$$= .286\frac{100\times 20^2}{2}$$

$$= 5{,}720 \text{ pounds}$$

$P$ is found by dividing the wall into a rectangle and a triangle and finding the weights and the center of gravity of each, and also that of the triangle of earth back of the wall, and then finding the combined weights and the center of gravity of the wall and earth. This is done by taking moments about any convenient point. In this problem the toe of the wall ($A$ in Fig. 52) is selected. Assume that the weight of the concrete is 150 pounds per cubic foot and the earth 100 pounds per cubic foot, and consider a section of wall as being one foot in length. The center of gravity of the wall may be obtained thus:

| Section | Volume Cubic Feet | Moment Arm in Feet about Point $A$ | Volume Moment |
|---|---|---|---|
| *A H C D* | $1'\times 3'\times 20' = 60.0$ | $3' \div 2 = 1.5'$ | $60\times 1.5' = 90.0$ |
| *H B C* | $\frac{1'\times 7'\times 20'}{2} = 70.0$ | $3' + \frac{1}{3}(7') = 5.33'$ | $70\times 5.33' = 373.1$ |
| | 130.0 | | 463.1 |

Distance from $A$ to center of gravity of the wall $=463.1 \div 130 = 3.56$ feet.

Weight of wall per lineal foot $= 130 \times 150 = 19{,}500$ pounds.

$\therefore$ For wall, static moment about $A = 19{,}500 \times 3.56 = 69{,}420$ foot-pounds.

In a like manner the static moment for the fill is found. In this problem only one triangle ($CFB$) of fill is to be considered. The center of gravity is located one-third of the distance $CF$ from the back line $FB$ of the triangle, or 7.67 feet from point $A$.

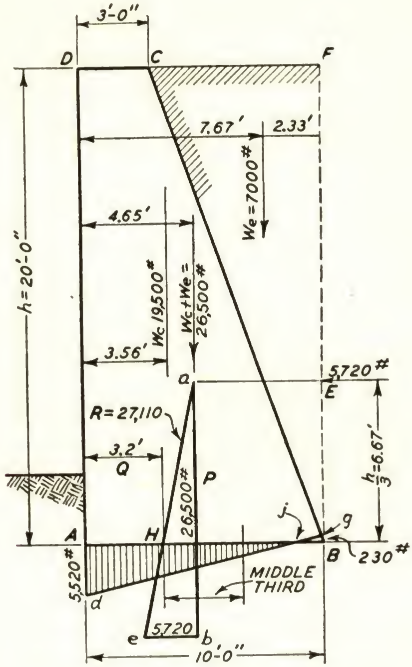



Fig. 52. Stress Diagram for Retaining Wall

Weight of earth per lineal foot $= \frac{7' \times 20'}{2} \times 100\# = 7{,}000$ pounds.

$\therefore$ For earth the static moment about $A = 7{,}000 \times 7.67 = 53{,}690$ foot-pounds.

The position of the center of gravity of the combined concrete and earth sections is determined by dividing the sum of the static moments by the sum of the weights:

$$\frac{69{,}420+53{,}690}{19{,}500+7{,}000} = \frac{123{,}110}{26{,}500} = 4.65 \text{ feet}$$

That is, the center of gravity of the combined vertical loads (concrete and fill) is located 4.65 feet from the toe of the wall (point $A$).

Produce the line $E$ to meet the vertical line passing through the combined center of gravity, that is, $W_c + W_e$. On this vertical line, lay off the value of $P = 26{,}500$ pounds, to any convenient scale. At the lower end of $P$ (point $b$)

draw a line parallel to line $E$, and on this line lay off (at the same scale) the value of $E$, which is 5,720 pounds. Draw line $ae$, which is the resultant of the two forces, and scale the stress which in this problem is 27,110 pounds. This line cuts the base at $H$ at a scaled distance of 3.20 feet from the toe, which is a point near but outside the edge of the middle third of the base. Therefore,

$$Q \text{ is less than } \frac{B}{3}$$

Substitute in Equation (**6f**) for the condition when $Q$ is less than $\frac{B}{3}$

$$\text{Pressure at toe} = \frac{2P}{3Q} = \frac{2\times 26,500}{3\times 3.20} = 5,520 \text{ pounds}$$

Lay off $Ad$, at any convenient scale, equal to 5,520 pounds and on the base lay off a distance equal to $3Q$ which is $3\times 3.20 = 9.60$ feet. Through this

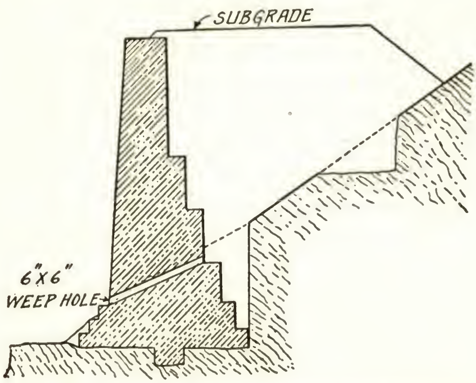



**Fig. 53. Retaining Wall for Railroad Embankment**

point $j$ draw line $dg$ and scale the force shown from $g$ to the base line, which will be about 230 pounds.

This shows an uplift at the heel of about 230 pounds, which may be ignored since the resultant is only about 1 inch outside the middle third. The pressure at the toe amounts to 5,520 pounds, which is a load any good soil will easily support; therefore, the wall should be stable.

The shape of the wall may be changed so that the resultant will cut the middle third. This may be done by increasing the thickness of the wall at the top and giving the face of the wall a batter as shown in Fig. 53. If it is sufficiently increased in thickness at the top, then the base may be reduced. This can be carried on until the shape of the wall becomes rectangular and the resultant will be well within the middle third. A wall designed under these conditions, if supported on good soil and well anchored, should be safe.

As already pointed out, many walls have given good service in which the thickness of the wall was from 0.4 to 0.45 of the height. Of course, much depends on the bearing and on the fill behind the wall. The wall resists the maximum load when a new fill is made, but if it is constructed against or close to a substantial bank, the load will probably be very small.

In investigating the wall for stability against sliding on its base, we will assume that the wall is to be built on dry clay with a horizontal thrust, $E$, of 5,720 pounds; the total weight is 26,500 pounds, and the coefficient of friction of masonry on dry clay is 0.50. Substituting in Equation (5)

$$W = \frac{2E}{f}$$

Substituting,
$$26{,}500 = \frac{2 \times 5{,}720}{.50}$$

Solving,
$$26{,}500 \times .50 = \frac{2 \times 5{,}720}{\cancel{.50}} \times \cancel{.50}$$

$$26{,}500 \times .50 = 2 \times 5{,}720$$

$$13{,}250 = 11{,}440$$

The approximate equality of the equations shows that there is a factor of almost 2¼ against sliding on such a base. On a base of wet clay the factor against sliding would be 1½ and the wall should be secured against sliding.

In Fig. 53 is shown a retaining wall for a railroad embankment. It will be noted that this wall is anchored to prevent it from sliding, the back of the wall is stepped so as to produce as small a pressure as possible. Also, the fill behind the wall is drained.

## BRIDGE PIERS AND ABUTMENTS

**Location.** The outline design of a long bridge which requires several spans involves many considerations:

(1) If the river is navigable, at least one deep and wide channel must be left for navigation. The placing of piers, the clear height of the spans above high water, and the general plans of all bridges over navigable rivers are subject to the approval of the United States Government.

(2) A long bridge always requires a solution of the general question of few piers and long spans, or more piers and shorter spans. No general solution of the question is possible, since it depends on the required clear height of the spans above the water, on the required depth below the water for a suitable foundation, and on several other conditions (such as swift current, etc.) which would influence the relative cost of additional piers or longer spans. Each case must be decided according to its particular circumstances.

(3) Even the general location of the line of the bridge is often determined by a careful comparison, not only of several plans for a given crossing, but even a comparison of the plans for several locations.

**Sizes and Shapes.** The requirements for the bridge seats for the ends of the two spans resting on a pier are usually such that a pier with a top as large as thus required, and with a proper batter to the faces, will have all the strength necessary for the external forces acting on the pier. For example, the channel pier of one of the large railroad bridges crossing the Mississippi River was capped by a course of stonework 14 feet wide and 29 feet long, besides two semicircles with a radius of 7 feet. The footing of this pier was 30 feet wide by 70 feet long, and the total height from subsoil to top was about 170 feet. This pier, of course, was unusually large. For trusses of shorter span, the bridge seats are correspondingly smaller. The elements which affect stability are so easily computed that it is always proper, as a matter of precaution, to test every pier designed to fulfill the other usual requirements, to see whether it is certainly safe against certain possible methods of failure. This is especially true when the piers are unusually high.

The requirements for supporting the truss are, fortunately, just such as give the pier the most favorable formation so that it offers the least obstruction to the flow of the current in the river. In other words, since the normal condition is for a bridge to cross a river at right angles, the bridge piers are always comparatively long, in the direction of the river, and narrow in a direction perpendicular to the flow of the current. The rectangular shape, however, is modified by making both the upper and the lower ends pointed. The pointing of the upper end serves the double purpose of deflecting the current, and thus offers less resistance to the flow of the water; and it also deflects the floating ice and timber, so that there is less danger of the formation of a jam during a freshet. The lower end should also be pointed in order to reduce the resistance to the flow of the water. The ends of the piers are sometimes made semicircular, but a better plan is to make them in the form of two arcs of circles which intersect at a point.

**Causes of Failure.** The forces tending to cause a bridge pier to fail in a direction perpendicular to the line of the bridge include the action of wind on the pier itself, on the trusses, and on a train which may be crossing the bridge. They will also include the maximum possible effect of floating ice in the river and of the current due to a freshet. It is not at all improbable that all of these causes

may combine to act together simultaneously. The least favorable condition for resisting such an effect is that produced by the weight of the bridge, together with that of a train of empty cars, and the weight of the masonry of the pier above any joint whose stability is in question. The effects of wind, ice, and current will tend to make the masonry slide on the horizontal joints. They will also increase the pressure on the subsoil on the downstream end of the foundation of a pier. They will tend to crush the masonry on the downstream side, causing the pier to tip over.

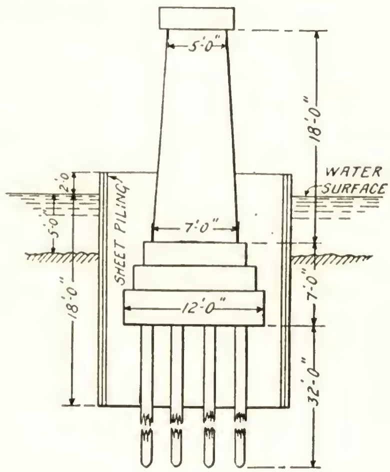



Fig. 54. Bridge Pier

Another possible cause of failure of a bridge pier arises from forces parallel with the length of the bridge. The stress produced on a bridge by the sudden stoppage of a train thereon, combined with a wind pressure parallel with the length of the bridge, will tend to cause the pier to fail in that direction, Fig. 54. Although these forces are never so great as the other external forces, yet the resisting power of the pier in this direction is so very much less than that in the other direction, that the factor of safety against failure is probably less, even if there is no actual danger under any reasonable values for these external forces.

**Abutment Piers.** A pier is usually built comparatively thin in the direction of the line of the bridge, because the forces tending

to produce overturning in that direction are usually very small. When a series of stone arches are placed on piers, the thrusts of the two arches on each side of a pier nearly balance each other, and it is only necessary for the pier to be sufficiently rigid to withstand the effect of an eccentric loading on the arches; but, if by any accident or failure, one arch is destroyed, the thrust on such a pier is unbalanced and the pier will probably be overturned by the unbalanced thrust of the adjoining arch. The failure of that arch would similarly cause the failure of the succeeding pier and arch. On this account a very long series of arches usually includes an abutment pier for every fourth or fifth pier. An abutment pier is

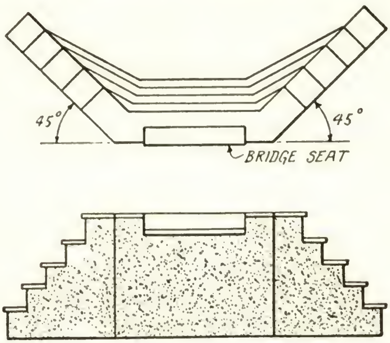



Fig. 55. Typical Abutment with Flaring Wing Walls

one which has sufficient thickness to withstand the thrust of an arch, even though it is not balanced by the thrust of an arch on the other side of the pier. Abutment piers are chiefly for arch bridges; but all piers should have sufficient rigidity in the direction of the line of the bridge so that any possible thrust which may come from the action of a truss of the bridge may be resisted, even if there is no counterbalancing thrust from an adjoining truss.

## ABUTMENTS

**Requirements of Design.** The term abutment usually implies not only a support for the bridge, but also what is virtually a retaining wall for the bank behind it. In the case of an arch bridge, the thrust of the arch is invariably so great that there is never any chance that the pressure of the earth behind the abutment will throw the abutment over, and therefore the abutment never needs

to be designed as a retaining wall in this case; but when the abutment supports a truss bridge which does not transmit any horizontal thrust through the bridge, the abutment must be designed as a retaining wall. The conditions of stability for such structures have already been discussed. This principle of the retaining wall is especially applicable if the abutment consists of a perfectly straight wall. There are other forms of abutments which, by their shape, act as retaining walls and tend to prevent failure.

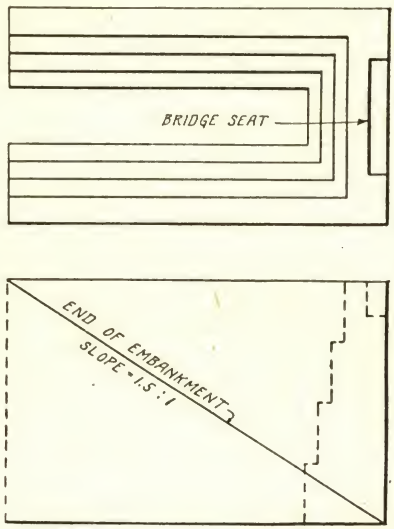



Fig. 56. U-Shaped Abutment

**Abutments with Flaring Wing Walls.** These are constructed substantially as shown in Fig. 55. The wing walls make an angle of about 30° to 45° with the face of the abutment, and the height decreases at such a rate that it will just catch the embankment formed behind it, the slopes of the embankment probably being at the rate of 1.5 : 1. If the bonding of the wing walls, and especially the bonding at the junction of the wing walls with the face of the abutment, are properly done, the wing walls will act virtually as counterforts and will materially assist in resisting the overturning tendency of the earth. The assistance given by these wing walls will be much greater as the angle between the wing walls and the face becomes larger.

**U-Shaped Abutments.** These consist of a head wall and two walls which run back perpendicular to the head wall, Fig. 56. This form of wall is occasionally used, but the occasions are rare when such a shape is necessary or desirable.

In Fig. 57 is shown a view of a super-highway bridge at St. Louis, Mo. In this illustration it will be noted that the abutments are parallel to the axis of the highway and form retaining walls for

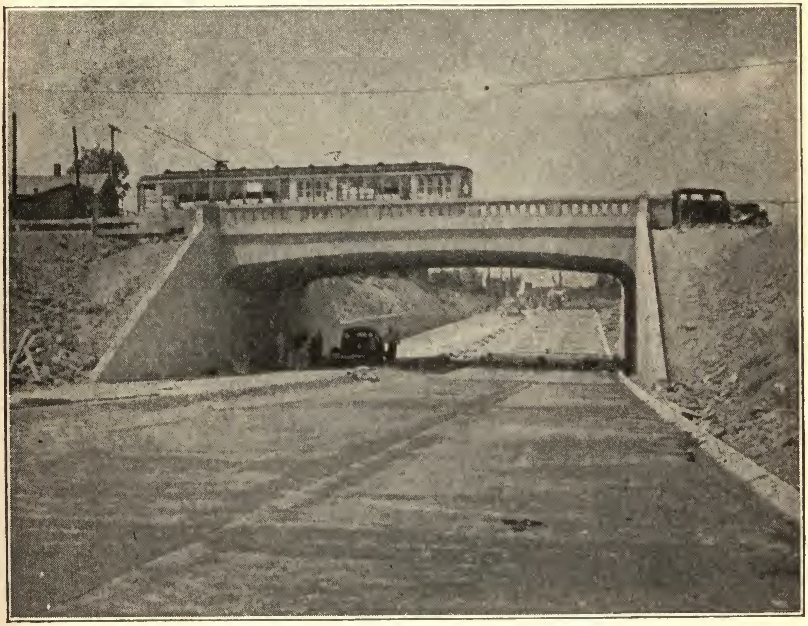
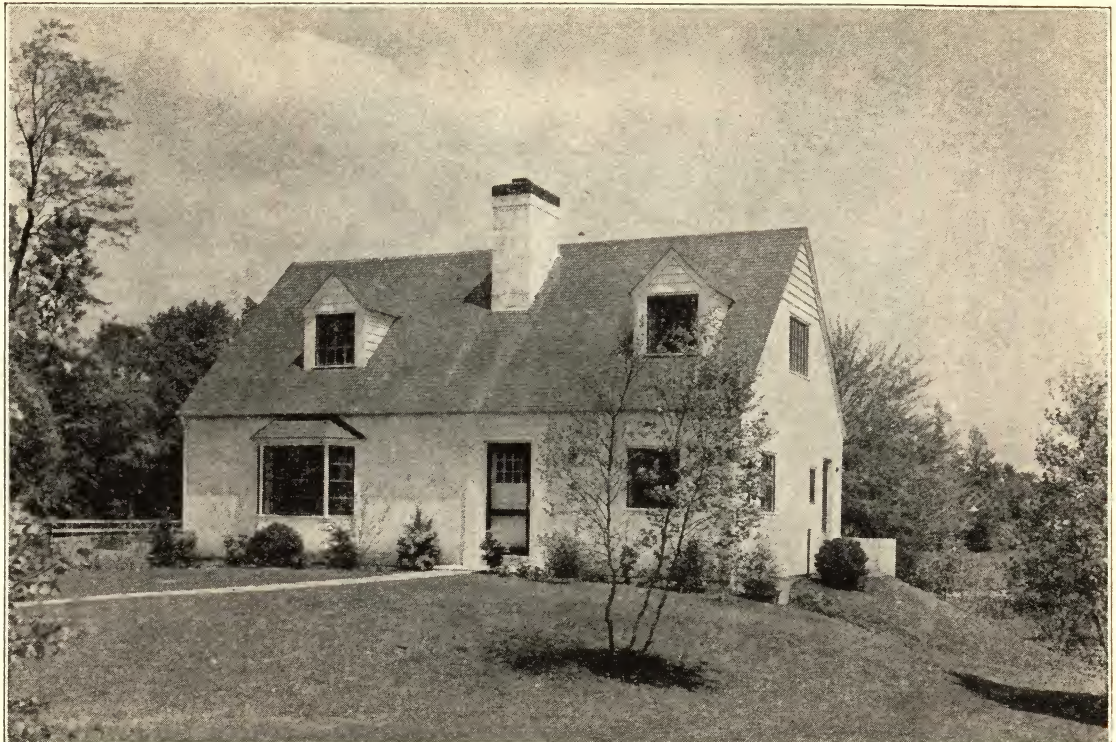


**Fig. 57. Super-Highway Bridge, St. Louis, Missouri**
*Courtesy of Lone Star Cement Company*

the highway as well as abutments for the bridge. The lines of the superstructure indicate it was designed as a rigid frame.

In rigid frame bridges the abutments are subjected to three kinds of stresses: (1) bending stresses produced by making the corners rigid; (2) direct compression due to the loads on the structure; and (3) stresses due to earth pressure. Because of the rigid corners the moments are greatly reduced near the center of the deck and the deck can be made much shallower than for a simple span bridge supported by bearing on abutments. Many bridges of this type have been constructed in the last few years.



WESTON HEATH CONCRETE MASONRY HOUSE IN WESTON, MASS.

*Courtesy of Portland Cement Association*

## CHAPTER VII

## CONCRETE WALKS

**Foundations.** The depth of the foundations for sidewalks should be not less than 12 inches in fairly mild climates, and for cold climates should be deeper. The fill under the pavement can be made with tamped cinders or other porous materials that will permit the water to seep through. Sidewalks should never be built on filled ground; however, this is necessary in many cases. The materials used for filling should be a good grade of clay, gravel, or cinder and should be well tamped. The width of the embankment on each side of the pavement should be at least 2½ to 3 feet, to protect the pavement. It is sometimes necessary to lay a line of tile drains to get rid of the water. This tile should be laid with open joints so that the water can enter the drains. All foundations for pavements should be well drained.

**Lines and Grades.** All curbs and pavements should be laid true to the lines established. Curbs and pavements out of line look very bad. The pavement should be sloped about ¼ inch for the water to run off.

**Paving.** The entire thickness of the concrete slab should be poured at one operation and finished as soon as the top can be worked. However, if a top course of one inch of mortar is desired, it should be poured immediately after the base and worked into the base so that the two materials will be bonded together. The old method of putting down pavements made of a cinder concrete base of very lean mixture and finished with one inch of rich cement mortar usually was found to be unsatisfactory.

The mixture for a good pavement should consist of 1 part cement, 2½ parts sand and 3½ parts stone or gravel. If the pavement is to be put down with a 1-inch concrete top, then the base should be of the mixture already called for, and the top should consist of 1 part cement, 1 part sand, and 2 parts of grits or granite chips. Sometimes a rich mixture is used, but for a satisfactory pavement the above

proportions should not be made leaner. The Portland Cement Association recommends a mixture of 1 part cement, 2¼ parts sand and 3 parts stone, the entire thickness to be poured at one operation. The thickness of the pavement should never be less than 4 inches; a 5-inch slab will be found more satisfactory; and for heavy traffic it

Fig. 58. Square Tamper

should be 6 inches in thickness. For residential work the pavements are usually 4 inches in thickness. Wherever automobiles are to pass over a sidewalk, the concrete should be 6 inches in thickness.

The concrete should be mixed as dry as possible to be tamped in place. In Fig. 58 is shown a square tamper often used for this work.

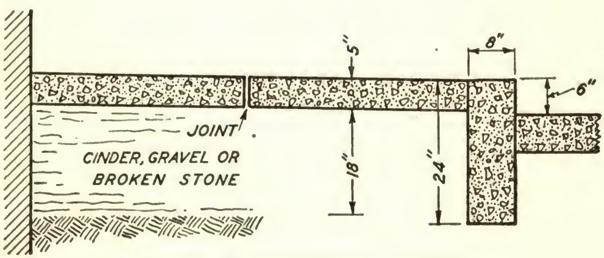



Fig. 59. Concrete Sidewalk and Curb

**Finish.** On completion of the tamping, the top should be finished at once and with the least amount of work possible. A wood trowel or an old piece of canvas should be used to secure the finish. In finishing the top course, no dry material such as sand or cement should be used. Any excess moisture should be taken up with burlaps or like material.

**Joints.** All sidewalks should be laid off in squares with an expansion joint between each block of concrete. Fig. 59 shows a section

of sidewalk and curb with a joint indicated. The size of the blocks will depend on the width of the pavement, but in general they should not be more than about 6 feet square. The joints should be of sufficient width to allow for expansion and contraction. These joints are constructed by putting a piece of wood ¼ inch or more in thickness between all blocks of concrete. This joint should extend to the bottom of the slab; otherwise, it will prove inadequate. These joints can be finished by the use of the jointers shown in Fig. 60.

**Reinforcing.** Sidewalks are often reinforced with wire mesh or bars to assist in producing the amount of expansion and contraction

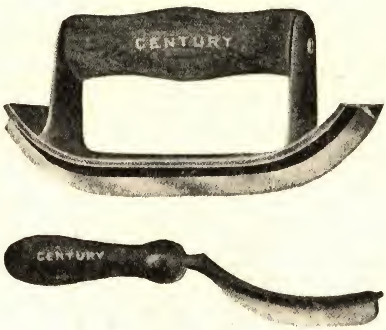



Fig. 60. Jointers

caused by the changes of temperature and also to make them stronger where poor foundations are encountered. Walks laid on filled ground should always be reinforced.

**Curing Concrete.** The finished concrete walk must be carefully cured. This may be done by covering the concrete with burlaps and keeping them wet for a period of several days. The burlaps should be placed as soon as workmen can walk on the concrete. It is important to start the curing early, as the evaporation in dry hot weather is very rapid and the moisture in the concrete should be retained as long as possible.

## CONCRETE CURB

The curb is usually built just in advance of the sidewalk. The foundation is prepared similarly to that of walks. The curb is divided into lengths similar to that of the walk; and the joints between the blocks, and also between the walk and the curb, are

made similar to the joints between the blocks of the walk. The concrete is generally composed of 1 part Portland cement, 2½ parts

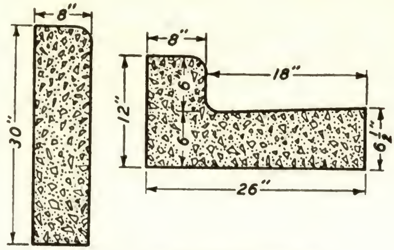



Fig. 61. Typical Curb Sections

sand, and 3½ parts stone, although a richer mixture is sometimes used. A facing of mortar or granolithic finish on the part exposed to wear will improve the wearing qualities of the curb.

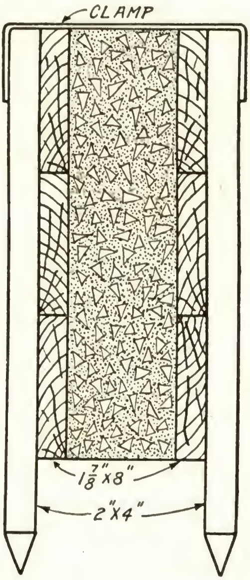



Fig. 62. Forms for Constructing Curb

**Types of Curbing.** There are two general types of curb used—a curb rectangular in section, and a combined curb and gutter; both types are shown in Fig. 61. The foundation for either type is constructed in the same manner. Both these types of curb are made

in place or molded and set in place like stone curb, but the former method is preferable. A metal corner is sometimes laid in the exposed edge of the curb to protect it from wear.

**Construction.** The construction of the rectangular section is a simple process, but requires care to secure a good job. This is usually about 8 inches wide and from 20 to 30 inches deep. After the foundation has been properly prepared, the forms are set in place. Fig. 62 shows the section of a curb 8 inches wide and 30 inches deep,

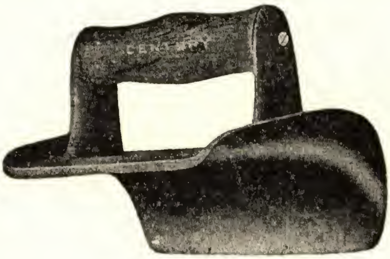



Fig. 63. Curb Edger


Fig. 64. Radius Tool

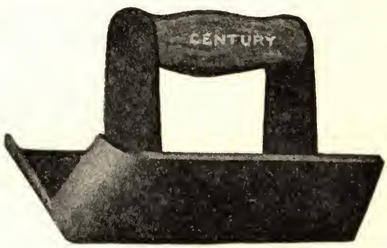



Fig. 65. Inside-Angle Tool

and the forms as they are often used. The forms for the front and back each consist of three planks 1⅞ inches thick and 8 inches wide, and are surfaced on the side next the concrete. They are held in place at the bottom by the two 2- by 4-inch stakes, and at the top the stakes are kept from spreading by a clamp. A sheet-iron plate ¼ inch thick is inserted every 6 feet, or at whatever distance the joints are made. After the concrete has been placed and rammed, and has set hard enough to support itself, the plate and front forms are removed, and the surface and top are finished smooth with a trowel, and with other tools such as shown in Figs. 63, 64, and 65. The joint is usually plastered over, and acts as an expansion joint.

The forms on the back are not removed until the concrete is well set. If a mortar or granolithic finish is used, a piece of sheet iron is placed in the form one inch from the facing, and mortar is placed between the sheet iron and the front form, and the coarser concrete is placed back of the sheet iron, Fig. 149. The sheet iron is then withdrawn and the two concretes thoroughly tamped.

Fig. 66 shows the section of a combined curb and gutter, and the forms that are necessary for its construction. This combination is often laid on a porous soil without any special foundation, with fair results. A 1⅞-inch plank 12 inches wide is used for the back form,

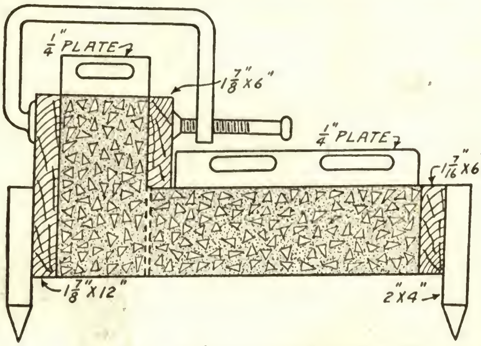



Fig. 66. Forms for Curb and Gutter

and is held in place at the bottom by pegs. The front form consists of a plank 1⅞ by 6 inches, and is held in place by pegs. Before the concrete is placed, two sheet-iron plates, cut as shown in the figure, are placed in the forms, six feet to eight feet apart. After the concrete for the gutter and the lower part of the curb is placed and rammed, a 1⅞-inch plank is placed against these plates and held in place by screw clamps, Fig. 66. The upper part of the curb is then molded. When the concrete is set enough to stay in place, the front forms and plates are removed, and the surface is treated in the same manner as described for the other type of curb.

**Cost.** The cost of concrete curb will depend upon the conditions under which it is made. Under ordinary circumstances, the contract price for rectangular curbing 8 inches wide and 30 inches deep will be about 95 cents per linear foot; or 80 cents per linear foot for curb 8 inches wide and 24 inches deep. Under favorable conditions on large jobs, 6-inch curbing can be constructed for 60 cents or

70 cents per linear foot. These prices include the excavation that is required below the street grade.

The cost of the combined curb and gutter is about 10 to 20 per cent more than that of the rectangular curbing. In addition to having a larger surface to finish, the combined curb and gutter requires more material, and therefore more work, to construct it.

In Fig. 67 a metal curb bar is shown. These bars are used to protect concrete curbs where they will have hard usage, and are especially valuable at corners of streets and entrances to factories. The bars are galvanized and the anchors are sheared from the webs.

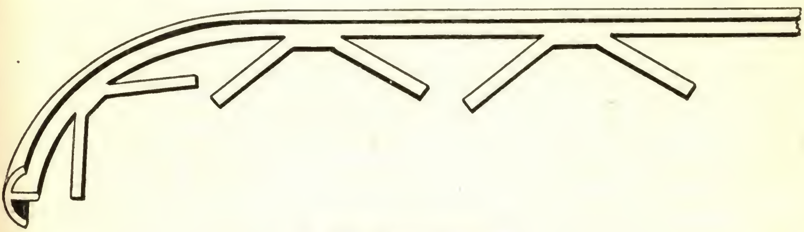


Fig. 67. Metal Curb Bar

## CONCRETE STONE AND BLOCKS

.Concrete blocks, tiles, and bricks are extensively used as masonry units. This material is used for backing up walls, partitions in buildings, and for the entire walls for factories, garages, residences, school buildings, etc.

Good materials have been produced in each of these classes of material, but due to lack of experience and false economies, some unsatisfactory work has been done. With the experience gained, superior materials are now being produced. With proper aggregates, good workmanship, and careful curing, good units for either decoration or masonry units can be produced at a reasonable cost. The artificial stone costs less than cut stone and is often substituted for it for that reason; but many times it is used because it produces, architecturally, a pleasing and satisfactory finish. The concrete blocks, especially the lightweight ones (for example, cinder concrete) cost less laid in the wall than brick or stone, and also have the advantage of furnishing additional insulation due to openings in the blocks.

**Building Blocks.** Concrete blocks are made of Portland cement, sand, and either stone, gravel, haydite, crushed slag or cinders. The

most popular block made at this time is the cinder block, which is composed of cement and cinders in varying ratios. Stone concrete blocks are sometimes made with a special finish which is about one-half inch in thickness; or the face of the block sometimes is formed to represent ashlar masonry when the blocks are for use in face work for houses or similar structures. However, this practice has been largely discontinued and the entire block is now made of one material and without an attempt at ornamentation.

Cinder blocks are composed of 1 part cement to 6, 7, 8, 9, or 10 parts well-graded cinders measured as loosely thrown into measuring receptacles. This is a large variation in the ratio of cement to cinders, but many of the blocks are sold under building laws that require a crushing strength of 700 pounds per square inch gross area at the age of 28 days; with the equipment in use today, this strength can be produced with a mix of 1 part cement and 11 parts cinders. For load-bearing blocks the richer mixtures must be used.

Cinder blocks are used for many purposes. They are extensively used for backing up brick and stone walls, for partitions in fireproofed buildings, walls for residences, school buildings, etc. For exposed walls, they are usually painted.

**Sizes of Blocks.** Cinder concrete blocks are made in standard sizes. These standards were adopted in 1932 and at the meeting in Chicago, Illinois, February 10, 1938, the Standing Committee in Charge of Simplified Practice of the Bureau of Standards decided to make no changes. In the May 1938 issue of *The News of the National Cinder Concrete Products Association* the following sizes of blocks and brick were tabulated and, as Tables 1 and 2, are reproduced by permission.

**Materials.** A standard brand of Portland cement should be used. The aggregates, fine and coarse, such as sand, stone, gravel, slag, and cinders should be of the best quality. The fine aggregates should pass a screen with 4 mesh to the inch and 15 per cent should be retained on a No. 8 screen. The maximum size of the coarse aggregate should not have a dimension of over ½ inch for blocks with webs of 1½ inches in thickness. It should all be retained on a screen having 4 mesh to the inch.

**Proportions.** The proportions of cement and aggregates for manufacturing blocks cannot be definitely stated, but the coarse

**Table 1. Load and Nonload Bearing Block and Tile***

| Width Inches | Height Inches | Length Inches | Width Inches | Height Inches | Length Inches |
|---|---|---|---|---|---|
| 12 | 7¾ | 15¾ | 6 | 11¾ | 23¾ |
| 10 | 7¾ | 15¾ | 4 | 11¾ | 23¾ |
| 8 | 7¾ | 15¾ | 3 | 11¾ | 23¾ |
| 6 | 7¾ | 15¾ | 2 | 11¾ | 23¾ |
| 4 | 7¾ | 15¾ | 7¾ | 5 | 11¾ |
| 3 | 7¾ | 15¾ | 5¾ | 5 | 11¾ |
| 12 | 8 | 12 | 3¾ | 5 | 11¾ |
| 8 | 8 | 12 | 7¾ | 3½ | 11¾ |
| 6 | 8 | 12 | 5¾ | 3½ | 11¾ |
| 4 | 8 | 12 | 3¾ | 3½ | 11¾ |
| 8 | 11¾ | 23¾ | ... | ... | ... |

*Tolerance: The permissible variation over or under the above dimensions shall not exceed ¼ inch.

**Table 2. Face and Common Concrete Brick†**

| Width Inches | Height Inches | Length Inches |
|---|---|---|
| 3¾ | 2¼ | 8 |

†Tolerance: The permissible variation over or under the above dimensions shall not exceed 1/16 inch in height, ⅛ inch in width, ¼ inch in length.

These standards were adopted in 1932. However, they recommend the addition of the following 12″ units as standard:

| Width Inches | Height Inches | Length Inches |
|---|---|---|
| 4 | 8 | 12 |
| 6 | 8 | 12 |
| 8 | 8 | 12 |
| 12 | 8 | 12 |

These units are now being used extensively over the country, and the Committee believes that they will become increasingly popular in the future. Consideration was also given to units 18″ long, but it was decided that inasmuch as interest in these units was diminishing, at present it would take no action but wait to observe what the trend would be during the coming year.

aggregates generally will not run over 40 to 50 per cent of the total aggregates used. The blocks, to be salable, must have sufficient strength to support the load for which they are intended and at the same time must have square corners and fairly smooth faces, except where stucco or plaster is to be applied. Trial mixes and crushing tests will be of great assistance in determining the best mix. A work-

able concrete must be used. If the aggregate is too coarse, there will be breakage in removing the blocks from the machine.

**Consistency.** The consistency of the concrete for blocks will depend somewhat on the materials used and the manner of tamping. Hand tamping will generally require more water in the mix than pressure tamping. Cinder blocks formed in pressure or vibrating machines are made of a very dry mixture that has no appreciable slump, in fact, the manufacturers claim they do not have any slump. The dry concrete materially assists in handling the block when being taken from the machine.

**Mixing.** The cement, aggregates and water must be well mixed to produce a satisfactory concrete. The time of mixing will depend somewhat on the materials used, but a period of 5 to 10 minutes usually is allowed, and for cinder blocks 15 minutes are allowed by some manufacturers.

**Curing.** Concrete blocks may be cured in open sheds, by steam heat, or by a combination of the two methods. To cure blocks properly, they must be in a temperature of at least 70°, have plenty of moisture and be protected from the weather for several days. Many manufacturers of cinder blocks cure their blocks in steam at a temperature of about 100° for a period of 24 to 48 hours and then place them in the yard piled closely, under a shed or canvas. They are then sprayed frequently with water to keep them moist. If blocks are entirely cured in the air under favorable conditions, they can be used safely at the age of 28 days. When cured in steam for 48 hours and then moved to the yard, the 28-day curing period may be reduced several days. Blocks should be air cured, in a temperature of not less than 70°, for 5 or 6 hours before being put in the steam.

## SPECIAL TYPES OF CONCRETE

**Gunite.** Gunite has been extensively used for repair work and reconditioning concrete in work that has become defective. It has been used often for relining reservoirs, refacing retaining walls and arches, repairing reinforced concrete buildings damaged in fires, etc. Also it has been much used on new work. This method is especially adapted to meet the conditions on many pieces of work. For example, where a floor must be reinforced, a steel frame may be placed under the old floor, or the beams in the old floor reinforced, and then fire-

proofed by the gunite method. It is also a good method to use in constructing thin walls.

Gunite is a surface coating composed of cement and sand which are mixed with water at the discharge nozzle and placed under pneumatic pressure with a cement gun. Being placed by impact makes it dense when it is properly applied. This method may be used horizontally, vertically, or for overhead work. It should be applied to a firm material.

Fig. 68. Cement Gun in Operation
*Courtesy of Cement Gun Company, Allentown, Pa.*

When old work is to be repaired by the gunite method, the damaged or deteriorated concrete should be carefully removed. The hard surfaces should be made rough and covered with wire mesh or with small reinforcing bars. To properly bond the new material to the old, it is often necessary to place dowels in the old concrete and anchor the reinforcing metal to them. The surface to be covered with gunite should be well soaked with water several hours before the guniting is started.

A standard brand of Portland cement should be used with a well-graded sand. The proportions used are usually 1 part cement to 3 parts sand. The sand must be graded well and no material over ¼ inch in diameter should be used. The thickness of the concrete to be

placed may vary from 1 inch to any desired thickness, and it can be applied in layers of ¼ inch or more; that is, the space to be protected by the gunite method may be built up in steps to any desired thickness. Concrete placed by this method should be well protected and cured.

Being placed by pressure, a certain amount of this material bounces off the surface and is called the *rebound.* It is a waste material and must be discarded. The amount of the rebound depends on the surface being concreted and will vary from 20 to 40 per cent or more. The rebound will be greater on overhead work than on vertical work.

Fig. 68 illustrates the use of the cement gun. Concrete beams have been wrapped with wire mesh and the beam is being protected by adding the necessary fireproofing by use of the gun. This material can be built up to the desired thickness. The gun is operated and controlled by the man holding the nozzle.

The cement gun is described in another chapter.

**Lightweight Concrete.** Lightweight concretes are often found desirable. They are used extensively in different forms for roofs of buildings, for fills on floors, and for the structural slabs and fireproofing of steel-frame buildings, etc.

The lightweight concretes may be divided into several classes, the first of which may be cinder concrete, which has a weight of about 110 pounds per cubic foot when used as a structural material. A second class consists of a Portland cement concrete in which a lightweight aggregate is used, such as haydite, slag, terra cotta burned especially for this purpose, or other suitable materials that are light in weight. With these materials, the regular mix of concrete is used to produce the strength required for the work.

A third classification may be described as a lightweight concrete which is produced by the addition of a chemical compound either to an ordinary stone concrete or to a mixture of cement and sand. The chemical compound results in a chemical reaction which forms gas and, in turn, expands the concrete, reducing its weight and strength. When it is used as a slab in a building, the regular concrete mix is used; when a nailing surface is required only cement and sand are used. The stone concrete, when expanded, will weigh about 110 pounds per cubic foot and should have a crushing strength of about 1000 pounds per square inch at the age of 28 days.

When used as a nailing surface or as a roof, the material may be cast in place or cast in slabs at a factory and shipped to the job. These slabs are reinforced with metal, for structural purposes should weigh 60 to 80 pounds per cubic foot, and should have a crushing strength of not less than 800 pounds per square inch at the age of 28 days. This lightweight concrete is a very porous concrete containing many bubble holes and often is known as bubble concrete. It is a much better insulator than dense concrete.

## CONCRETE FLOOR FINISH

**Qualification.** To secure a satisfactory wearing surface a concrete floor requires good materials and careful workmanship. The surfaces must be hard, so that the wear on them will not be excessive, and so that they will not dust. They should also be nonabsorptive and free from cracks, crazes or other defects.

**Surface Treatments.** Finishing floors by adding dry cement and sand is generally a source of trouble. These materials will often produce a crazed surface. Also, if the floors are not protected after being poured, some other building material such as mortar, dirt, etc., will be tramped and worn into the surface. Often this condition can be cured, or helped, by being thoroughly cleaned and well scrubbed with soap suds and then rubbed with wire brushes or fine steel wool. The action of the wire brushes or steel wool will remove the dirt and expose the real wearing surface. It should be carefully mopped and cleaned of all dirt. If the scrubbing, brushing, etc., do not produce results, the surface can be treated with various oils, gums, magnesium, fluosilicate, sodium silicates, aluminum, sulphate, etc. It is essential that the floors be thoroughly cleaned before such processes are applied. In paper *No. ST37*, published July 1937 by the Portland Cement Association, is given the treatment for fluosilicate. This method is quoted from the mentioned paper.

**Fluosilicate Treatment.** The fluosilicates of zinc and magnesium, when dissolved in water, have been used with success. Either of the fluosilicates may be used separately, but a mixture of 20 per cent zinc and 80 per cent magnesium appears to give the best results. In making up the solutions, ½ pound of the fluosilicate should be dissolved in one gallon of water for the first application, and 2 pounds to each gallon for subsequent applications. The solution may be mopped on, or applied with a sprinkling can and then spread evenly with mops. Two or more applications should be given, allowing the surface to dry between applications. About 3 or 4 hours are generally required for

absorption, reaction, and drying. Shortly after the last application has dried, care should be taken to mop the floor with water to remove incrusted salts; otherwise white stains may be formed.

**Finish of Structural Slab for Wearing Surface.** Often it is desirable to finish the top of a structural slab for a wearing surface. This is done usually for economy. Ordinarily, a floor finished at the time the structural concrete is poured will not be as level and smooth as a floor laid later; however, a satisfactory floor can be secured if care is used. If high spots are found after the floor is finished, they may be ground off. A specification calling for high spots to be ground off usually assures careful work, as the grinding is an expense to be avoided.

The structural slab should contain as small an amount of water as practicable to work the concrete. After pouring the slab, ½ inch to 1 inch of mortar may be added. This should be composed of 1 part cement, 1 part sand, and 1½ to 2 parts crushed stone or gravel. This should be worked into the slab well and should be considered as part of the slab. This mixture should be extremely dry, with a slump of not more than one-half inch. Before this mortar is applied, all surface water on the structural slabs should be mopped up. This surface should be screeded and worked with wood floats until an even surface is secured. The application of dry cement and sand to the top surface should never be permitted. This material is apt to dust after the floor is put in use, making an unsatisfactory condition. Steel trowels should be confined to a minimum amount of work so that a smooth hard surface is secured.

**Finish on a Hardened Structural Slab.** A finish 1 to 2 inches thick may be applied to the structural slab after it has hardened for some time. There are several companies making a specialty of finishes of this kind; some of them have patents on their methods of doing work or on their equipment. In general this finish consists of roughening up the structural slab when it is poured and later chipping off the top to a depth of $\frac{1}{16}''$ to $\frac{1}{8}''$ so that all soft material is removed. Before applying the finish, the surface is washed with clean water, soaked for several hours, but the surface must be free of all pools of water when the finish is applied. The material and general methods for doing this work are the same as required for resurfacing concrete floors described in the following paragraph.

**Resurfacing Concrete Floors.** Often it is necessary to resurface a concrete floor. This may be due to poor workmanship or materials when the floor was originally finished, hard trucking over the floor, or other hard uses. The old concrete must be removed to a depth of at least 1 inch, the concrete roughened with picks or air drills, and

Fig. 69. Floor Roller in Operation
*Courtesy of Lone Star Cement Company, New York*

thoroughly cleaned of all loose material. A hose should be turned on the floor under a good pressure in order to remove all the loose material and dust. The new concrete surface should consist of 1 part cement, 1 part sand, and 1½ to 2¼ parts crushed stone or gravel. This material should be free of dust, clay and loam, and the sand should pass a ¼-inch screen and not more than 10 per cent of it pass through a No. 50 sieve. The stone or gravel should be graded from ¼ to ⅜ inch. The concrete should be mixed with the least amount

of water possible to work the concrete. If it is going to be hand-floated, the water should be limited to about $4\frac{1}{2}$ gallons; if machine-floated, to $3\frac{1}{2}$ to 4 gallons of water per bag of cement. Some hours before the concrete is poured, the surface should be wet and the water permitted to stand. When the finish is poured, the surface

Fig. 70. Machine Floor Finisher in Operation
*Courtesy of Lone Star Cement Company, New York*

should still be damp from the water that has been previously used, but no pools should be permitted. The surfaces should be washed over with grout composed of 1 part cement and 1 part sand and, following this operation, the concrete is spread over the area to be refinished. This material should be well tamped and rolled. In Fig. 69 a roller weighing over 500 pounds is shown in operation.

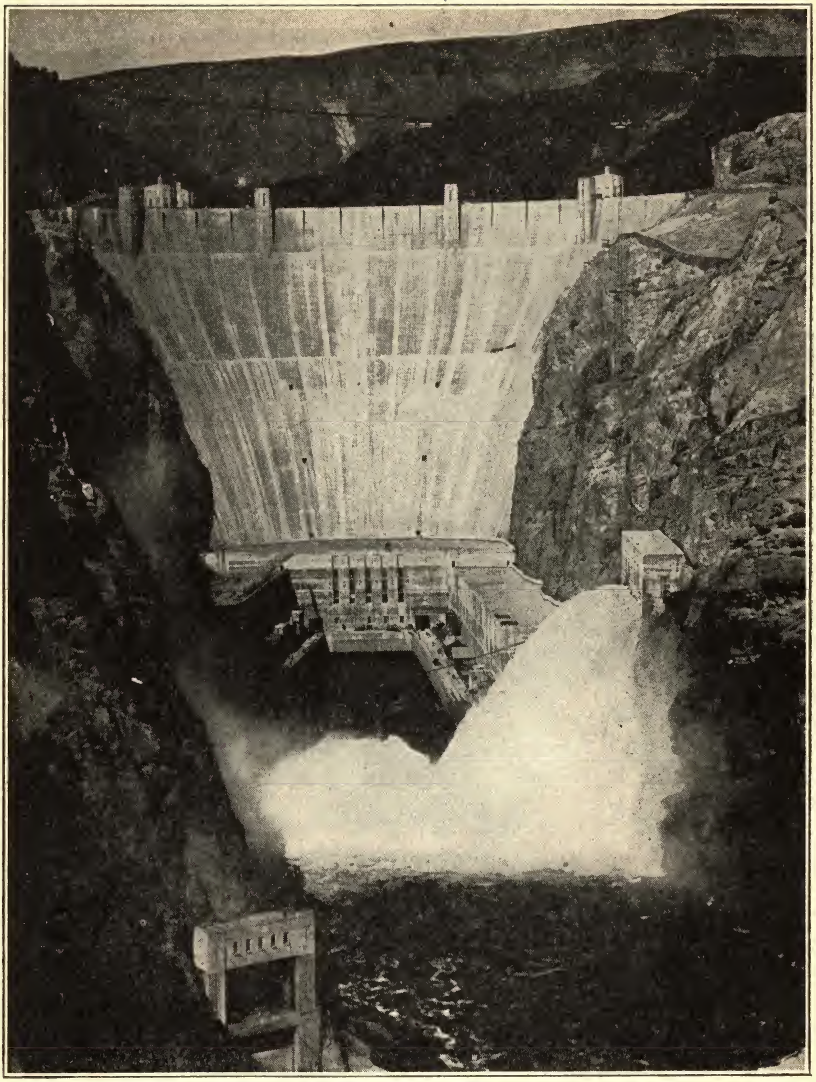
After thorough tamping, the floors should be carefully screeded and the surface worked over with a power machine or by hand. In

Fig. 70 is shown a machine for compacting and leveling off the top by means of a rotating steel disc. It brings to the surface sufficient mortar to be smoothed off with a steel trowel. As already stated, by use of this machine the amount of water can be reduced, and therefore there is less danger of scaling, dusting, or crazing in the finish. When finished by hand, this should be done carefully with

**Fig. 71. Finishing a Floor by Troweling**
*Courtesy of Lone Star Cement Company, New York*

wood floats and finally finished with a steel trowel. Fig. 71 shows a floor being troweled.

The curing of such a floor is a very important matter. The floor should not be permitted to dry out immediately after it is finished. The curing of the floor should be started at once. This may be done by placing burlaps over the floor and keeping them thoroughly soaked with water. This curing should be continued for several days, but should it be necessary to use the floor soon after it is placed (for example, in repair work in a factory) then quick-setting cement should be used.



THE CONCRETE ARCH, GRAVITY TYPE, AT BOULDER DAM IS THE HIGHEST IN THE WORLD

*Courtesy of Reclamation Bureau*

# CHAPTER VIII

## REVIEW OF BEAM DESIGN

**Beams.** A beam is a structural member which is subjected to loads and reactions acting transversely to its longitudinal axis. For the purpose of design, there are four general classes of beams: *simple, cantilever, continuous* and *restrained* or *fixed* beams. A better understanding of these terms and the definitions that follow can be obtained by carefully examining the "Beam Diagrams and Formulas for Various Loading Conditions."

A *simple* beam is a structural member supported at both ends. The load is usually applied perpendicular to its longitudinal axis.

A *cantilever* beam has one end rigidly anchored, or one end supported over two columns, the other projects outward for supporting a load.

A *continuous* beam is one supported by more than two supports.

A *restrained* beam is one rigidly connected at the ends.

**Slab.** In reinforced concrete work a slab is a flat plate of concrete reinforced with steel bars or mesh.

**Joist.** A joist is a small beam, usually placed 2 feet to 3 feet on centers, connected at the top with slabs 2 to 3 inches in thickness.

**Lintel.** A lintel is a beam that supports masonry over an opening.

**Spandrel.** A spandrel or spandrel beam is one that supports the exterior walls and the adjacent floor area of a building.

**Girder.** A beam supporting one or more beams is called a girder.

**Span.** A span is the distance from center to center of the supports of a slab, beam, or girder.

**Column.** A column is a vertical member subject to axial compression and having a length which is several times greater than the least lateral dimension. Columns are commonly used to support the floors of a building and for many other purposes.

**Pedestal.** A short compression member between a column and the top of a footing, used to distribute the column load on the footing.

**Wall Footing.** A wall footing is a slab of plain concrete, reinforced concrete, or other material, used to transmit the wall load to the soil on which the wall is constructed. The width of the footing is greater than the thickness of the wall.

**Column Footing.** A column footing is a flat plate of material built under a column to transmit the load from the column to the soil.

**Stress in Beams.** In designing reinforced concrete structures sufficient concrete and steel bars must be used to support safely the load for which it is designed. The dimensions of the concrete should conform to economical design, and bars of the proper size and shape should be placed correctly.

The stresses to be calculated for beams are: the bending moment, shear, and the reaction on the supports. These stresses will be explained briefly as a guide to the reader.

**Bending Moment.** The external loads on a beam tend to bend it and produce forces in the beam which are called bending stresses. The bending moment at any point in a beam is the summation of the moments, about that point, of all the external forces on one side of the given point. The bending moment in a beam varies from point to point and the maximum bending moment must be found for every beam. The maximum bending moment occurs at the point where the shear is zero or where its value changes from positive to negative or vice versa.

**Resisting Moment.** The external loads applied to a beam produce stresses in the fibers of the beam. The summation of the moments of these fiber stresses about the neutral axis of the beam is the resisting moment of the beam. The resisting moment must be equal to or greater than the maximum bending moment.

**Reactions.** The reactions of a beam are the supporting forces at the ends of the beam, and these forces are considered as acting upward. The sum of the reactions must be equal to the load to be supported. In the case of a simple beam uniformly loaded or loaded at the center, the reactions at each end are equal and each reaction is equal to one-half of the total load.

**Shear.** A load placed on a beam acts downward; the reaction acts upward. These two forces acting in opposite directions produce a tendency to cut the beam in two parts. This action is called shear.

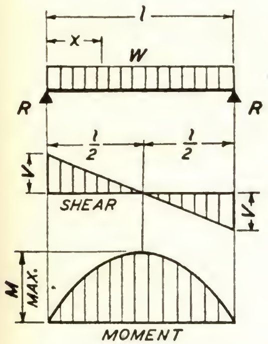
The shear at the end of a beam is always equal to the reaction of the beam. In a simple beam uniformly loaded, the maximum shear is at the ends and is equal at each end. At the center the shear is zero.

## BEAM DIAGRAMS AND FORMULAS FOR VARIOUS LOADING CONDITIONS

$W$ = load in pounds
$R$ = reaction in pounds
$l$ = span in inches
$V$ = vertical shear in pounds
$M$ = bending moment in inch-pounds
$x$ = any point on the beam, in inches

### 1. Simple Beam

*Uniformly Distributed Load*




Maximum shear, at ends, $V = \frac{W}{2}$

Shear at any point $V = \frac{W}{2l}(l - 2x)$

Reaction, $R = \frac{W}{2}$

Bending Moments:

At center, $M\ max. = \frac{Wl}{8}$

At any point, $M = \frac{Wx}{2l}(l - x)$

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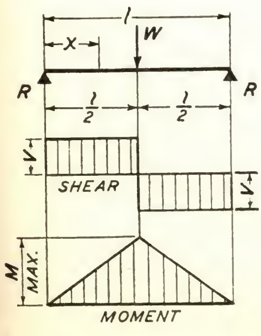
### 2. Simple Beam

*Concentrated Load at Center*




Maximum shear, at ends, $V = \frac{W}{2}$

Shear at any point, $V = \frac{W}{2}$

Reaction, $R = \frac{W}{2}$

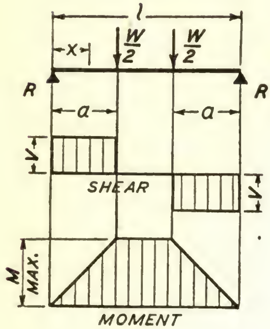
Bending Moments:

At center, $M\ max. = \frac{Wl}{4}$

When $x$ is less than $\frac{l}{2}$, $Mx = \frac{Wx}{2}$

## BEAM DIAGRAMS AND FORMULAS FOR VARIOUS LOADING CONDITIONS—Continued

### 3. Simple Beam




*Two Concentrated Loads, Symmetrically Placed*

Shear between load and supports $= V = \frac{W}{2}$

Shear between loads, $V = 0$

Reaction, $R = \frac{W}{2}$

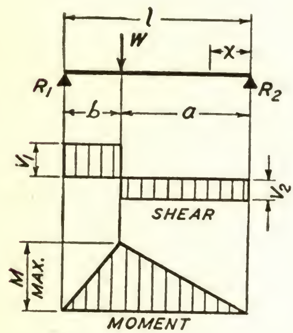
Bending Moments:

Between loads, $M = \frac{Wa}{2}$

$M = \frac{Wx}{2}$ if $x$ is less than $a$

---

### 4. Simple Beam




*Concentrated Load at Any Point*

$R_1 = \frac{Wa}{l}$, maximum if $a$ is greater than $b$

$R_2 = \frac{Wb}{l}$, maximum if $b$ is greater than $a$

Shears same as reactions.

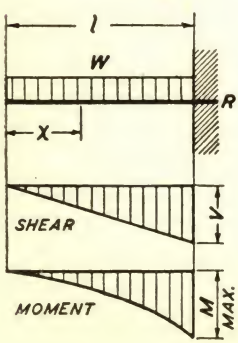
Bending Moments:

$M\ max. = \frac{Wab}{l}$

$Mx = \frac{Wbx}{l}$, if $x$ is less than $a$

---

### 5. Cantilever Beam




*Uniformly Distributed Load*

Maximum shear, $V = W$

Shear at any point $V = W\frac{x}{l}$

Reaction, $R = V$

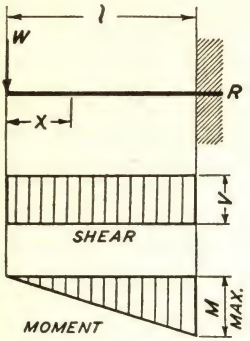
Bending Moments:

Maximum at support, $M\ max. = \frac{Wl}{2}$

At any point, $Mx = \frac{Wx^2}{2l}$

## BEAM DIAGRAMS AND FORMULAS FOR VARIOUS LOADING CONDITIONS—Continued

### 6. Cantilever Beam




*Concentrated Load at Free End*

Maximum shear, $V = W$

Shear at any point, $V = W$

Reaction, $R = W$

Bending Moments:

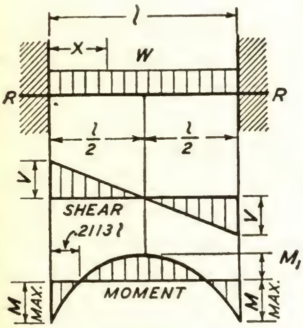
At support, $M\ max. = Wl$

At any point, $M = Wx$

---

### 7. Beam Fixed at Both Ends

*Uniformly Distributed Load*




Maximum shear, at ends, $V = \frac{W}{2}$

Shear at any point, $V = \frac{W}{l}\left(\frac{l}{2} - x\right)$

Reaction, $R = \frac{W}{2}$

Bending Moments:

At end, $M\ max. = \frac{Wl}{12}$

At center, $M = \frac{Wl}{24}$

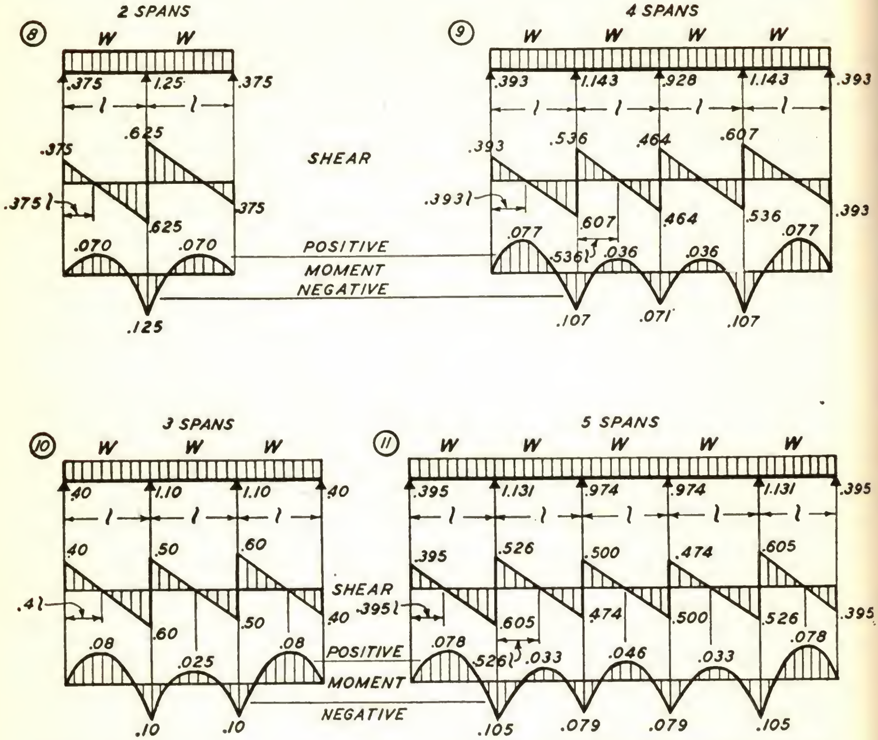
At any point, $M = \frac{W}{12}\left(6x - l - \frac{6x^2}{l}\right)$

---

In examining the diagrams for moments, shears and reactions, the great difference between the moment for a simple beam (diagram 1) and the moments for a beam with fixed ends (diagram 7) is noted. For the simple beam the maximum moment at the center is $\frac{Wl}{8}$ and at the ends it is zero. In the case of the fixed beam, the moment at the center is only $\frac{Wl}{24}$ and at the ends it is $\frac{Wl}{12}$. In designing a rein-

## CONTINUOUS BEAM DIAGRAMS, FREE ENDS UNIFORMLY DISTRIBUTED LOAD

Reactions and shears in terms of $W$
Moments in terms of $Wl$




forced concrete beam with fixed ends, half of the steel area required for the center moment must extend into the supports, the other half may be turned up at the point where the moment changes from plus to minus (point of contraflexure) and additional bars placed in the top of the beam for the end moments. In practice, the conditions for such a beam are seldom found.

An explanation of the diagrams for continuous beams for five spans will be given to assist the reader in reviewing this subject. The maximum positive moment, which occurs in the end span, is $M = .078Wl$. In the center span it is $M = .046Wl$, while in the

intermediate spans between the ends and the center it is $M=.033Wl$. The maximum negative moment occurs over the support between the first and second spans and between the fourth and fifth spans, and $M=.105Wl$. The other negative moments are $M=.079Wl$.

In examining the diagram for five spans it will be noted that the load on the beam is uniform, but that the shear is not the same at all points. At the end supports it is $V=.395W$; at the left side of the second support it is $V=.605W$. Also it is noted that $.395+.605=1.000$ or unity.

### BENDING MOMENTS

The bending moments commonly used in designing reinforced concrete are as follows:

Beams and slabs of one span, $M=\frac{Wl}{8}$

Beams and slabs continuous for two spans:

Maximum positive near center, $M=\frac{Wl}{10}$

Negative moment over the center support, $M=\frac{Wl}{8}$

Beams and slabs continuous for more than two spans:

The maximum positive moment near the centers of end spans and negative moment over first interior support is

$$M=\frac{Wl}{10}$$

The maximum positive moment near the center and the maximum negative moment at the support of interior bays is

$$M=\frac{Wl}{12}$$

Experts, under favorable conditions of loading, materials, and expert workmanship, use $M=\frac{Wl}{16}$ for wholly continuous beams.

The recommended moments given here do not agree with the theoretical diagrams for continuous beams, but do agree for the simple span. These recommended moments are more conservative than the moments shown in the diagrams and are the ones that should be used.

## REINFORCED CONCRETE DESIGN

**Historical Review.** In 1904 a Joint Committee on Concrete and Reinforced Concrete, consisting of representatives of the American Society for Testing Materials, American Society of Civil Engineers, American Railway Engineers Association, American Concrete Institute, and the Portland Cement Association, was organized to standardize the science. The original committee presented its final report in 1916 on Recommended Practice for Concrete and Reinforced Concrete. In 1919 another committee from the same societies was formed to prepare specifications for concrete and reinforced concrete. With the 1916 report as a basis, this committee formulated the specifications and made its report in 1924. This report was generally adopted. In 1927 the Building Code Committee of the American Concrete Institute and the Committee on Standard Practice of the Concrete Reinforcing Steel Institute united and wrote the 1928 Joint Standard Building Code for Reinforced Concrete, usually referred to as the 1928 Joint Code of the American Concrete Institute. This code is based on the 1924 Joint Committee report. The points of difference are largely a matter of simplification of formulas. Recently this committee submitted a revised building code which was adopted as a tentative standard by the American Concrete Institute in 1936, but it has not been finally adopted. A third Joint Committee was organized in 1930 to study the advances made in the use and design of concrete and reinforced concrete since the previous report. In 1937 this Committee issued a Progress Report and submitted Standard Specifications for Concrete and Reinforced Concrete.

The recommendations of these specifications and codes have been used largely in this book.

**General Theory of Flexure.** The theory of flexure for beams, as developed in texts on strength of materials, applies only to homogeneous beams (such as steel or wood beams) and cannot be applied directly to reinforced concrete beams. Special rules and formulas for designing reinforced concrete work have been devised. These are somewhat complicated, but are easy of solution with the help of charts and tables. The derivation of the formulas on which the rules for designing are based, will be made as clear and simple as possible. The following notations and symbols are used throughout the text.

### Symbols Defined

$b$ = Breadth of concrete beam or width of flange of T-beam

$d$ = Depth from compression face to center of gravity of the steel

$A_s$ = Area of the steel

$p$ = Ratio of area of steel to area of concrete above the center of gravity of the steel $= \dfrac{A_s}{bd}$

$E_s$ = Modulus of elasticity of steel = 30,000,000#/□″

$E_c$ = Modulus of elasticity of concrete

$n = \dfrac{E_s}{E_c}$ = Ratio of the moduli

$f_s$ = Tensile stress per unit of area in steel

$f_c$ = Compressive stress per unit of area in concrete at the outer fiber of the beam

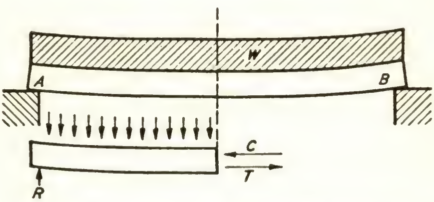



Fig. 72. Diagram of Beam Carrying Uniformly Distributed Load

$\epsilon_s$ = Deformation per unit of length in the steel

$\epsilon_c$ = Deformation per unit of length in outer fiber of concrete

$k$ = Ratio of dimension from neutral axis to outer fiber of the concrete to the total effective depth $d$

$j$ = Ratio of dimension from steel to center of compressive stresses to the total effective depth $d$

$z$ = Distance from compressive face to center of compressive stresses

$C$ = Summation of horizontal compressive stresses

$M$ = Resisting moment of a section

$T$ = Summation of horizontal tensile stresses.

**Statics of Plain Homogeneous Beams.** As a preliminary to the theory of the use of reinforced concrete in beams, a very brief discussion will be given of the statics of an ordinary homogeneous beam, made of a material whose moduli of elasticity in tension and compression are equal. Let $AB$, Fig. 72, represent a beam carrying a uniformly distributed load $W$; then the beam is subjected to transverse stresses. Take one half of the beam as a "free body" in space, Fig. 72. Then this half of the beam is in equilibrium under the action of the external load on this part of the beam and the internal

fiber stresses of the rest of the beam acting on this part. In Fig. 72 the load $W$ is represented by the series of small, equal, and equally spaced vertical arrows pointing downward. The reaction of the the support against the beam is an upward force shown at the left. The fiber stresses are represented by their resultants $C$ and $T$. Taking moments at the right end of the "free body," the external loads produce a clockwise moment. As the free body is in equilibrium this moment must be balanced by a counterclockwise moment. This balancing moment is produced by the force $C$ acting to the left and the force $T$ acting to the right, as shown in Fig. 72. This means that the force $C$ is a compressive force, and the force $T$ is a tensile force. As $C$ and $T$ are the only horizontal forces acting, and the body is in equilibrium, they must be equal and consequently form a couple.

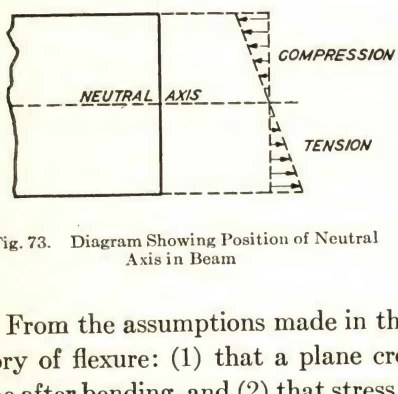



Fig. 73. Diagram Showing Position of Neutral Axis in Beam

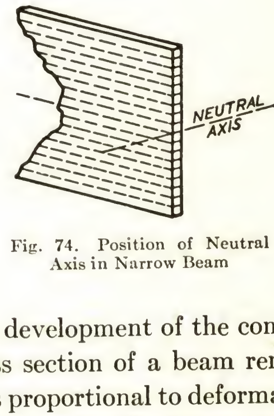



Fig. 74. Position of Neutral Axis in Narrow Beam

From the assumptions made in the development of the common theory of flexure: (1) that a plane cross section of a beam remains plane after bending, and (2) that stress is proportional to deformation, it is known that the fiber stress in a beam varies directly as the distance from the neutral axis, the stress in the outer fibers being the greatest. The intensity of this stress is given by the familiar formula $f=\frac{Mc}{I}$ where $M=$ bending moment, $c=$ distance of fiber from neutral axis, and $I$ is the moment of inertia of the section with respect to the neutral axis. In the case of simple bending, the neutral axis or point of zero stress is at the center of gravity of the section. For a rectangular section this is at the center of the height. Then for the beam shown in Fig. 72 the stresses are as shown in Fig. 73. Fig. 74 shows a narrow portion of the beam, having full length and depth but so narrow it includes only one set of fibers, one above the other. It

also shows the location of the neutral axis. Since the section was taken at the center of the beam (Fig. 72), the shear on the section is zero. Shearing stresses will be discussed later.

A beam may be constructed of plain concrete; but its strength will be very small, since the tensile strength of concrete is comparatively insignificant. Reinforced concrete utilizes the great tensile strength of steel in combination with the compressive strength of concrete. It should be realized that two of the most essential qualities are *compression* and *tension*, and that, other things being equal, the cheapest method of obtaining the necessary compression and tension is the most economical.

**Economy of Concrete for Compression.** The ultimate compressive strength of concrete is generally 2,000#/□″, or over. Just for numerical comparison, use a working stress of 800#/□″. On the average, concrete costs about 70 cents per cubic foot, including the cost of forms. On the other hand, steel placed in the work costs about 5 cents per pound. It will weigh 480# per cubic foot; therefore, the steel costs $24.00 per cubic foot, or 34 times as much as an equal volume of concrete or an equal cross section per unit of length. But the steel can safely withstand a compressive stress of 16,000#/□″, which is 20 times the safe working load on concrete. Since, however, a given volume of steel costs 34 times an equal volume of concrete, the cost of a given compressive resistance in steel is $^{34}/_{20}$ or 1.7 times the cost of that resistance in concrete. Of course, the unit prices of concrete and steel will vary with circumstances. The advantage of concrete over steel for compression may be somewhat greater or less than the ratio given above, but the advantage is almost invariably with the concrete. There are many other advantages which will be discussed later.

**Economy of Steel for Tension.** The ultimate tensile strength of ordinary concrete is rarely more than 200#/□″. With a factor of safety of 4, this would allow a working stress of only 50#/□″. This is generally too small for practical use and certainly too small for economical use. On the other hand, steel may be used with a working stress of 20,000#/□″, which is 400 times that allowable for concrete. Using the same unit values for the cost of steel and concrete as given in the previous paragraph, even if steel costs 34 times as much as an equal volume of concrete, its real tensile value economically is $^{400}/_{34}$

or 11.8 times as great. Any reasonable variation from the above unit values cannot alter the essential truths of the economy of steel for tension and of concrete for compression. In a reinforced-concrete beam, the steel is placed in the tension side of the beam. Usually it is placed 1 to 2 inches from the outer face, with the double purpose of protecting the steel from corrosion or fire, and also to better insure the union of the concrete and the steel. But the concrete below the steel is not considered in the numerical calculations. The concrete between the steel and the neutral axis performs the very necessary function of transmitting the stress from the steel to the concrete. Although the concrete in the lower part of the beam is, theoretically, subject to the tension of transverse stress and does actually contribute its share of the tension when the stresses in the

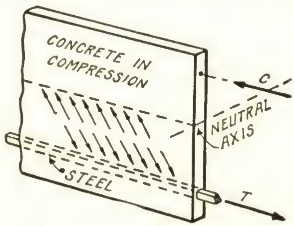



Fig. 75. Diagram Showing Transmission of Tension in Steel to Concrete

beam are small, the proportion of the necessary tension which the concrete can furnish when the beam is heavily loaded is so very small that it is usually ignored, especially since such a policy is on the side of safety, and also since it greatly simplifies the theoretical calculations and yet makes very little difference in the final result. Therefore, it is considered that in a section of the beam, Fig. 75, the concrete above the neutral axis is subject to compression, and that the tension is furnished entirely by the steel.

**Elasticity of Concrete.** In computing the transverse stresses in a wood beam or steel **I** beam, it is assumed that the material obeys Hooke's law, i.e., that the intensity of the stress is proportional to the strain within the elastic limit. The ratio of the stress to the strain is therefore uniform and this ratio is called the modulus of elasticity. Experiments have shown this to be so nearly true that it is accepted as a mechanical law. Unfortunately, concrete is not so uniform in its behavior. Unlike steel, the elastic line for concrete is curved almost

from the beginning, deformation increasing faster than the stress. On the average, the stress-strain curve in compression may be taken as a parabola with the vertex at the point of ultimate strength. However, within the range of working stresses, the variation of the curve from a straight line is so small that it is the custom to assume that the modulus of elasticity is a constant within the limits of the working stresses.

The test to determine the elasticity may be made as follows: a gage reading to —one ten-thousandth part of an inch is fastened to a short concrete column section by means of pointed screws of hardened steel. These screws are set a known distance apart, usually 8 inches. The column section is then put in a testing machine and the load is applied in increments. At each increment of load the gage is read, indicating the amount that the column has been shortened. The load is applied until the concrete fails. The values tabulated below are representative of the behavior of ordinary concrete.

If the load is released when the stress is about 600#/□″, the specimen usually will have a small set indicating imperfect elasticity. On reapplying the load a straighter line than the first will be obtained, with much less permanent set following the release of the load. After several repetitions, the elastic line will become a straight line up to the load applied, and there will be no additional set. However, there is a stress, about 50 per cent of the ultimate, beyond which repetition of stress will continue to add to the deformation.

Few tests have been made to determine the elasticity of concrete in tension, but the tests made indicate that the moduli of elasticity in tension and compression are practically equal, and are so considered. Fig. 76 illustrates a typical stress-strain curve for concrete in compression and tension. The straight line in Fig. 76 represents the initial modulus of elasticity.

| Pressure, per Square Inch | Compression, Proportion of Total Length | Pressure, per Square Inch | Compression, Proportion of Total Length |
|---|---|---|---|
| 200# | .00007 | 1,400# | .00060 |
| 400# | .00015 | 1,600# | .00071 |
| 600# | .00022 | 1,800# | .00086 |
| 800# | .00030 | 2,000# | .00102 |
| 1,000# | .00038 | 2,200# | .00126 |
| 1,200# | .00048 | 2,380# | .00170 |

It was formerly quite common to base the computation of formulas on the assumption that the curve of compression is a parabola. The development of the theory is correspondingly complex, but as stated, for a compression of 600 or even 1,000#/□″, the parabolic curve is not very different from a straight line. A comparison of the results based on the strict parabolic theory with those based on the more simple straight-line formulas shows that the difference is small and often not greater than the uncertainty as to the true strength of the concrete. The straight-line theory will, therefore, be used exclusively in the demonstrations which follow.

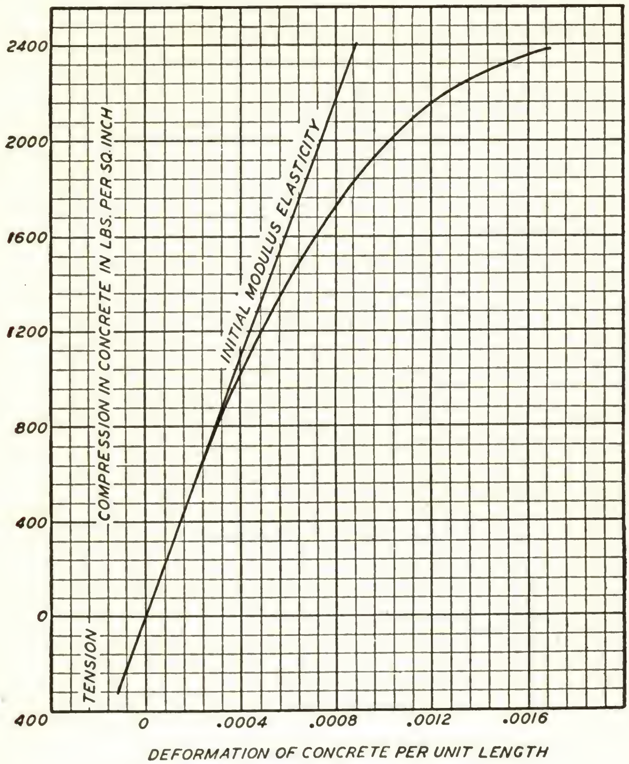



Fig. 76. Stress-Deformation Curve of Concrete

**Theoretical Assumptions.** The theory of reinforced-concrete beams is based on the assumptions that:

(1) Any section of the beam which is plane before bending is plane after bending.

(2) The deformation produced in the material is proportional to the stress applied.

(3) The concrete and steel stretch together without breaking the bond between them. This is absolutely essential.

(4) The loads are applied at right angles to the axis of the beam. The usual vertical gravity loads supported by a horizontal beam fulfill this condition.

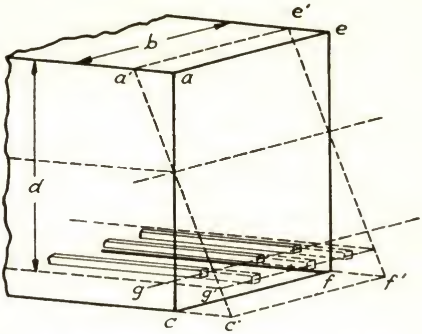



Fig. 77. Plane Section of Beam before and after Bending

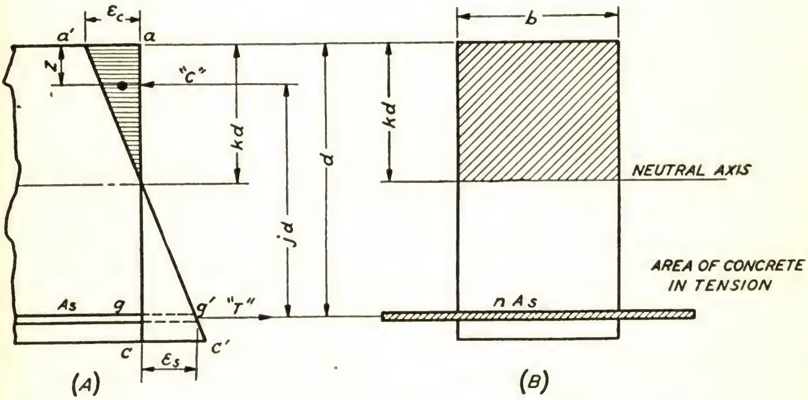



Fig. 78—Beam Subjected to Moment—(*A*) Deformation Diagram; (*B*) Transformed Section

(5) There is no resistance to free horizontal motion. This condition is seldom, if ever, exactly fulfilled in practice. The more rigidly the beam is held at the ends, the greater will be its strength above that computed by the simple theory.

In Fig. 77 is shown in a very exaggerated form, the essential meaning of assumption (*1*). The section *aefc* in the unstrained condition is changed to the plane *a'e'f'c'* when the load is applied. The compression at the top equals *aa'* equals *ee'*. The neutral axis is

unchanged. The concrete at the bottom is stretched an amount equal to $cc'$ equals $ff'$, while the stretch in the steel equals $gg'$. The compression in the concrete between the neutral axis and the top is proportional to the distance from the neutral axis.

In Fig. 78, at ($A$), is given a side view of the beam, with special reference to the deformation of the fibers. Since the fibers between the neutral axis and the compressive face are compressed proportionally, then, if $aa'$ represents the linear compression of the outer fiber, the shaded lines represent, at the same scale, the compression of the intermediate fibers.

**Summation of Compressive Forces.** The summation of compressive forces evidently equals the sum of all the compressions, varying from zero to the maximum compressive stress $f_c$ at the extreme upper fiber, where the linear compression is $\epsilon_c$. The average unit compressive stress is, therefore, $\frac{1}{2}f_c$. Since $k$ is the ratio of the distance from the neutral axis to the upper fiber to the total effective depth $d$, that distance equals $kd$; the breadth of the beam is $b$. Therefore the summation of all the compression in all the fibers is the summation of their number (represented by their area) times their *average* intensity, and

$$C = \tfrac{1}{2}f_c bkd \tag{7}$$

**Center of Gravity of Compressive Forces.** The center of gravity of compressive forces is sometimes called the *centroid of compression.* It here coincides with the center of gravity of the triangle, which is at one-third the height of the triangle from the upper face. Therefore

$$z = \tfrac{1}{3}kd \tag{8}$$

The ratio of the dimension from the steel to the center of the compressive stress to the dimension $d$ equals $j$ and, therefore, the dimension between the centroids of the tensile and the compressive forces equals $jd$, which equals $(d-z)$.

**Position of the Neutral Axis.** From the laws of mechanics, it is known that $C$ must be equal and opposite to $T$, Fig. 78, at ($A$). Disregarding the tension furnished by the concrete, below the neutral axis, $T$ equals $A_s f_s$ equals $pbdf_s$. (See the definition of $p$.) Also from Equation (7), $C$ equals $\frac{1}{2}f_c bkd$. Therefore

$$pbdf_s = \tfrac{1}{2}f_c bkd$$

Dividing by $pbdf_c$
$$\frac{f_s}{f_c}=\frac{k}{2p} \tag{9}$$

It follows from the assumption of plane sections, see Figs. 77 and 78, that
$$\frac{\epsilon_s}{\epsilon_c}=\frac{d-kd}{kd}=\frac{(1-k)}{k} \tag{10}$$

Also by definition of the modulus of elasticity
$$\frac{\epsilon_s}{\epsilon_c}=\frac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}}$$

and as $\frac{E_s}{E_c}=n$, also by definition,

Then
$$\frac{\epsilon_s}{\epsilon_c}=\frac{f_s}{nf_c} \tag{11}$$

Substituting the value of $\frac{f_s}{f_c}$ from Equation (9) in Equation (11)
$$\frac{\epsilon_s}{\epsilon_c}=\frac{k}{2pn} \tag{12}$$

Equating (10) and (12)
$$\frac{k}{2pn}=\frac{1-k}{k} \tag{13}$$

Multiplying by $2kpn$,
$$k^2=2pn-2kpn \tag{13a}$$

Transposing
$$k^2+2kpn=2pn$$

adding $p^2n^2$ to both sides,
$$k^2+2kpn+p^2n^2=2pn+p^2n^2$$

Taking the square root,
$$k+pn=\sqrt{2pn+p^2n^2}$$

or
$$k=\sqrt{2pn+p^2n^2}-pn \tag{14}$$

**The Transformed Section.** The general method and equations used for homogeneous beams may be applied in deriving the formulas for reinforced concrete by use of the "transformed section": In the "transformed section" the steel is replaced by a hypothetical concrete which has the same modulus of elasticity as the compression concrete and in addition has the ability to carry tension. The steel is replaced by an amount of this hypothetical concrete equal to $n$ times the area of the steel. The concrete section replacing the steel has the same

**TABLE XV**

**Value of *k* for Various Values of *n* and *p***

**(Straight-Line Formulas)**

| $n$ | $p$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | .020 | .018 | .016 | .014 | .012 | .010 | .008 | .006 | .004 | .003 |
| 10 | .464 | .446 | .427 | .407 | .385 | .358 | .328 | .292 | .246 | .216 |
| 12 | .493 | .476 | .457 | .436 | .412 | .385 | .353 | .314 | .266 | .235 |
| 15 | .531 | .513 | .493 | .471 | .446 | .418 | .384 | .343 | .291 | .258 |
| 20 | .580 | .562 | .542 | .519 | .493 | .463 | .428 | .384 | .328 | .292 |
| 30 | .649 | .631 | .611 | .588 | .562 | .531 | .493 | .446 | .384 | .344 |

deformation as the steel and acts in the same plane as the steel. Fig. 78 at ($B$), shows the *transformed section* of a beam.

The neutral axis passes through the center of gravity of the cross section. Then in Fig. 78, at ($B$), taking moments of the areas about the neutral axis

$$bkd \times \frac{kd}{2} = nA_s(d-kd)$$

Substituting $pbd$ for $A_s$

$$\frac{bk^2d^2}{2} = npbd^2(1-k)$$

dividing by $\frac{bd^2}{2}$ $\qquad k^2 = 2pn - 2kpn$

This equation is identical with Equation (**13a**). Consequently, solving this expression for $k$ will result in Equation (**14**) or

$$k = \sqrt{2pn + p^2n^2} - pn$$

The values of $k$ for various values of $n$ and $p$ are given in Table XV. These values cover the range of the values usually used in design.

The dimension $jd$ from the center of the steel to the centroid of the compression in the concrete equals ($d$–$z$). See Fig. 78, at ($A$), and the paragraph under Equation (8).

Therefore

$$j = \frac{d-z}{d} = \frac{d - \frac{1}{3}kd}{d} = 1 - \frac{1}{3}k \qquad \textbf{(15)}$$

The corresponding values for $j$ have been computed for the several values of $p$ and $n$, as shown in Table XVI.

These several values for $k$ and $j$ which correspond to the various

### TABLE XVI

**Value of *j* for Various Values of *n* and *p***
**(Straight-Line Formulas)**

| $n$ | $p$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | .020 | .018 | .016 | .014 | .012 | .010 | .008 | .006 | .004 | .003 |
| 10 | .845 | .851 | .858 | .864 | .872 | .881 | .891 | .903 | .918 | .928 |
| 12 | .836 | .841 | .848 | .855 | .863 | .872 | .882 | .895 | .911 | .922 |
| 15 | .823 | .829 | .836 | .843 | .851 | .861 | .872 | .886 | .903 | .914 |
| 20 | .807 | .813 | .819 | .827 | .836 | .846 | .857 | .872 | .891 | .903 |
| 30 | .784 | .790 | .796 | .804 | .813 | .823 | .836 | .851 | .872 | .885 |

values for $p$ and $n$ are shown in Fig. 79, which is especially useful when the required values of $k$ and $j$ must be obtained by interpolation.

**Example 1.** Assume $n=15$ and $p=.01$; how much are $k$ and $j$?

*Solution.* In Fig. 79 follow up the vertical line on the diagram for the steel ratio, $p=.010$, to the point where it intersects the $k$ curve for $n=15$; the intersection point is $\frac{9}{10}$ of one of the smallest divisions above the .40 line, as shown on the scale at the left; each small division is .020, and, therefore, the reading is $\frac{9}{10}\times.020=.018$ plus .400 or .418, the value of $k$. Similarly the .010 $p$ line intersects the $j$ curve for $n=15$ at a point slightly above the .860 line or at .861.

**Example 2.** Assume $n=16$ and $p=.0082$; how much are $k$ and $j$?

*Solution.* One must imagine a vertical line (or perhaps draw one) at $\frac{2}{5}$ of a space between the .0080 and .0085 vertical lines for $p$. This line would intersect the line for $n=15$ at about .388; and the line for $n=18$ at about .416; one-third of the difference (.028) or .009, added to .388 gives .397, the interpolated value. Although this is sufficiently close for practical purposes, the precise value (.398) may be computed from Equation (14). Similarly the value of $j$ may be interpolated as .867. Although the values of these ratios have been computed to three significant figures (thousandths), the uncertainties as to the actual character and strength of the concrete used will make it useless to obtain these ratios closer than the nearest hundredth.

**Values of Ratio of Moduli of Elasticity.** The modulus of elasticity for steel is fairly constant at between 29,000,000 and 30,000,000#/□″. The latter value is used in all calculations in reinforced concrete design. The value of the modulus of elasticity for concrete, unlike steel, varies considerably as determined for different grades of concrete. The value of 3,000,000#/□″ is perhaps the most accepted value for ordinary stone concrete at 28 days. As a rule, the older the concrete, the harder and stiffer it becomes, with a consequent increase in the modulus. In design calculations, a lower value of the modulus is used. The modulus of elasticity is commonly taken to have the value 1,000$S$, where $S$ is the ultimate strength at 28 days.

## TABLE XVII

### Modulus of Elasticity of Some Grades of Concrete

| Kind of Concrete | Age (Days) | Mixture | $E_c$ | $n$ |
|---|---|---|---|---|
| Cinder Concrete......... | 28 | 1:2:4 | 1,000,000 | 30 |
| 1,500# Stone Concrete.... | 28 | 1:3:5 | 1,500,000 | 20 |
| 2,000# Stone Concrete.... | 28 | 1:2:4 | 2,000,000 | 15 |
| 2,500# Stone Concrete.... | 28 | 1:2½:3½ | 2,500,000 | 12 |
| 3,000# Stone Concrete.... | 28 | 1:1¾:3 | 3,000,000 | 10 |

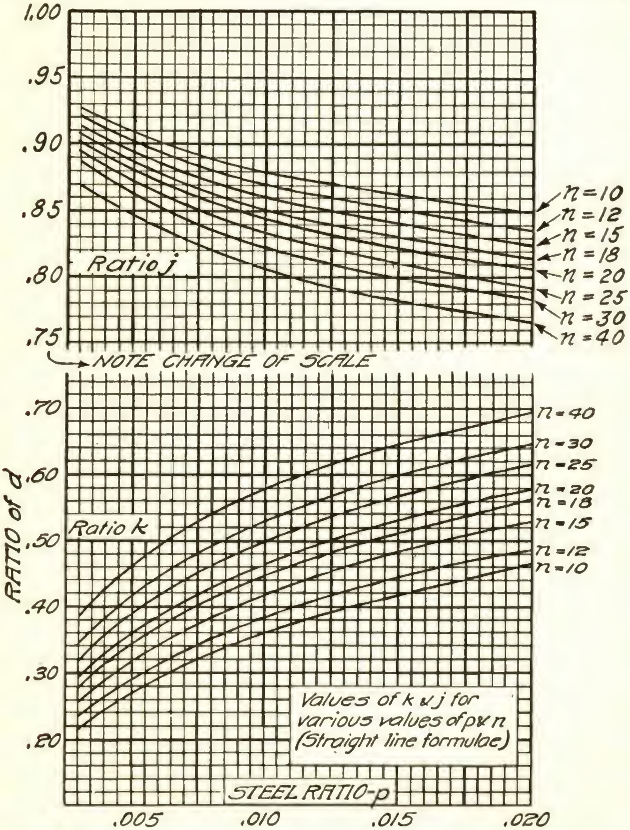



Fig. 79. Curves Giving Values of $k$ and $j$ for Various Values of $p$ and $n$. Values used for these curves will be found in Tables XV and XVI

This formula gives approximately the same values as those of the Joint Committee, and these values, which will be used in this text, are given in Table XVII.

**Ratio of Steel.** The previous calculations have been made as if the ratio of the steel might be varied almost indefinitely. While there is considerable freedom of choice, there are limitations beyond which it is useless to pass; and there is always a most economical ratio, depending on the conditions. The theoretical value of $p$ will now be determined. Substituting in Equation (9) the value of $k$ in Equation (14), and solving for $p$

$$p=\frac{f_c}{2f_s}\left(\sqrt{2pn+p^2n^2}-pn\right)$$

or

$$p=\frac{f_c}{2f_s}\sqrt{2pn+p^2n^2}-\left(\frac{f_c}{2f_s}\right)pn$$

and

$$p+\frac{f_cpn}{2f_s}=\frac{f_c}{2f_s}\sqrt{2pn+v^2n^2}$$

Multiplying by $2\frac{f_s}{f_c}$,

$$\frac{2pf_s}{f_c}+pm=\sqrt{2pn+p^2n^2}$$

Squaring both sides,

$$\left(\frac{2pf_s}{f_c}\right)^2+\frac{4p^2f_sn}{f_c}+p^2n^2=2pn+p^2n^2$$

Multiplying by $f_c^2$,

$$4p^2f_s^2+4p^2f_snf_c=2pnf_c^2$$

Dividing by $4p$,

$$pf_s^2+pf_snf_c=\tfrac{1}{2}nf_c^2$$

Dividing by $f_s(f_s+nf_c)$,

$$p=\frac{nf_c^2}{2f_s(f_s+nf_c)}=\frac{f_c}{2f_s}\;\frac{f_cn}{(f_s+f_cn)} \tag{16}$$

Equation (16) shows that the ratio of steel cannot be selected at random, if the steel and concrete are to be stressed to their maximum allowable fiber stresses under the design load. When the ratio of steel used is that given by Equation (16), it is called the "theoretical" ratio and the design is said to be a balanced design. For example, consider a "2,500-lb." concrete whose modulus of elasticity is 2,500,000#/□″, with a working compressive stress $f_c=1000$#/□″ in

conjunction with a tensile stress of 20,000#/□″ in the steel. The values of $f_c$, $f_s$, and $n$ are 1,000; 20,000 and 12 respectively, which, substituted in Equation (16) give

$$p=\frac{1,000}{2\times 20,000}\times\frac{1,000\times 12}{(20,000)+(1,000\times 12)}=\frac{3}{320}=.0094$$

The "theoretical" ratio is not, necessarily, the most economical or the most desirable percentage to use. For a beam of given size, some increase of strength may be obtained by using a higher ratio of steel; or for a given strength, or load capacity, the depth may be somewhat decreased by using a higher ratio of steel. The decrease in height, making possible a decrease in the total height of the building for a given clear headroom between floors, *may* justify the increase in the ratio of steel, but that is a matter of economics.

**Resisting Moment.** The moment which resists the action of the external forces is measured by the product of the distance from the center of gravity of the steel to the centroid of compression of the concrete (the distance, $jd$, Fig. 78 at ($A$)), times the total compression of the concrete, or times the tension in the steel. As the compression in the concrete and the tension in the steel are equal, it is only a matter of convenience to express this product in terms of the tension in the steel. Therefore, adopting the notation already mentioned, since the steel tension equals the area $A_s$ times the unit stress $f_s$,

$$M=A_sf_s(jd) \tag{17}$$

But since the computations are frequently made in terms of the dimensions of the concrete and of the ratio of the reinforcing steel, and since $A_s=pbd$, it may be more convenient to write the equation

$$M=(pbdf_s)jd \tag{18}$$

Equation (7) gives the total compression in the concrete. Multiplying this by the distance from the steel to the centroid of compression $jd$, produces another equation for the moment

$$M=\frac{1}{2}(f_cbkd)jd \tag{19}$$

When the ratio of steel used agrees with that computed from Equation (16), then Equations (18) and (19) will give identically the same results; but when the ratio of steel is selected arbitrarily, as is frequently done, then the proposed section should be tested by both equations. When the ratio of steel is larger than that required by Equation (16),

the concrete will be compressed more than is intended before the steel attains its normal tension. On the other hand, a lower ratio of steel will require a higher unit tension in the steel before the concrete attains its normal compression. When the discrepancy between the ratio of steel assumed and the theoretical value is very great, the stress in the steel, or the concrete, may become dangerously high when the stress in the other element, on which the computation may have been made, is only normal.

Using Equation (16) for 1:2:4 concrete, with $n$ equal to 15, and with working values of $f_c$ as 800#/□″ and $f_s$ as 20,000#/□″, the ratio of steel equals

$$p=\frac{800}{2\times 20{,}000}\times\frac{800\times 15}{(20{,}000)+(800\times 15)}=.0075$$

By interpolation in Table XV, $k$ equals .375 and $j$ therefore equals .875. Substituting these values in Equations (18) and (19) respectively,

$$M=(.0075\times 20{,}000\times .875)bd^2 \text{ and } M=(\tfrac{1}{2}\times 800\times .375\times .875)bd^2, \text{ or}$$
$$M=131bd^2$$

The ratio of steel computed from Equation (16) has been called *the theoretical ratio*, because, when the beam is loaded with a definite maximum load, it is the ratio which will develop the maximum allowable stresses in the concrete and the steel at the same time. The real meaning of this is best illustrated by a numerical example using another ratio.

Assume that the ratio of steel is exactly double that used above, or that $p$ equals $2\times .0075=.015$. From Table XV when $n$ equals 15 and $p$ equals .015, by interpolation $k$ equals .482, then $z$ equals .161 and $j$ equals .839. Substituting these values in both Equation (18) and Equation (19), $M_s=(.015\times 20{,}000\times .861)bd^2=258bd^2$ and $M_c=(\frac{1}{2}\times 800\times .482\times .839)bd^2=162bd^2$. The interpretation of these two equations, and also of the equation $M=131bd^2$ is as follows:

Assume a beam of definite dimensions $b$ and $d$, made of concrete whose modulus of elasticity is $\frac{1}{15}$ that of the modulus of elasticity of the reinforcing steel; assume that it is reinforced with steel having a cross-sectional area equal to $.0075bd$. Then, when it supports a load which develops a moment of $131bd^2$, the tension in the steel will equal 20,000#/□″, and the compression in the concrete will equal 800#/□″ at the outer fiber. Assume that the area of the steel is

exactly doubled. One effect of this is to lower the neutral axis ($k$ is increased from .375 to .482) and more of the concrete is available for compression. The load may be increased about 24 per cent, or until the moment equals $162bd^2$, before the compression in the concrete reaches 800#/□″. Under these conditions the steel has a tension of about 12,860#/□″, and its full strength is not utilized.

If the load were increased until the moment is $258bd^2$, then the steel would be stressed to 20,000#/□″, but the concrete would be compressed to about 1,280#/□″, which would, of course, be unsafe with such a grade of concrete.

If the compression in the concrete is to be limited to 800#/□″, then the load must be limited to that which will give a moment of $162bd^2$. Even for this the steel is doubled in order to increase the load 24 per cent. Whether this is justifiable depends on several circumstances—the relative cost of steel and concrete, the possible necessity for keeping the dimensions of the beam within certain limits, etc.

A larger value of $n$ will indicate higher values of $k$, which will indicate higher moments; but $n$ cannot be selected at pleasure. It depends on the character of the concrete used; and, with $E_s$ constant, a large value of $n$ means a small value for $E_c$, which also means a small value for $f_c$ the permissible compression stress. Whenever the ratio of steel is greater than the *theoretical* ratio, as is usual, then Equation (19) should be used. When in doubt, both Equations (18) and (19) should be tested, and that one giving the lower moment should be used.

When $p$ equals .010, $n$ equals 15, $f_c$ equals 800#/□″ and $f_s$ equals 20,000#/□″ as before, $k$ equals .418, $z$ equals .139 and $j$ equals .861. Then, since $p$ is greater than the theoretical value, .0075, using Equation (19)

$$M = (\tfrac{1}{2} \times 800 \times .418 \times .861) = 144bd^2$$

**Example 1.** What is the working moment for a slab with 5″ thickness to the steel, the concrete having the properties just described?

*Solution.* The concrete described is capable of resisting a moment, $M = 144bd^2$. For a section of slab 12″ wide, $b$ will equal 12″ and $d$ has been given as 5″. Substitute these values in the equation $M = 144bd^2$. Then $M = 144 \times 12 \times (5)^2 =$ 43,200″#, the permissible moment for a section of slab 12″ wide having the same properties as the concrete described above, and with a depth to the steel of 5″.

**Example 2.** A slab having a span of 8′ is to support a uniformly dis-

TABLE XVIII

Value of $p$ for Various Values of $(f_s \div f_c)$ and $n$

Formula: $p = \frac{1}{2} \times \frac{1}{R}\left(\frac{n}{R+n}\right)$ in which $R = (f_s \div f_c)$

| $(f_s \div f_c)$ | $n$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 10 | 12 | 15 | 18 | 20 | 25 | 30 | 40 |
| 10 | .0250 | .0273 | .0300 | .0321 | .0333 | .0357 | .0375 | .0400 |
| 12.5 | .0178 | .0196 | .0218 | .0236 | .0246 | .0267 | .0282 | .0304 |
| 15 | .0133 | .0148 | .0167 | .0182 | .0190 | .0208 | .0222 | .0242 |
| 17.5 | .0104 | .0116 | .0132 | .0145 | .0152 | .0168 | .0180 | .0199 |
| 20 | .0083 | .0094 | .0107 | .0118 | .0125 | .0139 | .0150 | .0167 |
| 25 | .0057 | .0065 | .0075 | .0084 | .0089 | .0100 | .0109 | .0123 |
| 30 | .0042 | .0048 | .0056 | .0062 | .0067 | .0076 | .0083 | .0095 |
| 40 | .0025 | .0029 | .0034 | .0039 | .0042 | .0048 | .0054 | .0062 |
| 50 | .0017 | .0019 | .0023 | .0026 | .0029 | .0033 | .0037 | .0044 |

tributed load of 150#/□′. The concrete is to be as described above, and the ratio of steel is to be .010. What is the required thickness $d$ to the steel?

*Solution.* Assume that the slab will be 5″ thick and allow (5″×12# = 60#) 60#/□′ for the weight of the slab itself. The total load to be carried by the slab is 150#/□′ + 60#/□′ = 210#/□′. A section of slab 12″ wide has an area of 8□′, and the total load on this section will be 1′×8′×210# = 1,680#. Assuming the slab as free-ended, the moment produced by the load is $\frac{Wl}{8}$ or $\frac{1680 \times 8 \times 12}{8} = 20{,}160''\#$. The concrete has a resisting moment, $M = 144bd^2 = 20{,}160''\#$. As the section is 12″ wide, $b$ equals 12″ and $M = 144 \times 12 \times d^2 = 20{,}160''\#$. Solving this equation, $d^2 = \frac{20{,}160}{144 \times 12} = 11.67$ and $d = \sqrt{11.67} = 3.42''$. Then allowing ¼″ for half the thickness of steel and 1″ for concrete below the steel, the total thickness of the slab will be 3.42″ + 0.25″ + 1.00″ = 4.67″ or say 4¾″. Its weight, allowing 12#/□′ per inch of depth, would be 57#/□′, thus agreeing safely with the estimated allowance for dead load. If the computed thickness and weight had proved to be materially more than the original allowance, another calculation would be necessary, assuming a somewhat greater dead load. This increase of dead load would of itself produce a somewhat greater moment, but the increased thickness would develop a greater resisting moment. Experience will enable one to make the preliminary estimate so close to the final that not more than one trial calculation should be necessary.

**Working Values for the Ratio of the Steel Tension to the Concrete Compression.** It is often more convenient to obtain working values from tables or diagrams rather than to compute them each time from equations.

Solving Equation (16) for several combinations of values of

$(f_s \div f_c)$ and $n$, the values are tabulated in Table XVIII. These values are also shown in Fig. 80. For other combinations than those used

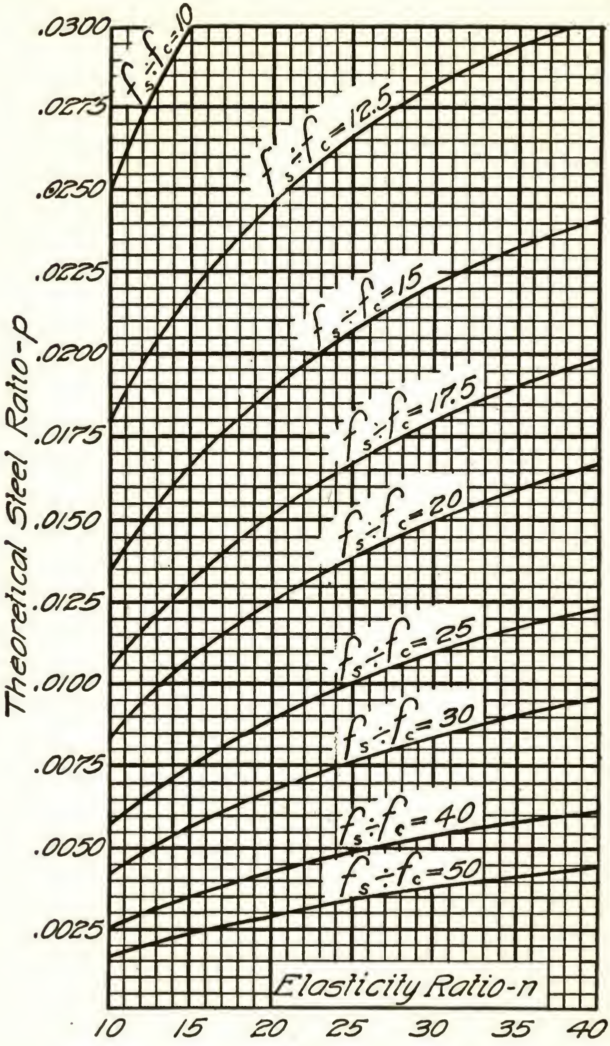



Fig. 80. Curves Showing the Relation of $(f_s \div f_c)$ to $p$ and $n$

in Table XVIII, the values of $p$ may be obtained with great accuracy provided that $(f_s \div f_c)$ corresponds with some curve already on the diagram. If it is necessary to interpolate for some value of $(f_s \div f_c)$ of

which the curve has not been drawn, it must be recognized that the space between the curves increases rapidly as $(f_s \div f_c)$ is smaller. For example, to interpolate for $(f_s \div f_c)$ equals 32, the point must be below the 30 curve by considerably more than 0.2 of the interval between the 30 and the 40 curve.

The relative elasticities $(n)$ of various grades of concrete and steel are usually roughly proportional to the relative working values, as expressed by $(f_s \div f_c)$. In other words, if $n$ is large, $(f_s \div f_c)$ is correspondingly large unless the working value for $f_s$ or for $f_c$ is for some reason made abnormally low. Therefore, there will be little if any use for the values given in the lower left-hand and upper right-hand corners of Table XVIII.

**Determination of Values for Frequent Use.** The moment of resistance of a beam equals the total tension in the steel, or the total compression in the concrete (which are equal), times $jd$. Therefore we have the choice of two values as given in Equations (18) and (19).

$$M_c = \frac{1}{2}(f_c bkd)jd \tag{20}$$

$$M_s = A_s f_s(jd) = (pbdf_s)jd$$

If the theoretical ratio $p$ has already been determined from Equation (16), then either equation may be used, as most convenient, since they will give identical results. If the ratio has been arbitrarily chosen, then the least value must be determined. For any given steel ratio and any one grade of concrete, the factors $\frac{1}{2}f_c kj$ or $pf_s j$ are constant and Equation (20) may be written.

$$M_c = K_c bd^2$$

$$M_s = K_s bd^2$$

or, in general, $$M = Kbd^2$$

when the theoretical ratio of steel is used. Diagrams for quickly determining $K$ are given in Figs. 82, 83 and 84.

**Constants Commonly Used in Design.** At the present time the majority of concrete structures are designed on the basis of using concrete with a minimum ultimate strength in 28 days of 2,000#/□″, and a working stress of 800#/□″. In designing these structures, the steel is usually calculated to take a fiber stress of 20,000#/□″. However, there are still many cities and towns that limit the steel stress

### TABLE XIX

**Commonly Used Constants for Three Grades of Concrete**

| Grade of Concrete | $f_c = .40f'_c$ | $E_c$ | $n$ | $f_s$ | Steel Ratio $p$ | $k$ | $j$ | $K=\frac{M}{bd^2}$ |
|---|---|---|---|---|---|---|---|---|
| 2000-lb. | *800 | *2,000,000 | 15 | *18,000 | .0089 | .400 | .867 | 138.6 |
| | 800 | 2,000,000 | 15 | 20,000 | .0075 | .375 | .875 | 131.2 |
| 2500-lb. | 1000 | 2,500,000 | 12 | 20,000 | .0094 | .375 | .875 | 164.1 |
| 3000-lb. | 1200 | 3,000,000 | 10 | 20,000 | .0113 | .375 | .875 | 196.9 |

*Values in this column are given in pounds per square inch.

to 18,000#/□″. For the 2,000-lb. concrete with $f_s = 18000\#/□''$, the resisting moment is expressed by the equation

$$M = 138bd^2 \tag{A}$$

Where the permissible stress in the steel can be increased to 20,000#/□″ the resisting moment equals

$$M = 131bd^2 \tag{B}$$

**Note.** In designing rectangular beams and slabs, Equations (A) and (B) generally are used.

While the usual designs are made on the basis of 2,000# concrete the use of higher strength concrete is coming more into use, particularly where a strict and accurate control of the quality and quantity of aggregates can be secured as well as the amount of mixing water definitely measured. The development of scales for weighing aggregates, the demand of specifications for complete mixing plants on large jobs and the many central mixing and batching plants have greatly raised the standard of the grade of concrete that can be obtained. The compacting of concrete by means of vibrators has materially increased the density of concrete over the old hand spading methods.

Accordingly, Table XIX has been prepared showing the constants for three grades of concrete. These are the constants needed for computing the strength of slabs, beams, and girders, and they will be used in the succeeding problems in the text.

**Working Stresses.** The working stresses given in Tables XIX-A and XIX-B are those in common use. The concrete stresses are given in terms of $f_c'$, the ultimate strength at 28 days. Variations in these stresses are found in building codes and engineering specifications.

## TABLE XIX-A

### Allowable Unit Stresses in Concrete

| Description | | Allowable Unit Stresses | | | |
|---|---|---|---|---|---|
| | | For Any Strength of Concrete as Fixed by Test $n=\frac{30000}{f'_c}$ | When Strength of Concrete is Fixed by the Water-Content | | |
| | | | $f'_c=$ 2000 p. s. i. $n=15$ | $f'_c=$ 2500 p. s. i. $n=12$ | $f'_c=$ 3000 p. s. i. $n=10$ |
| **Flexure:** $f_c$ | | | | | |
| Extreme fiber stress in compression | $f_c$ | $0.40f'_c$ | 800 | 1000 | 1200 |
| Extreme fiber stress in compression adjacent to supports of continuous or fixed beams or of rigid frames | $f_c$ | $0.45f'_c$ | 900 | 1125 | 1350 |
| **Shear:** $v$ | | | | | |
| Beams with no web reinforcement and without special anchorage of longitudinal steel | $v_c$ | $0.02f'_c$ | 40 | 50 | 60 |
| Beams with no web reinforcement, but with special anchorage of longitudinal steel | $v_c$ | $0.03f'_c$ | 60 | 75 | 90 |
| Beams with properly designed web reinforcement but without special anchorage of longitudinal steel | $v$ | $0.06f'_c$ | 120 | 150 | 180 |
| Beams with properly designed web reinforcement and with special anchorage of longitudinal steel | $v$ | $0.09f'_c$† | 180 | 225 | 270 |
| Flat slabs at distance $d$ from edge of column capital or dropped panel | $v_c$ | $0.03f'_c$ | 60 | 75 | 90 |
| Footings where longitudinal bars have no special anchorage | $v_c$ | $0.02f'_c$ | 40 | 50 | 60 |
| Footings where longitudinal bars have special anchorage | $v_c$ | $0.03f'_c$ | 60 | 75 | 90 |
| **Bond:** $u$ | | | | | |
| In beams and slabs and one-way footings: | | | | | |
| Plain bars or structural shapes | $u$ | $0.04f'_c$ | 80 | 100 | 120 |
| Deformed bars | $u$ | $0.05f'_c$ | 100 | 125 | 150 |
| In two-way footings: | | | | | |
| Plain bars or structural shapes | $u$ | $0.03f'_c$ | 60 | 75 | 90 |
| Deformed bars | $u$ | $0.0375f'_c$ | 75 | 94 | 112 |
| (Where special anchorage is provided, double these values in bond may be used) | | | | | |
| **Bearing:** $f_c$ | | | | | |
| Where a concrete member has an area at least three times the area of the loaded portion | $f_c$ | $0.375f'_c$ | 750 | 938 | 1125 |
| **Axial compression:** $f_c$ | | | | | |
| In columns with lateral ties | $f_c$ | $0.225f'_c$ | 450 | 563 | 675 |
| In columns with continuous spirals enclosing a circular core*. | | | | | |
| Ratio of longitudinal reinforcement $p=0.01$ | | $300+0.14f'_c$ | 580 | 650 | 720 |
| 0.02 | | $300+0.18f'_c$ | 660 | 750 | 840 |
| 0.03 | | $300+0.22f'_c$ | 740 | 850 | 960 |
| 0.04 | | $300+0.26f'_c$ | 820 | 950 | 1080 |
| 0.05 | | $300+0.30f'_c$ | 900 | 1050 | 1200 |
| 0.06 | | $300+0.34f'_c$ | 980 | 1150 | 1320 |
| (Spiral reinforcement not to be less than ¼ the longitudinal.) | | | | | |

*Unit stress in spirally reinforced columns $=(300+(0.10+4p)f'_c)$.

†This value of $v_c$ may be increased to $0.12\,f'_c$ but when the value exceeds $0.09\,f'_c$ web reinforcement should provide for total shear.

The following unit stresses in reinforcing steel shall not be exceeded: (See Table XIX-B.)

### TABLE XIX-B

**Allowable Unit Stresses in Reinforcement**

| | |
|---|---|
| Tension: | |
| Intermediate grade billet steel | $(f_s) = 20{,}000$ p. s. i. |
| Rail steel bars | $(f_s) = 20{,}000$ p. s. i. |
| Web reinforcement | $(f_s) = 16{,}000$ p. s. i. |
| Structural steel shapes | $(f_s) = 18{,}000$ p. s. i. |
| Other steel reinforcement 50 per cent of the yield point stress, but not to exceed | $(f_s) = 20{,}000$ p. s. i. |
| Compression: | |
| Bars | $nf_c$ |
| Structural steel section in composite columns | 15,000 lb. p. s. i. |
| Cast-iron section in composite columns | 9,000 lb. p. s. i. |

## CHAPTER IX

## SLABS AND SLAB TABLES

**Slab Tables.** The necessity of frequently computing the required thickness of slabs renders useful the data given in Table XX, which has been worked out on the basis of several combinations of values of $f_c$ and $f_s$. Municipal building laws specify the unit values which must be used and even the moment formula. For example, slabs are usually continuous over beams and even the wall ends of slabs are so restrained at the wall that the working moment is less than $Wl \div 8$ and, therefore, the formula $Wl \div 10$ is specifically permitted in many municipal regulations. Table XX is computed on that basis, but the tabulated unit loads may be easily changed to the basis of $Wl \div 8$ or $Wl \div 12$.

In the seventh column of the table are shown the unit weights of various slab thicknesses on the basis of 150 pounds per cubic foot for stone concrete. These weights may need to be altered if a concrete of different weight is used, or if an extra top coat of concrete, which cannot be considered to be structurally a part of the slab, is laid on afterward. The "thickness of concrete below the bottom of the steel" is such as is approved by good practice, but in case municipal regulations or other reasons should require other thicknesses of concrete below the steel, Table XX may still be used by considering the *effective* thickness $d$, plus half the thickness of the bars, and by varying, as need be, the weight of the slab to determine the net load.

The blanks in the upper right-hand corner of each section of the table indicate that for those spans and slab thicknesses the slabs cannot safely carry much more than their own weight and that even the weights nearest the blanks are too small for practical working floor loads, or even roof loads. The blanks in the lower left-hand corner of each section of the table indicate that for these combinations of span, load, and slab thickness, the shearing strength would be insufficient for the load which its transverse strength would enable

# TABLE XX—Working Loads on Floor Slabs. $M = Wl \div 10$

For $M = Wl \div 8$, subtract 20 per cent from total loads (total load = net load + wt. of slab).
For $M = Wl \div 12$, add 20 per cent to total loads.

| Total Thickness of Slab (Inches) | Thickness of Concrete Below Bottom of Steel (Inches) | Effective Thickness "d" (Inches) | Steel: Size | Steel: Spacing (Inches) | $M$ for $b$ 12″ (In.-Lbs.) | Weight of Slab per Sq. Ft. (Lbs.) | Net Superimposed Load, in Pounds per Square Foot, Not Including Weight of Slab, for the Given Spans in Feet: 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | "2000-lb." CONCRETE. $f_c = 800$; $f_s = 18{,}000$; $n = 15$; $p = .00889$; $M \div bd^2 = K = 138.6$; $j = .867$; $k = .400$ | | | | | | | | | | | | | | | | | |
| 3½ | 1 | 2.31 | ⅜″ φ | 5¼ | 8880 | 44 | 419 | 252 | 162 | 107 | 112 | 47 | 30 | ..... | ..... | ..... | ..... | ..... |
| 4 | 1 | 2.81 | ⅜″ φ | 4¼ | 13140 | 50 | 634 | 388 | 254 | 174 | 121 | 85 | 60 | 40 | 26 | ..... | ..... | ..... |
| 4½ | 1 | 3.31 | ⅜″ φ | 3¾ | 18240 | 56 | 894 | 552 | 366 | 254 | 182 | 132 | 96 | 70 | 50 | 34 | 22 | ..... |
| 5 | 1 | 3.75 | ½″ φ | 5¾ | 23410 | 63 | ..... | 718 | 479 | 335 | 242 | 178 | 132 | 98 | 73 | 52 | 37 | 24 |
| 5½ | 1 | 4.25 | ½″ φ | 5 | 30060 | 69 | ..... | 933 | 627 | 442 | 323 | 240 | 182 | 138 | 105 | 79 | 59 | 42 |
| 6 | 1 | 4.75 | ½″ φ | 4½ | 37550 | 75 | ..... | ..... | 794 | 563 | 414 | 311 | 238 | 183 | 142 | 110 | 85 | 64 |
| 7 | 1 | 5.75 | ½″ □ | 4¾ | 55030 | 88 | ..... | ..... | ..... | 848 | 628 | 478 | 371 | 291 | 230 | 184 | 146 | 116 |
| 8 | 1 | 6.69 | ⅝″ φ | 5 | 74490 | 100 | ..... | ..... | ..... | ..... | 869 | 666 | 521 | 414 | 331 | 267 | 216 | 176 |
| **2** | "2000-lb." CONCRETE. $f_c = 800$; $f_s = 20{,}000$; $n = 15$; $p = .0075$; $M \div bd^2 = K = 131.2$; $j = 0.875$; $k = .375$ | | | | | | | | | | | | | | | | | |
| 3½ | 1 | 2.31 | ⅜″ φ | 6¼ | 8400 | 44 | 394 | 236 | 150 | 99 | 65 | 42 | 26 | ..... | ..... | ..... | ..... | ..... |
| 4 | 1 | 2.81 | ⅜″ φ | 5¼ | 12430 | 50 | 597 | 364 | 238 | 161 | 112 | 78 | 54 | 36 | 22 | ..... | ..... | ..... |
| 4½ | 1 | 3.31 | ⅜″ φ | 4¼ | 17250 | 56 | 843 | 519 | 343 | 237 | 168 | 121 | 88 | 63 | 44 | 29 | 17 | ..... |
| 5 | 1 | 3.75 | ½″ φ | 7 | 22140 | 63 | ..... | 675 | 450 | 313 | 225 | 165 | 122 | 89 | 65 | 46 | 31 | 19 |
| 5½ | 1 | 4.25 | ½″ φ | 6 | 28440 | 69 | ..... | 879 | 599 | 415 | 301 | 224 | 168 | 127 | 96 | 71 | 52 | 36 |
| 6 | 1 | 4.75 | ½″ φ | 5½ | 35520 | 75 | ..... | ..... | 747 | 529 | 387 | 290 | 221 | 170 | 131 | 100 | 76 | 57 |
| 7 | 1 | 5.75 | ½″ □ | 5¾ | 52050 | 88 | ..... | ..... | ..... | 797 | 590 | 447 | 346 | 270 | 213 | 169 | 134 | 105 |
| 8 | 1 | 6.69 | ⅝″ φ | 6 | 70460 | 100 | ..... | ..... | ..... | ..... | 817 | 625 | 487 | 385 | 308 | 247 | 200 | 161 |
| **3** | "2500-lb." CONCRETE. $f_c = 1000$; $f_s = 20{,}000$; $n = 12$; $p = .0094$; $M \div bd^2 = K = 164.1$; $j = 0.875$; $k = .375$ | | | | | | | | | | | | | | | | | |
| 3½ | 1 | 2.31 | ⅜″ φ | 5 | 10510 | 44 | 504 | 306 | 199 | 135 | 93 | 64 | 44 | 28 | ..... | ..... | ..... | ..... |
| 4 | 1 | 2.81 | ⅜″ φ | 4 | 15550 | 50 | 759 | 468 | 310 | 214 | 152 | 110 | 80 | 57 | 40 | 27 | ..... | ..... |
| 4½ | 1 | 3.31 | ½″ φ | 6¼ | 21570 | 56 | 1066 | 663 | 443 | 311 | 225 | 166 | 124 | 93 | 69 | 50 | 36 | 24 |
| 5 | 1 | 3.75 | ½″ φ | 5½ | 27690 | 63 | ..... | 860 | 578 | 408 | 297 | 222 | 168 | 128 | 97 | 74 | 55 | 40 |
| 5½ | 1 | 4.25 | ½″ φ | 4¾ | 35570 | 69 | ..... | 1118 | 754 | 536 | 394 | 297 | 227 | 176 | 137 | 106 | 82 | 63 |
| 6 | 1 | 4.75 | ½″ φ | 4¼ | 44430 | 75 | ..... | ..... | 954 | 681 | 503 | 382 | 295 | 231 | 182 | 144 | 114 | 90 |
| 7 | 1 | 5.75 | ½″ □ | 4½ | 65110 | 88 | ..... | ..... | ..... | 1020 | 760 | 582 | 455 | 360 | 289 | 233 | 189 | 153 |
| 8 | 1 | 6.69 | ⅝″ φ | 4¾ | 88130 | 100 | ..... | ..... | ..... | ..... | 1048 | 806 | 634 | 507 | 410 | 334 | 275 | 227 |

it to carry and, therefore, although those slabs would carry a great load, those combinations of span and slab thickness are uneconomical and should not be used.

**Example 1.** Using stone concrete such that $f_c=800\#/\square''$, $n=15$, and $f_s=18{,}000\#/\square''$, and with a required working load of $200\#/\square'$, what span may be chosen, using a 6″ slab.

*Solution.* This requires Section 1 of Table XX. Note that a 6″ slab on a span of 10′ will carry 238# net per square foot, which is a little more than what is required. The exact span will be found at which the net load carried will be 200#. To interpolate, it should be noted from Table XX that the difference $311-238$, or 73, is greater than the difference $238-183$, or 55, which in turn is greater than the difference $183-142$, or 41. This shows that the *reduction* in net pounds carried grows less and less for each even-foot increase in span, or each successive *unit* increase, whether foot, half-foot, or inch. From this it is known, without precise calculations, that the decrease in net load for an increase in span from 10′ to 10′6″ will be greater than the decrease in net load for an increase of span from 10′6″ to 11′. The successive load decreases of 38 and 17 (238 down to 200, and 200 down to 183) suggest at once that the increase from 10′ to the required span is more than the increase from that required span up to 11′. As an approximation, estimate that 10′9″ is very nearly the required span. This value may be checked by applying the basic formula, noting however that, since the table gives *net* loads, we must call the total load carried by the beam $200+75=275$ pounds. Then, for a 6″ slab, $d=4.75$ and $M=37{,}550''\#$. Since $M=Wl\div 10$, and $l=10'9''=129''$, $W=\dfrac{10M}{l}=\dfrac{10}{129}\times 37{,}550=2{,}910\#$, which is the gross load on a strip 1′ wide and 10′9″ long. Divide 2,910 by 10.75, which gives 271, the gross load per square foot. Deducting $75\#/\square'$ for the weight of the slab leaves $196\#/\square'$, the net load, which checks very nearly. Of course the exact span ($l$) which will carry the desired net load (200) could be directly computed by an inversion of the formula, but the above method shows how the span *may* be obtained by an interpolation from the table with sufficient accuracy without taking the time to compute a precise value from a formula.

**Example 2.** Assume a slab made of "2000-lb." concrete, $f_s=20{,}000\#/\square''$; the span has been determined already as 7′; the floor is to be covered with 2″ of cinder-concrete fill between the wood sleepers and a wood floor, the weight of fill and flooring being $23\#/\square'$; the live load is to be $150\#/\square'$. Required, the slab thickness.

*Solution.* For such concrete, use Section 2, Table XX. $150+23=173\#/\square'$, which must here be considered as the net load to be carried. Under 7′ span is found 161 for a 4″ slab and 237 for a 4½″ slab. Four inches is too thin and 4½″ is somewhat needlessly thick. Since 173 is nearer to 161 than to 237, cut the thickness to 4¼″. The detail of the interpolation, elaborated in Example 1, shows this to be justifiable. Similarly interpolate the bar spacing, and specify ⅜″ round bars, spaced 4¾″.

It should be noted that the three sections of Table XX are based on the moment formula $Wl\div 10$, and that when the conditions require the use of $Wl\div 8$ (or $Wl\div 12$), the table may be used by

subtracting (or adding) 20 per cent to the total loads. When the slab has only a single span between two walls, use $Wl \div 8$. When there are two or more continuous spans, the spans being made continuous over interior beams or girders, use $Wl \div 10$ for the outer spans and $Wl \div 12$ for any interior span.

Fig. 81A illustrates the details of a reinforced concrete slab between reinforced concrete beams. Half of the bars are bent. For example the bent bar labeled $E$ in diagram, turns up over both beams at the fifth points and extends to the quarter points of the adjoining spans.

The straight bars in the bottom can be used in long lengths, but when splices are necessary they should occur over the beams (see point $x$) and the ends should be well lapped. By turning up every other bar, the same area of steel that is found in the center of the span is secured in the top of the slab over the beams. The design is based on the bending moment being the same at these points.

The bars are turned up at the fifth point of the span because the moment changes from positive to negative near that point; they extend to the quarter point to secure anchorage beyond the point where they are needed. See diagrams shown in Chapter VIII under "Review of Beam Design."

When used with beam and girder construction, the minimum thickness for floor slabs is 4 inches and for roofs 3½ inches. The maximum spacing for tension bars is three times the thickness of the slab. When joist and steel tile construction is used, with joists 25 to 30 inches on centers, the slab is made 2, 2½, or 3 inches in thickness, depending largely on the spans of the joists. For joist and terra cotta tile construction, slabs are made 1½, 2, 2½ or 3 inches in thickness, depending on the depth of joist and the amount of compression required. The slab bars in thin slabs are always straight and are usually 8 to 12 inches on centers, depending on conditions and thickness of slabs.

Temperature or shrinkage bars are required for all slabs, as specified hereafter in this chapter.

**Fireproofing Structural Steel.** The skeleton framework for buildings is often made of structural steel with reinforced concrete slabs used for the structural floor between the beams. The fireproofing of the steel work is done with concrete. The top of the slab is usually

placed two inches above the steel beams as a fireproofing measure and also to permit small electric conduits to pass over the beams.

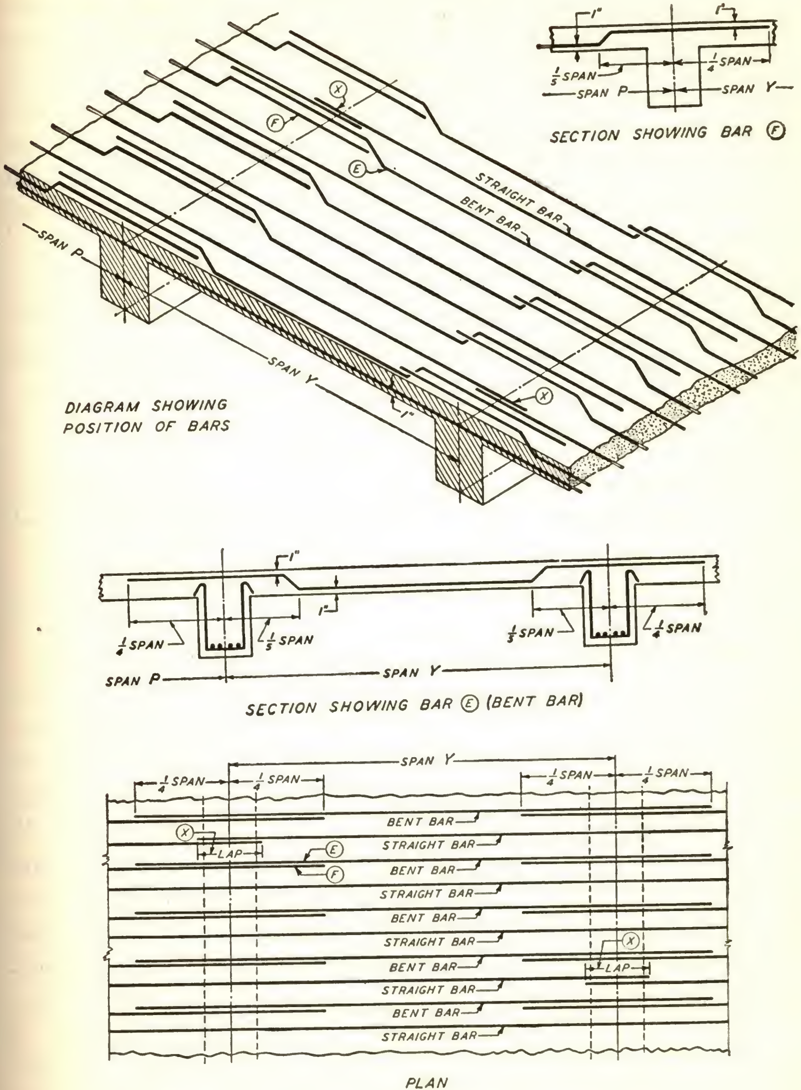



Fig. 81A. Detail of Slab in Reinforced Concrete Construction

Ordinary steel beams are fireproofed with 1½ inches of material on the side and bottom. Large girders are protected with at least 2 inches

of concrete on the side and bottom. In Fig. 81B is shown a slab supported by I-beams, and the details for bending the bars.

These slabs may be made either of stone concrete or cinder concrete. If cinder concrete is used, the reinforcing steel usually consists of wire mesh. The span for cinder concrete slabs usually is limited to about 8 feet. In Fig. 81B alternate bars are turned up so that there is the same area of steel over the beam as in the center of the slab. A continuous clip is placed on the bottom flange when 12-inch or smaller beams are used; for larger beams ties passing around the beams are necessary to hold the concrete.

**Slabs Reinforced in Two Directions.** When the floor beams of a floor are spaced about equally in two directions, so that they form, between the beams, panels which are nearly square, a material

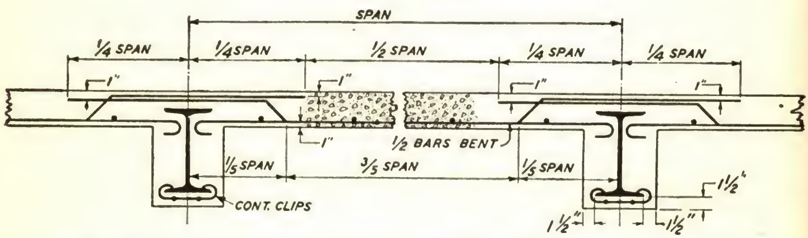



Fig. 81B. Detail for Reinforced Concrete Slab and Fire Protection for I-Beams

saving can be made in the thickness of the slab by reinforcing it with bars running in two directions. The theoretical computation of these slabs is exceedingly involved and complicated. It is common practice to consider that if the length of the slab exceeds one and one-half times its width, the entire load is carried in the short direction. In the case of square panels and uniformly distributed load, half the live and dead load is assumed as being resisted by the bars in each band. In rectangular panels of length $l$ and breadth $b$, the portion of the load which is assumed as being supported by the slab in the short direction, is equal to $\left(\frac{l}{b}-\frac{1}{2}\right)$ times the total load. The remainder of the load is assumed as being supported by the slab in the long direction. The reinforcement in the long direction should in no case be less than that specified for shrinkage and temperature reinforcement. The table gives the load carried in the short direction,

as a percentage of the total load, for various values of the ratio $\frac{l}{b}$:

| Ratio $l:b$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|---|---|---|---|---|---|
| Proportion of load carried by "$b$" bars | 50% | 60% | 70% | 80% | 90% | 100% |

**Example.** Design a slab $12' \times 12'$ to support a live load of $150\#/\square'$, supported on all four sides by beams, slab being fully continuous, $M = \frac{Wl}{12}$, in which $f_s = 18{,}000\#/\square''$, $f_c = 800\#/\square''$ and $n = 15$.

*Solution.* The span being the same in both directions, the moment therefore will be the same. Take a strip one foot wide through the center. Then the load will be $150\#/\square' + 56\#/\square'$ (assume $4\frac{1}{2}''$ slab) $= 206\#/\square'$, half of which will be supported in each direction. Then $\frac{206\#}{2} \times 12' = 1236\#$, total load on the strip.

$$M = \frac{Wl}{12} = \frac{1236 \times 12 \times 12}{12} = 14{,}832''\#$$

From Equation (A) $\quad M = 138bd^2$
then $\quad M = 14{,}832 = 138 \times 12 \times d^2$
$d^2 = 8.96$
$d = 3''$ nearly

The thickness of the slab will be $3''$ plus half the thickness of the steel (say $\frac{1}{2}''$), plus $1''$ of fireproofing $= 4\frac{1}{2}''$.

Steel area, $A_s = \frac{14{,}832}{18{,}000 \times 3 \times .87} = .32/\square''$ or $\frac{3}{8}''\ \phi$ bars, $4''$ center to center in two directions.

In the example just given it will be noted that the strip of floor in each direction was taken through the center. Undoubtedly the maximum moments occur at the center and taper off to zero or near zero close to the beams. This means that the reinforcing steel may have a greater spacing as the beams are approached.

**Temperature Stresses and Shrinkage.** The modulus of elasticity of ordinary concrete is approximately $3{,}000{,}000\#/\square''$, while its ultimate tensile strength is about $200\#/\square''$. Therefore, a pull producing an elongation of about $\frac{1}{15{,}000}$ of the length would nearly, if not quite, rupture the concrete. The coefficient of expansion of concrete has been found to be almost identical with that of steel, or .0000065 for each degree Fahrenheit. Therefore, if a block of concrete were held at the ends with absolute rigidity, while its temperature was lowered about 10 degrees, the stress developed in the concrete

would be very nearly, if not quite, at the rupture point. Fortunately, the ends will not usually be held with such rigidity; but, nevertheless, it does generally happen that, unless the entire mass of concrete is permitted to expand and contract freely so that the temperature stresses are small, the stresses will usually localize themselves at the weak point of the cross section, wherever it may be, and will there develop a crack, provided the concrete is not reinforced with steel. If, however, steel is well distributed throughout the cross section of the concrete, it will prevent the concentration of the stresses at local points, and will distribute it uniformly throughout the mass.

Reinforced concrete beams, girders, columns, etc., usually are provided with enough steel so that temperature cracks are minimized and it is unnecessary to make any special provision against cracks. In floor or roof slabs which structurally require reinforcing in only one direction, it is found that cracks will parallel the main reinforcement unless temperature reinforcement is provided at right angles to the main reinforcing. Such reinforcement should provide for the following minimum ratio of reinforcement area to concrete area, but in no case should such reinforcing bars be placed farther apart than five times the slab thickness nor more than 18″.

Floor slabs where plain bars are used, .0025

Floor slabs where deformed bars are used, .0020

Floor slabs where wire fabric is used, largest mesh 12″, .0018

Roof slabs where plain bars are used, .0030

Roof slabs where deformed bars are used, .0025

Roof slabs where wire fabric with 12″ maximum mesh is used. .0022

Retaining walls, the balustrades of bridges, and other similar structures, which may need but little or no reinforcement for structural purposes, should likewise be provided with temperature reinforcement. The amount of this reinforcement should not be less than 0.0025 times the cross-sectional area. Bearing walls preferably should have an area of steel in each direction, vertical and horizontal, at least equal to 0.0025 times the cross-sectional area. Walls 8″ or more in thickness should have half the steel at each face of the wall. These temperature bars, in addition to preventing cracks, help to tie the structure together, and so add somewhat to its strength and its ability to resist disintegration due to vibration and shock.

## DESIGN BY USE OF CHARTS AND TABLES

**Explanation of Diagram.** A very large proportion of concrete work is done with a grade of concrete such that we may call the ratio $n$ of the moduli of the steel and the concrete either 12 or 15. The working values of the stresses in the steel and the concrete,

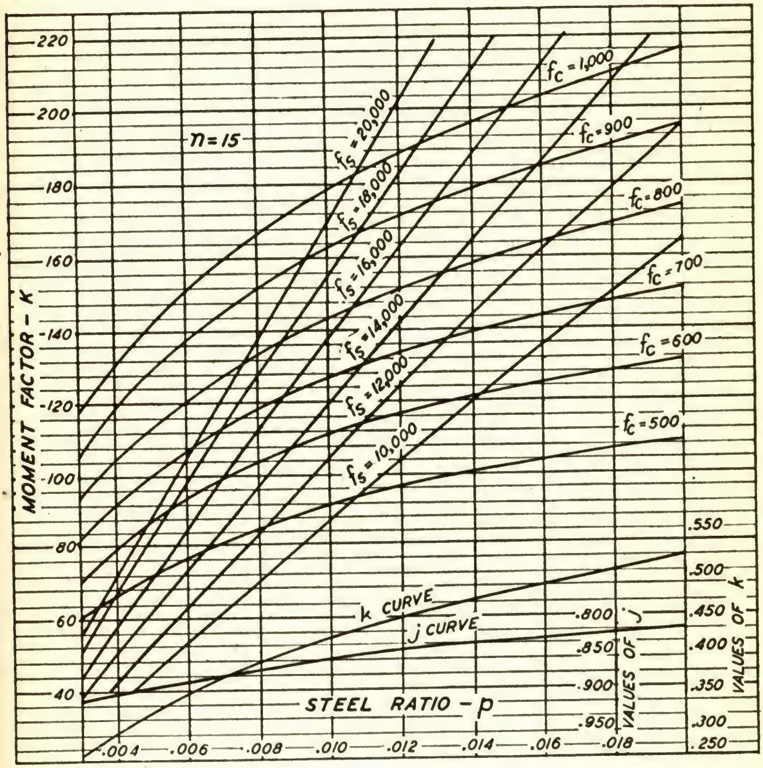



Fig. 82. Curves Showing Values of Moment Factor $K$ for $n=15$

$f_s$ and $f_c$, are determined either by public regulation or by the engineer's estimate of the proper values to be used. The diagrams, Figs. 82, 83 and 84, fully cover the whole range of practicable values for steel and for stone concrete. In the previous problems all values have been calculated on the basis of formulas. By means of these diagrams all needed values, on the basis of the other factors, may be read from the diagram with sufficient accuracy for practical work.

In addition, the diagrams enable one to note readily the effect of any proposed change in one or more factors.

**Example 1.** If a beam, made of concrete such that $n=15$, is to be so loaded that when the stress in the steel ($f_s$) is 18,000#/□″, the stress in the concrete ($f_c$) shall be 800#/□″, the steel ratio ($p$) must be .0089. This is found on the diagram, Fig. 82 for $n=15$, by following the line $f_s=18{,}000$ to its intersection with the line $f_c=800$. The intersection point, measured on the steel ratio scale at

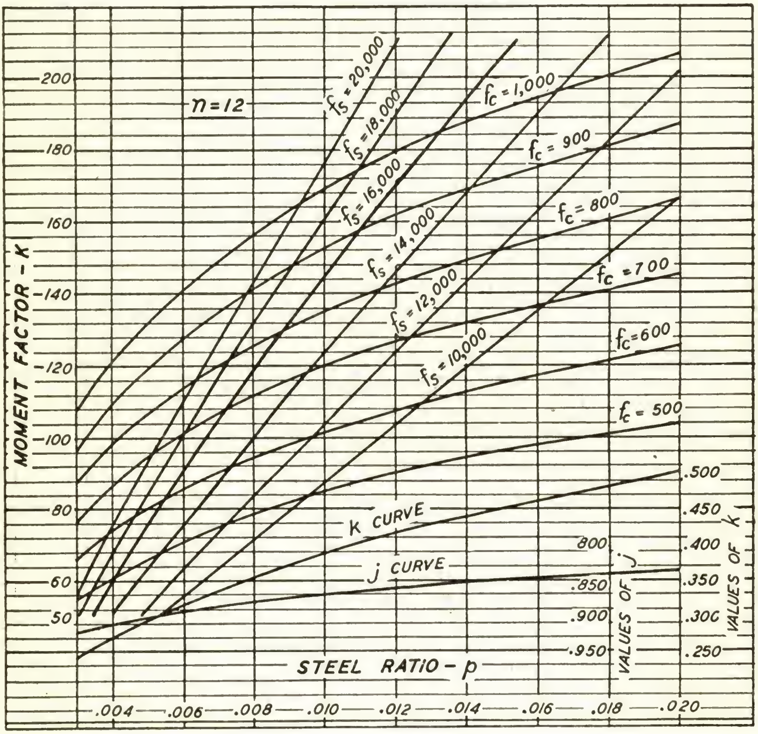



Fig. 83. Curves Showing Values of Moment Factor $K$ for $n=12$

the bottom of the diagram, reads .0089. Also, running horizontally from the intersection point to the scale at the left, read $K=138$ which is the factor for $bd^2$ in the moment equation, Equation (**20**). Incidentally, the corresponding values of $k$ and $j$ for this steel ratio may be obtained, with greater convenience, from this diagram, although they are also obtainable from the more general diagram, Fig. 79.

**Example 2.** Assume that it is decided to increase the steel ratio in the beam in Example 1 to 1.2 per cent. The vertical line for steel ratio equal to .012, intersects the line $f_c=800$ at a point where $K=152$, but the point is about half-way between the lines $f_s=14{,}000$ and $f_s=16{,}000$ indicating that, using that steel

ratio, the stress in the steel for a proper stress in the concrete is far less than the usual working stress, and that it would be about 15,000#/□″. If the load were

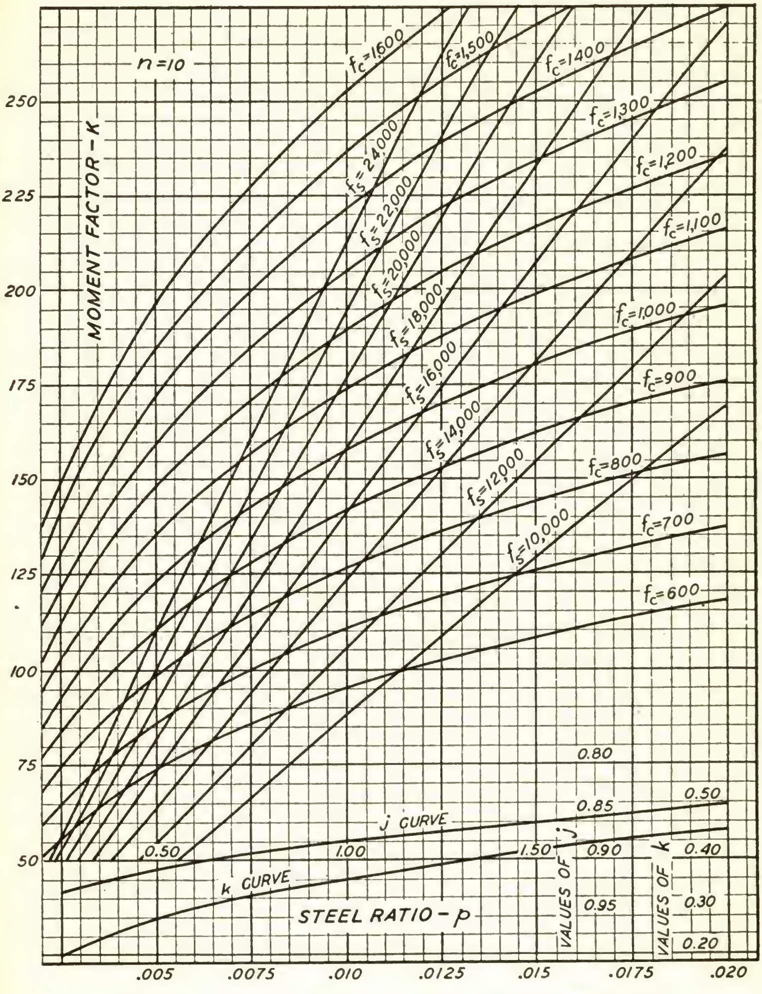



Fig. 84. Curves Showing Values of Moment Factor $K$ for $n = 10$

increased so that $f_s$ equals 18,000, $f_c$ would be about 1000, which is greater than the proper working value for that grade of concrete.

**Example 3.** Assume $p = .006$; $f_c = 800$ and $n = 15$; how much are $K$ and $f_s$? $K$ equals 122 and $f_s$ equals 23,000 which is impracticably high. The diagram, Fig. 82, shows plainly that for low steel ratios the values of $f_s$ are abnormally

high for ordinary values of $f_c$; on the other hand, for high steel ratios, the ordinary values of $f_c$ cannot utilize the full working strength of the steel.

**Table for Computation of Simple Beams.** In Table XXI has been computed, for convenience, the working total load (including the weight of the beam) on rectangular beams one inch wide and of various depths and spans. For other widths of beams, multiply the tabular load by the width of the beam in inches. Table XXI has been computed on the basis that $M = Wl \div 8$. For other loading conditions (continuous beams, etc.) corresponding allowances must be made. Table XXI is based on a grade of concrete such that $M$ equals $100bd^2$; for any other grade of concrete, determine the corresponding factor of $bd^2$, or, in other words, in Equation **(20)** compute the value of $\frac{1}{2}f_ckj$ or of $pf_sj$, whichever is least. Multiply the tabular load by the percentage of that factor to 100. The blanks in the upper right-hand corner of Table XXI are similar to the corresponding blanks of the other sections of Table XX, the beams cannot safely carry their own weight. And, as before, the values immediately adjacent to the blanks are of little or no use, since the possible load, after deducting the weight of the beam, would be too small for practical use. The values in the lower left-hand corner should be used with great caution. Many of the beams of such relative span and depth would fail from vertical shear or diagonal tension before tabulated loads were reached. But, since the liability to failure from the vertical shear is dependent on the nature of the web reinforcement, the line of demarcation is not easily drawn, as was done in Table XX.

**Example 1.** Assume the concrete described in Section 1, Table XX, which has the factor 138.6; how much load will be carried by a beam made of such concrete, when the beam is 8″ wide, 16″ effective depth, and 18′ span?

*Solution.* In Table XXI under 18′ span and opposite 16″ effective depth, find 948#, the load for a beam 1″ wide. An 8″ beam will carry 8×948, or 7,584#. 138.6 per cent of 7,584 is 10,510#, the load for that particular grade of concrete. The weight of the concrete, assuming a total depth of 18″, is $\frac{8}{12} \times \frac{18}{12} \times 18 \times 150 =$ 2700#. Deducting this from 10,510# gives the net load as 7,810#.

**Example 2.** Assume that $f_c = 1{,}000\#/\square''$, $f_s = 20{,}000\#/\square''$, $n = 12$ and $p = .012$ how much load will be carried by a beam 6″ wide, 12″ effective depth, and 14′ span?

*Solution.* From the ratio diagram, Fig. 80, it is seen that for $f_s \div f_c = 20$ and $n = 12$, $p = .0094$ and since this is less than the chosen steel ratio, $p = .012$, use the first part of Equation (20). For $n = 12$ and $p = .012$, $k = .412$ and

## TABLE XXI

### Gross Load on Rectangular Beam One Inch Wide

For other widths, multiply by width of beam. Based on formula, $M = 100bd^2$
For any other combination of unit values, multiply by percentage of its formula factor to 100

| Effective Depth Depth of Beam "d" (Inches) | Span in Feet | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 4 | 267 | 213 | 178 | 152 | 133 | 119 | 107 | 97 | 89 | 82 | 76 | 71 | 67 | .... | .... | .... | .... |
| 5 | 417 | 333 | 278 | 238 | 208 | 186 | 167 | 151 | 139 | 128 | 119 | 111 | 104 | 98 | 93 | .... | .... |
| 6 | 600 | 480 | 400 | 343 | 300 | 267 | 240 | 218 | 200 | 185 | 171 | 160 | 150 | 141 | 133 | 126 | 120 |
| 7 | 817 | 653 | 544 | 466 | 408 | 363 | 327 | 297 | 272 | 251 | 233 | 218 | 204 | 192 | 181 | 172 | 163 |
| 8 | 1067 | 853 | 711 | 609 | 533 | 474 | 427 | 388 | 356 | 328 | 304 | 284 | 267 | 251 | 237 | 224 | 213 |
| 9 | 1350 | 1080 | 900 | 771 | 675 | 600 | 540 | 491 | 450 | 415 | 385 | 360 | 337 | 317 | 300 | 284 | 270 |
| 10 | 1667 | 1333 | 1111 | 952 | 833 | 741 | 667 | 606 | 556 | 513 | 476 | 444 | 417 | 392 | 370 | 351 | 333 |
| 11 | 2017 | 1613 | 1344 | 1151 | 1008 | 896 | 807 | 733 | 672 | 620 | 575 | 538 | 504 | 474 | 448 | 424 | 403 |
| 12 | 2400 | 1920 | 1600 | 1371 | 1200 | 1067 | 960 | 872 | 800 | 738 | 685 | 640 | 600 | 564 | 533 | 505 | 480 |
| 13 | 2817 | 2253 | 1878 | 1610 | 1408 | 1252 | 1127 | 1024 | 939 | 867 | 805 | 751 | 704 | 663 | 626 | 593 | 563 |
| 14 | 3267 | 2613 | 2178 | 1866 | 1633 | 1452 | 1307 | 1188 | 1089 | 1005 | 933 | 871 | 817 | 768 | 726 | 687 | 653 |
| 15 | 3750 | 3000 | 2500 | 2141 | 1875 | 1667 | 1500 | 1364 | 1250 | 1154 | 1070 | 1000 | 937 | 882 | 833 | 789 | 750 |
| 16 | 4267 | 3413 | 2844 | 2436 | 2133 | 1896 | 1707 | 1551 | 1422 | 1313 | 1218 | 1138 | 1067 | 1004 | 948 | 898 | 853 |
| 17 | 4817 | 3853 | 3211 | 2752 | 2408 | 2141 | 1927 | 1751 | 1606 | 1482 | 1376 | 1284 | 1204 | 1133 | 1070 | 1014 | 963 |
| 18 | 5400 | 4320 | 3600 | 3085 | 2700 | 2400 | 2160 | 1964 | 1800 | 1661 | 1542 | 1440 | 1350 | 1271 | 1200 | 1136 | 1080 |
| 19 | 6017 | 4813 | 4011 | 3437 | 3008 | 2674 | 2407 | 2188 | 2006 | 1852 | 1718 | 1604 | 1504 | 1416 | 1337 | 1266 | 1203 |
| 20 | 6667 | 5333 | 4444 | 3809 | 3333 | 2963 | 2667 | 2422 | 2222 | 2050 | 1904 | 1778 | 1667 | 1569 | 1481 | 1404 | 1333 |

**Note.** For any beams corresponding to values from the lower left-hand corner of the table, the possible failure by diagonal shear should be carefully tested.

## TABLE XXII

**Required Width in Inches of Beams Having 2, 3, 4 or 5 Bars in One Row, and with 1½ Inches Concrete Protection on Each Side**

| | No. of bars in one row | Size and Shape of Bars | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | ½″○ | ½″□ | ⅝″○ | ¾″○ | ⅞″○ | 1″○ | 1″□ | 1⅛″□ | 1¼″□ |
| Not anchored | 2 | 5 | 5¼ | 5¼ | 5⅝ | 6 1/16 | 6½ | 7 | 7½ | 8 |
| | 3 | 6½ | 7 | 6⅞ | 7½ | 8¼ | 9 | 10 | 10⅞ | 11¾ |
| | 4 | 8 | 8¾ | 8½ | 9⅜ | 10 7/16 | 11½ | 13 | 14¼ | 15½ |
| | 5 | ...... | ...... | 10⅛ | 11¼ | 12⅝ | 14 | 16 | 17⅝ | 19¼ |
| | Each extra bar | 1½″ | 1¾″ | 1⅝″ | 1⅞″ | 2 3/16″ | 2½″ | 3″ | 3⅜″ | 3¾″ |
| | No. of bars in one row | Size and Shape of Bars | | | | | | | | |
| | | ½″○ | ½″□ | ⅝″○ | ¾″○ | ⅞″○ | 1″○ | 1″□ | 1⅛″□ | 1¼″□ |
| Bars anchored | 2 | 5 | 5¼ | 5¼ | 5½ | 5¾ | 6 | 6½ | 6 15/16 | 7⅜ |
| | 3 | 6½ | 7 | 6⅞ | 7¼ | 7⅝ | 8 | 9 | 9¾ | 10½ |
| | 4 | 8 | 8¾ | 8½ | 9 | 9½ | 10 | 11½ | 12 9/16 | 13⅝ |
| | 5 | ...... | ...... | 10⅛ | 10¾ | 11⅜ | 12 | 14 | 15⅜ | 16¾ |
| | Each extra bar | 1½″ | 1¾″ | 1⅝″ | 1¾″ | 1⅞″ | 2″ | 2½″ | 2 13/16″ | 3⅛″ |

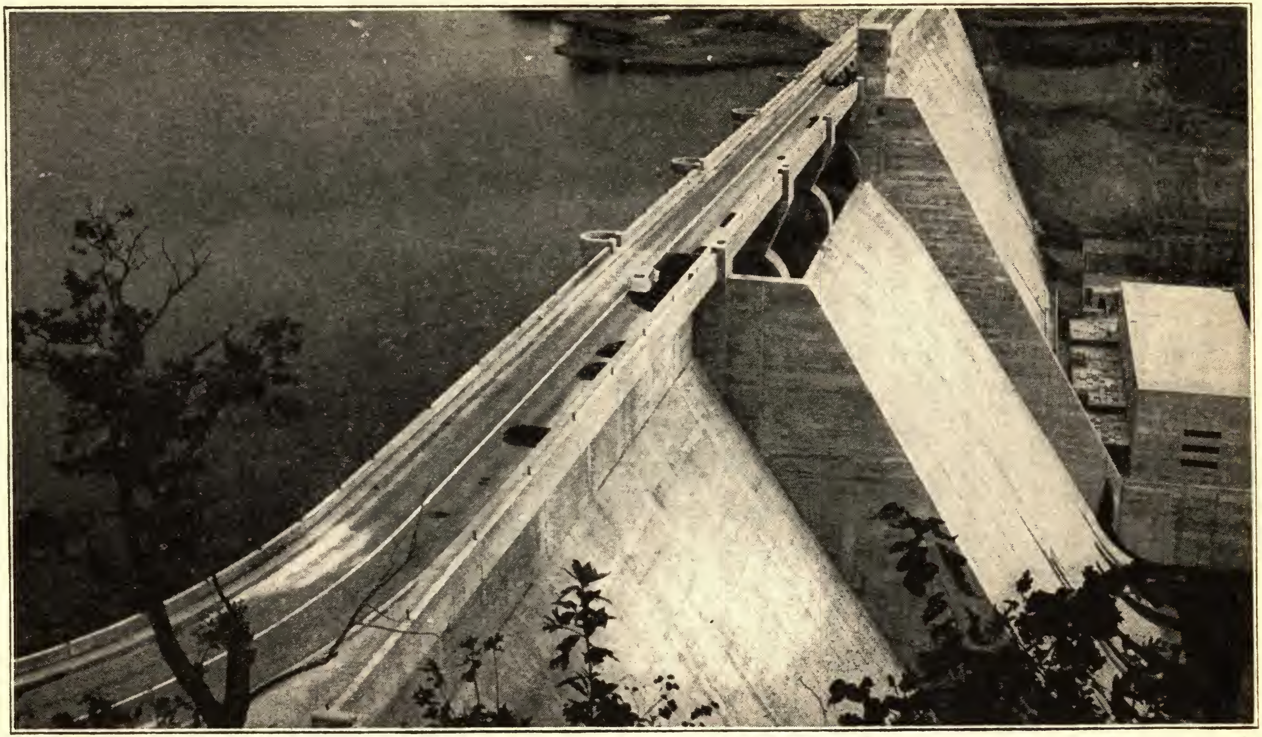
$j=.863$. See Tables XV and XVI. Then $\frac{1}{2}f_ckj=\frac{1}{2}\times1{,}000\times.412\times.863=177.8$ the factor of $bd^2$. The load on a beam 1″ wide, 12″ effective depth, and 14′span is 685#. See Table XXI. For 6″ wide it would be 4,110#. 177.8 per cent of this is 7,308#. The weight, allowing 2″ below the steel, is $\frac{6}{12}\times\frac{14}{12}\times14\times150$, or 1,225#. The net load is, therefore, 7,308−1,225, or 6,083#.

**Width of Beams.** The width of a beam required to hold the needed quantity of steel reinforcement requires consideration. The minimum center to center distance between parallel bars should be two and one-half times the diameter for round bars or three times the side dimensions for square bars; if the ends of bars are anchored either by a hook or an added length of bar whose bond resistance equals the stress in the bar, the center to center spacing may be made equal to two diameters for round bars or to two and one-half times the side dimension for square bars, but in no case shall the clear spacing between bars be less than one inch, nor less than one and one-third times the maximum size of the coarse aggregate. Bars at

the upper face of any member shall be embedded a clear distance of not less than one diameter, nor less than one inch.

Table XXII gives the total width of beams required for from two to five rods in one row for the standard sizes of reinforcing rods. It has been computed on the basis just given, for a coarse aggregate with a maximum size of ¾″, and with an allowance of 1½″ outside the outer longitudinal bars for fire protection. This amount of fire protection can sometimes be decreased to 1″ when the fire hazard is limited, but, on the other hand, if the aggregate is liable to disruptive action by heat, the thickness of concrete must be increased and must be reinforced by metal mesh. Table XXII may be used on the basis of other protective thickness by increasing or decreasing the widths shown in the table by twice the change from the 1½″ of protection.

The widths given in Table XXII are exact widths as computed according to the requirements given in the preceding paragraphs. In making the forms for beams, commercially dressed lumber is used, and while the nominal widths are 4″, 6″, 8″, 10″, 12″, etc., each actual width is ½″ less. So that a beam designed as an 8″x12″ beam may measure only 7½″x11½″ when built from commerically dressed lumber. This is an important point to remember in designing a reinforced concrete structure. As a general rule it is more economical to make the width or depth of a beam slightly larger than to insist on beams being made to exact dimensions, for the cost of making beams neat sizes is greater than the cost of the extra concrete used in making the beams a little larger. It will also be advisable to keep this point in mind when use is made of Table XXII in determining the needed width of beams.



THE NORRIS DAM, A TENNESSEE VALLEY PROJECT, IS A NOTEWORTHY EXAMPLE OF CONCRETE CONSTRUCTION

*Courtesy of Portland Cement Association*

## CHAPTER X

## BOND STRESSES

**Resistance to the Slipping of the Steel in the Concrete.** The previous discussion has considered merely the tension and compression in the upper and lower sides of the beam. A plain simple beam resting freely on two end supports has neither tension nor compression in the fibers at the ends of the beam. The horizontal tension and compression, found at or near the center of the beam, entirely disappear by the time the end of the beam is reached. This is done by transferring the tensile stress in the steel at the bottom of the beam to the compression fibers in the top of the beam, by means of the intermediate concrete. This is, in fact, the main use of the concrete in the lower part of the beam.

It is, therefore, necessary that the bond between the concrete and the steel shall be sufficiently great to withstand the tendency to slip. The required strength of this bond is evidently equal to the difference in the tension in the steel per unit of length. For example, suppose that we are considering a bar 1″ square in the middle of the length of a beam. Let the bar be under an actual tension of 15,000#/□″. Since the bar is 1″ square, the actual total tension is 15,000#. Suppose that, at a point 1″ beyond, the moment in the beam is so reduced that the tension in the bar is 14,900# instead of 15,000#. This means that the difference of pull (100#) has been taken up by the concrete. The surface of the bar for that length of 1″ is 4□″. This will require an average adhesion of 25#/□″ between the steel and the concrete in order to take up this difference of tension. The adhesion between concrete and plain bars is usually considerably greater than this, and there is, therefore, but little question about the bond in the center of the beam. But near the ends of the beam, the change in tension in the bar is far more rapid, and it then becomes questionable whether the bond is sufficient.

**Virtue of "Deformed" Bars.** The fact that the adhesion of the concrete to the steel is a critical feature under some conditions,

called attention to the desirability of using "deformed" bars, which furnish a mechanical bond. Microscopical examination of the surface of steel, and of concrete which has been molded around the steel, shows that the adhesion depends chiefly on the roughness of the steel, and that the cement actually enters into the microscopical indentations in the surface of the metal. Since a stress in the metal even within the elastic limit necessarily reduces its cross section somewhat, the so-called adhesion will be more and more reduced as the stress in the metal becomes greater. This view of the case has been verified by experiments by Professor Talbot, who used bars made of tool steel in many of his tests. These bars were exceptionally smooth; and concrete beams reinforced with these bars failed generally on account of the slipping of the bars. Special tests to determine the bond resistance showed that it was far lower than the bond resistance of ordinary plain bars. The designing of the various deformed bars is only a development of this same principle. The accidental roughness of rolled bars is purposely magnified and the resistance is correspondingly increased. The deformed bars have a variety of shapes; and since they are not prismatic, it is evident that, apart from adhesion, they cannot be drawn through the concrete without splitting or crushing the concrete immediately around the bars. The choice of form is chiefly a matter of designing a form which will furnish the greatest resistance, and which at the same time is not unduly expensive to manufacture.

Tests on plain bars have shown that until the load produces a bond stress of 200#/□″ or over, there is no apparent slipping. As the load is increased, the adhesion between the concrete and steel is broken and slipping begins. This is resisted by the friction between the steel and concrete. This frictional resistance increases until a slip of about 1/100″ occurs, when it is a maximum. Beyond this point the frictional resistance falls off. The behavior of deformed bars is similar to that of plain bars to the point where the slip amounts to 1/100″. Beyond that point, however, the frictional resistance between the deformed bar and concrete continues to increase until failure occurs. The failure occurs either by splitting of the concrete or shearing of the rods. The working stress in bond is $0.04f_c'$ for plain bars and $0.05f_c'$ for deformed bars, where $f_c'$ is the 28-day strength of the concrete. Table XXIII has been computed on this basis.

## TABLE XXIII

### Bond Adhesion of Plain and of Deformed Bars per Square Inch of Surface, and Also per Inch of Length

| | Bond Adhesion per Square Inch of Surface | | | |
|---|---|---|---|---|
| | $f_c'=1500$ | $f_c'=2000$ | $f_c'=2500$ | $f_c'=3000$ |
| Plain bars @ 0.04 $f_c'$ .......... | 60 | 80 | 100 | 120 |
| Deformed bars @ 0.05 $f_c'$ ...... | 75 | 100 | 125 | 150 |

Bond Adhesion per Inch of Length

| Size of Bar | ¼″ ○ | ⅜″ ○ | ½″ ○ | ½″ □ | ⅝″ ○ | ¾″ ○ | ⅞″ ○ | 1″ ○ | 1″ □ | 1⅛″ □ | 1¼″ □ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Surface; Sq. In. per Linear Inch | .785 | 1.18 | 1.57 | 2.00 | 1.96 | 2.36 | 2.75 | 3.14 | 4.00 | 4.50 | 5.00 |
| @ 60 lb. per sq. in. | 47 | 71 | 94 | 120 | 118 | 141 | 165 | 188 | 240 | 270 | 300 |
| 75 | 59 | 88 | 118 | 150 | 147 | 177 | 206 | 236 | 300 | 337 | 375 |
| 80 | 63 | 94 | 126 | 160 | 157 | 188 | 220 | 251 | 320 | 360 | 400 |
| 100 | 78 | 118 | 157 | 200 | 196 | 236 | 275 | 314 | 400 | 450 | 500 |
| 120 | 94 | 142 | 188 | 240 | 235 | 283 | 330 | 377 | 480 | 540 | 600 |
| 125 | 98 | 147 | 196 | 250 | 245 | 295 | 344 | 393 | 500 | 562 | 625 |
| 150 | 118 | 177 | 236 | 300 | 294 | 354 | 412 | 471 | 600 | 675 | 750 |

**Computation of the Bond Required in Bars.** From theoretical mechanics, it is known that the total shear at any section equals the difference in moment for a section of infinitesimal length. This may be seen from Fig. 85, where $T$ is tension in steel at left end of section, and toward the center of the span; $T'$ is tension in steel at right end of section; then $T-T'$ is the difference in tension, which is the amount of tension taken up by the concrete in the length $x$. The total horizontal shear acting on any horizontal section below the neutral axis is measured by this difference in tension. If $v$ is the unit horizontal shear, and $b$ the width of the beam, then

$$(T-T')=vbx$$

Taking moments about $a$, Fig. 85

$$Vx=(T-T')jd$$

Substituting the value just found for $(T-T')$ in this equation and solving for $v$

$$v = V \div bjd \tag{21}$$

Since the variation of $j$ is very little for the usual variations in percentage of steel and quality of concrete, it is a common practice to consider that, *as applied to this equation,* $j$ has the uniform value of .875 or ⅞. This would reduce Equation (21) to

$$v = \frac{8}{7}V \div bd = \frac{8V}{7bd} \tag{22}$$

The bond stress between steel and concrete for the length $x$ is equal to $(T - T')$. From the equation preceding Equation (23) $(T - T') =$

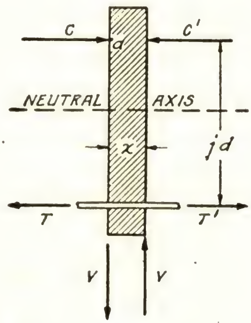



Fig. 85. Diagram for Calculating Unit Shear in a Beam

$Vx \div jd$, or for a unit length $(T - T') = V \div jd$. Let $u$ denote the permissible bond stress, and $\Sigma o$ the summation of the perimeters of all the bars in a section, then $u\Sigma o$ is the total bond stress developed in a unit length and this must equal $V \div jd$. Then

$$u = \frac{V}{jd\Sigma o} = \frac{8V}{7d\Sigma o} \tag{23}$$

Substitute the value of $V$ obtained from Equation (22) in Equation (23). Then

$$u = \frac{8V}{7d\Sigma o} = \frac{8}{7d\Sigma o} \times \frac{7Vbd}{8} = \frac{Vb}{\Sigma o} \tag{23a}$$

**Anchorage.** After a reinforcing bar has served its purpose of taking the tensile stress in a beam, etc., it is necessary to extend the bar past the point of theoretical zero stress in order to develop the bar and enable it to resist being pulled out. The distance that

the bar extends beyond the theoretical point of zero stress is known as anchorage. The length of embedment required is such that the resistance to pulling out developed by the bar, using the allowable stress in bond, is equal to or greater than the stress in the bar.

The simplest case of anchorage is that of a bar embedded in concrete and subject to a pull, Fig. 86A. The length of embedment required so that the bar will not be pulled out from the concrete is obtained as follows: Let $L$ designate the required length of embedment; $u$ the allowable bond stress; $d$ the diameter of the bar; $\Sigma o$ the perimeter of the bar, and $P$ the amount of the pull on the bar. For a round bar $P=f_s\times\frac{\pi d^2}{4}$. The bar develops a resistance to this pull equal to $u\times \Sigma o\times L$.

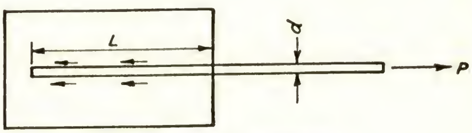



Fig. 86A. Anchorage

For a round bar $\Sigma o=\pi d$. So

$$u\pi dL=f_s\times\frac{\pi d^2}{4}$$

Solving for $L$
$$L=\frac{f_s d}{4u} \tag{24}$$

For a square bar $P=f_s\times d^2$ and $\Sigma o$ becomes $4d$. Then

$$u\times 4d\times L=f_s\times d^2$$

and
$$L=\frac{f_s d}{4u}$$

which is the same as for a round bar.

To provide for ordinary anchorage requirements, an embedment of at least 10 bar diameters is required. In noncontinuous beams at least half the positive reinforcement should be extended at the same face of the beam into the support sufficiently to provide an embedment of at least 10 bar diameters.

In continuous beams at least one fourth of the area of positive

reinforcement should extend into the support at the same face of the beam a distance sufficient to provide an embedment of at least 10 bar diameters.

All negative reinforcement in a continuous, restrained or cantilever beam should have a length of embedment beyond the face of the supporting member sufficient to develop the full maximum stress at an average bond stress of not less than $.04f_c'$ for plain bars or $.05f_c'$ for deformed bars. In addition, at least one third of the negative reinforcement should extend to the point of inflection and any

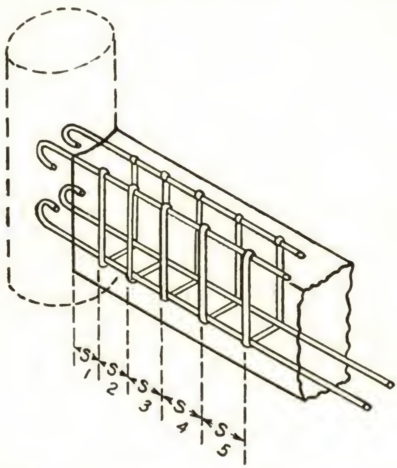



Fig. 86B. Typical Reinforced Concrete Beam, with Anchored Longitudinal Bars and Vertical Stirrups

bars not so extended are to be bent down and made continuous with the positive reinforcement or else anchored in a region of compression.

**Special Anchorage.** When reinforcing bars have special anchorage it is permissible to increase the allowable shear and bond stresses. This is indicated in the table of allowable stresses, Table XIXA. Special anchorage may be provided by an additional length of bar or by means of a 180° hook. Fig. 86B shows a beam in which the longitudinal bars are anchored by means of hooks. The radius of the hook should be not less than 4 diameters.

In footings all bars should be anchored by means of hooks at the ends of the bars. The total length of footing bars should not be less than the width of the footing plus 20 bar diameters.

In simple or freely supported beams, at least half the tension steel should extend along the tension side of the beam to provide an anchorage beyond the face of the support sufficient to develop one-third the allowable working stress in tension at an average bond stress of not more than $0.04f_c'$ for plain bars or $0.05f_c'$ for deformed bars.

In continuous and restrained beams, at least one-third the area of the negative reinforcement must be anchored beyond points of inflection, and one-third the area of the positive reinforcement must be anchored beyond the face of the support so as to develop one-third the maximum working stress in tension, with the bond stress not more than $0.04f_c'$ for plain bars or $0.05f_c'$ for deformed bars. This requirement may be simplified as follows:

Call $D$ the diameter of a bar, then "one-third the maximum working stress" is $\frac{1}{3}f_s$; this times the area $\frac{1}{4}\pi D^2 = \frac{1}{12}f_s\pi D^2$. This stress must be resisted by the bond stress which, in a unit length of $D$, has an allowable value of $.05f_c'(\pi D)$. Then the number of unit lengths of bar required to develop such a bond stress is $(\frac{1}{12}f_s\pi D^2) \div (.05f_c'\pi D) = 1.67(f_s \div f_c') \times D$. Note that $\pi$ cancels out; also that if square bars were considered, the calculation would be the same without using $\pi$. Note also that .05, the value for deformed bars, has been used; for plain bars, .04 would be used, and the coefficient would be 2.08 instead of 1.67. We may therefore write the *anchorage bar equation*

For plain bars, number of diameters for anchorage $= 2.08(f_s \div f_c')$ (24A)

For deformed bars, number of diameters for anchorage $= 1.67(f_s \div f_c')$

**Example.** For $f_c' = 2{,}000$ and $f_s = 18{,}000$, and using deformed bars, say $1\frac{1}{8}''$ square, the bars must be extended $1.67\ (18{,}000 \div 2{,}000)D = 15 \times 1\frac{1}{8} = 17''$.

## VERTICAL SHEAR AND DIAGONAL TENSION

**Diagonal Tension.** From Mechanics it is known that a homogeneous beam has inclined tensile stresses. Similarly, these stresses exist in a reinforced concrete beam. So when a concrete beam is reinforced uniformly with only horizontal bars and subjected to load, the concrete will begin to crack under very small loads. These cracks will start at the center and as the load is increased they will spread to each end, and the beam will fail under a load much smaller than that for which it was designed. As can be seen from Fig. 87 the cracks at the center are nearly vertical, but away from the center they

become inclined more and more. These cracks are called *diagonal tension cracks.* Large numbers of tests show that these diagonal tension cracks will develop and cause failure regardless of the amount of the horizontal reinforcement.

A beam is made safe against failure due to diagonal tension by the use of stirrups and of main longitudinal bars bent up at an angle after they are not needed to carry the horizontal fiber stresses.

In order to provide the proper amount of this reinforcement, it is necessary to know the intensity of the diagonal tension. Unfortunately a direct and accurate calculation of this is not possible. For purposes of design, it has been found satisfactory to use the calculated vertical shearing stresses as a measure of the diagonal tension stresses,

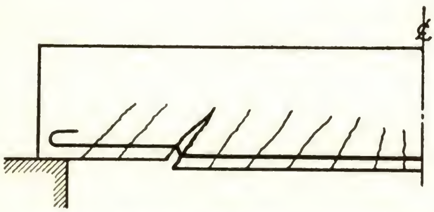


Fig. 87. Failure of Beam Reinforced Only with Horizontal Bars

and to consider the diagonal tension stresses as being a maximum along a 45-degree line and having an average value of $v=\frac{V}{bjd}$ the same as Equation (21), or transposing Equation (21) $V=bjdv$. The use of these assumptions is justified, for, if on any section below the neutral axis there existed no normal tension, the maximum diagonal tension would act at 45 degrees and would be equal to the shear at that particular section. As the shear is used as a measure of the diagonal tension, hereafter the term shear will generally be used when diagonal tension is actually meant.

As previously stated, the concrete can and does take up part of the diagonal tension stress. Calling $V_c$ the portion of the total shear which can be carried by the concrete, $v_c$ the corresponding unit value, and $j=7/8$, then

$$V_c=7/8\,bdv_c \qquad (25)$$

The allowable value of $v_c$ depends on the anchorage of the longi-

tudinal reinforcement. As shown in Table XIX-A, it is $0.02 f_c'$ where bars have no special anchorage and $0.03 f_c'$ where bars do have special anchorage. In Fig. 88A is shown the shear diagram for a beam loaded uniformly. The shear varies uniformly from a maximum at the ends to zero in the center. The line *ab* represents the amount of shear carried by the concrete, and the triangle *fba* represents the amount of shear which will have to be provided by steel, in the form of stirrups or bent up longitudinal reinforcement.

Let $V'$ equal the excess of the total shear over that permitted

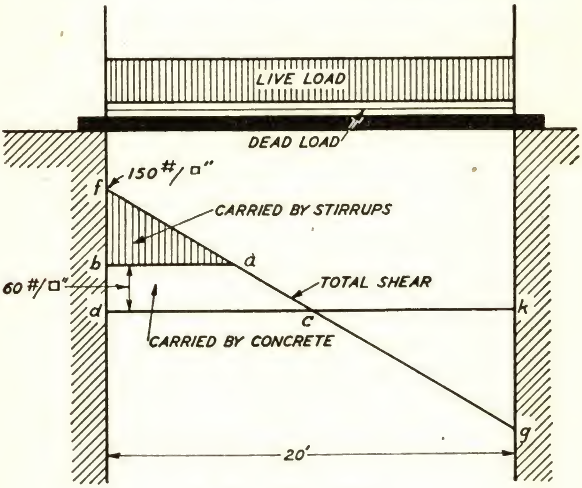



Fig. 88A. Shear Diagram

on the concrete. The allowable unit stress in stirrup bars, $f_v$, is 16,000#/□". Let $A_v$ = total area of steel in stirrups in a distance $s$, the spacing of the stirrups; this means twice the area of a stirrup bar for a **U** stirrup or four times the area for a **UU** stirrup. Then the area of steel required in stirrups shall be computed by Equation (26)

$$A_v = \frac{V's}{14{,}000d} \tag{26}$$

**Spacing the Stirrups.** In designing the stirrups for a reinforced concrete beam, the size of stirrup to be used is first selected and then the spacing along the beam is calculated. Having decided on the size of the stirrup bars (for example, a **U** stirrup of ⅜" round bars having an area of $2 \times 0.11 = 0.22 = A_v$) the number of stirrups

becomes known. This is equal to the area of the shear curve (using unit shear) to be carried by the stirrups, represented by triangle *fba,* Fig. 88A, multiplied by the width of the beam and divided by the allowable load on one stirrup. Of course, this must be a whole number usually the next higher integer. Suppose this gives 5 stirrups. Now triangle *fba,* Fig. 88B, must be divided into 5 equal areas. The stirrups should pass through the center of gravity of each of these areas. The area of triangle $f'b'a$ is one-fifth of the total area *fba,* the area of triangle $f''b''a$ is two-fifths, etc., and since the triangles are all similar, their respective areas vary as the square of their homologous dimensions. In general if $L$ denotes the length of the base of the shear

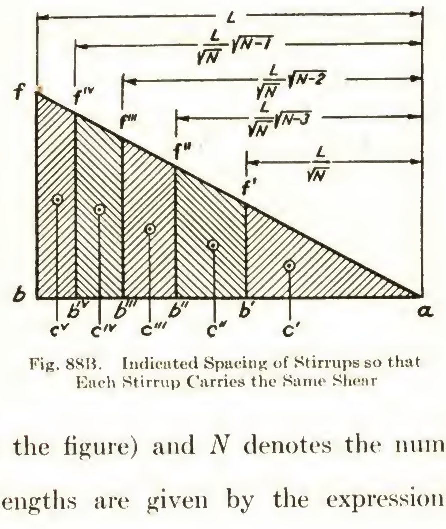



Fig. 88B. Indicated Spacing of Stirrups so that Each Stirrup Carries the Same Shear

triangle (*ab* in the figure) and $N$ denotes the number of stirrups, the required lengths are given by the expressions: $\frac{L}{\sqrt{N}}\sqrt{N-1}$; $\frac{L}{\sqrt{N}}\sqrt{N-2}$; $\frac{L}{\sqrt{N}}\sqrt{N-3}$; $\frac{L}{\sqrt{N}}\sqrt{N-4}$; and so on, the first length $ab'$ always being equal to $\frac{L}{\sqrt{N}}$ (See Fig. 88B). Then $ab'=\frac{ab}{\sqrt{5}}$; $ab''=ab\frac{\sqrt{2}}{\sqrt{5}}$; $ab'''=ab\frac{\sqrt{3}}{\sqrt{5}}$; $ab^{iv}=ab\frac{\sqrt{4}}{\sqrt{5}}$. From these, $bb^{iv}$, $bb'''$, $bb''$, and bb$'$ become known. The distance $bc'=bb'+\frac{1}{3}b'a$ .

The centers of gravity of trapezoids such as those shown in Fig. 88B are so near the center that they may be so considered.

Then $bc'' = bb'' + \frac{1}{2}b''b'$. The distance from $b$, the end of the beam, to each of the other centers of gravity may be obtained similarly. When the shearing stress is not greater than $0.06f_c'$ the distance $s$ between two successive stirrups, measured perpendicular to the direction of the stirrup, should not exceed $\frac{3}{4}d$; and where the unit shearing stress exceeds $0.06f_c'$, it should not be greater than $\frac{3}{8}d$.

**Resisting Shear by Bending Up the Longitudinal Bars.** Resistance to diagonal tension is efficiently provided by bending up the main longitudinal reinforcement bars as soon as they can be spared from their primary work of resisting transverse moment. As these bars are bent up, usually at 45 degrees, from the bottom near the ends of the beam, they will be nearly normal to the line of diagonal tension cracks and so will be very effective in resisting the diagonal tension. From this standpoint it would be advisable to design beams with a large number of small bars, so that they can be bent up at proper intervals and still have left at least one pair of bars to extend straight through to the ends of the beam. But the use and bending up of a large number of small bars adds considerably to the cost of the beams. Therefore, although one or two pairs of bars are turned up diagonally near the end of each beam where the diagonal tension is the greatest, stirrups are for the most part used to resist diagonal tension.

Where there is a series of parallel bent up bars at varying distances from the support, they are considered as inclined stirrups and the area required is determined by Equation (**26**).

Where one bar or one group of bars is bent up in a single plane and used for web reinforcement, the required area of the bars is computed by Equation (**27**).

$$A_v = \frac{V'}{16000 \sin \alpha} \qquad (27)$$

In which $V'$ is the total shear carried by the bent up bars, and $\alpha$ is the angle with the horizontal by which the bars are bent upward, Fig. 89A.

In Equation (**27**) $V'$ should not exceed $0.035f_c'bd$, nor $\alpha$ be less than 15°. The usual value of $\alpha$ is 45°. Only the center three-fourths of the inclined portion of such a bar or group of bars shall be considered effective in resisting shear. Between the face of the support and the area reinforced by the bent up bar or group of bars,

other web reinforcement should be provided, except when the distance is less than $\frac{d}{2}$ and the beam is designed for uniform load only.

Care should be taken to anchor properly stirrups or bent up bars serving as diagonal tension reinforcement. Bent up bars frequently lack sufficient bond strength to make them effective, and in this event should have hooks provided at the ends.

It is important that web reinforcement be anchored at both ends by one of the following methods or combinations thereof. Only anchorage meeting the requirements of Method *1*, *2*, or *3* is to be used for shearing unit stresses in excess of $0.08f_c'$.

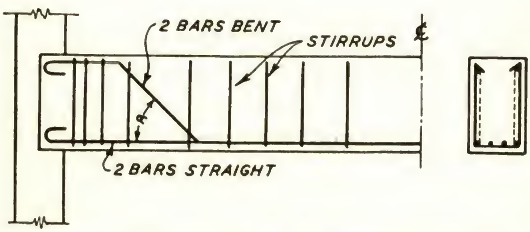



Fig. 89A. Group of Bars Used as Web Reinforcement

1. Provide continuity with the main longitudinal reinforcement.

2. Bend the web reinforcement around the longitudinal bar or steel shape.

3. Use a hook which has a radius of bend not less than 4 times the diameter of the web bar.

4. Provide a length of embedment sufficient to develop the stress in the stirrup by bond, provided that the other end of the stirrup is anchored as in Method *1*.

**Example 1.** (Shear provided by stirrups only.) A beam on a span of 20′ supports a uniform load of 3,150# per foot (including the weight of the beam). The beam is 12″ wide and 20″ deep to the center of the steel. Assuming that the concrete carries shear equivalent to 60#/□″, determine the number and spacing of the stirrups required in this beam.

*Solution.* The total load on the beam is $3150 \times 20 = 63{,}000\#$, and the end shear is one half of that, or 31,500#. Using Equation (22) the unit shear $v = \frac{8V}{7bd} = \frac{8 \times 31{,}500}{7 \times 12 \times 20} = 150\#/\square''$. As the concrete can only carry shear equivalent to a unit shear of 60#/□″, stirrups will be required to take the rest of the shear. Using Equation (25) and the unit stress of 60#/□″, the concrete can safely carry

a total shear of $V = 60 \times 7/8 \times 12 \times 20 = 12{,}600\#$. As the beam is loaded with a uniform load, the shear will vary from zero at the center to a maximum of $150\#/\square''$ at the end, and the shear curve is a triangle. (See Fig. 88A.) Then the length for which the concrete can safely carry the shear equals $\frac{60}{150} \times 10 \times 12 = 48''$ from the center of the beam, each way, and stirrups will be required for the balance of the span or for 72″ from each end. Now the total shear carried by the stirrup is represented in Fig. 88A by the area *abf* and is equal to $\frac{(150-60) \times 72 \times 12}{2} = 38{,}880\#$. Using $3/8''\phi$ stirrups, one stirrup will furnish $2 \times 0.11 = 0.22\square''$ of area. As $f_s$ for stirrups is limited to $16{,}000\#/\square''$, one $3/8''\phi$ stirrup can safely carry $0.22 \times 16{,}000 = 3{,}520\#$. Then to carry $38{,}880\#$ will require $\frac{38{,}800}{3520} = 11.1$ or 12 stirrups.

If the calculation is made in terms of the total shear instead of the unit load, use can be made of Equation (26) $A_v = \frac{V's}{14{,}000d}$. The average shear to be carried by the stirrups over the 72″ is $\frac{1}{2} \times 18{,}900 = 9{,}450\#$. Then $A_v = \frac{9{,}450 \times 72}{14{,}000 \times 20} = 2.43\square''$. So stirrups furnishing $2.43\square''$ of area must be supplied. Dividing by 0.22, the area of one stirrup, gives $\frac{2.43}{0.22} = 11.1$ or 12 stirrups as before.

To determine the spacing of the stirrups the method outlined under "Spacing the Stirrups" should be studied. In Fig. 88A the end shear is denoted by $fd = 150\#/\square''$, the shear carried by the concrete is indicated by $bd = 60\#/\square''$ and $fb$ indicates the shear to be carried by the stirrups $= 90\#/\square''$. The base $ba$ has already been determined as 72″. Then the triangle $fba$ must be divided into 12 equal parts. Accordingly, the distance from $a$ along $ab$ to the first triangle will be $ab \div \sqrt{12} = 72 \div \sqrt{12} = 20.8''$. The second distance equals $20.8\sqrt{2} = 29.4''$ from $a$. The third distance $= 20.8\sqrt{3} = 36.0''$ and correspondingly the other distances are 41.6, 46.4, 50.9, 55.0, 58.8, 62.4, 65.8 and 69.0 inches. The widths of the areas then are: 20.8, 8.6, 6.6, 5.6, 4.8, 4.5, 4.1, 3.8, 3.6, 3.4, 3.2 and 3.0 inches. The center of gravity of the areas from the end of the beam will be 1.5, 4.6, 7.9, 11.4, 15.1, 19.0, 23.3, 28.0, 33.2, 39.3, 46.9 and 58.1 inches. Adding 2/3 of $20.8'' = 13.9''$ to the last term checks the 72″. The stirrup spacing then will be (from the end of the beam toward the center) 1½, 3, 3, 3½, 4, 4, 4, 5, 5, 6, 7 and 11 inches. It is also necessary to anchor the stirrups at both ends. This is done by means of a hook, which may be turned either in or out depending on the conditions in the beam. The usual shapes to which stirrups are bent are shown in Fig. 89B.

**Example 2.** (Shear provided for by bent up bars.) Assume that the beam in Example 1 is reinforced with four $1''\phi$ bars. Two of the four bars are bent up at an angle of 45° at a distance equal to 1/6 of the span from the support. Determine whether the bars furnish sufficient steel area to take care of the diagonal tension if the concrete carries $60\#/\square''$ as before.

*Solution.* The span is $20' = 240''$. The point of bend is then $1/6 \times 240'' = 40''$ from the support. Allowing 2″ from the top of the beam to the center of the steel, the vertical and horizontal projection of the bent bars is equal to 18″. Then the bars will resist the diagonal tension for a distance of 18″ beginning 22″

from the support. Referring to the shear diagram, Fig. 88A, the unit shear at 22″ from the support $=\left(\frac{120-22}{120}\times 150\right)=122\#/\square''$ and at 40″ from the support is $\left(\frac{120-40}{120}\times 150\right)=100\#/\square''$ and the average unit shear for the distance of 18″ is $\frac{122+100}{2}=111\#/\square''$. As the concrete carries $60\#/\square''$, the steel will have to carry $51\#/\square''$. The amount of shear to be carried by the steel is equal to $51\times$

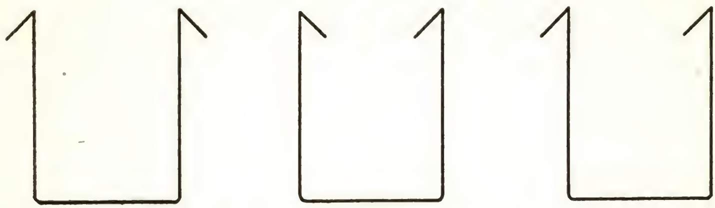


Fig. 89B. Three Types of Stirrups

$12\times 18=11{,}020\#=V'$. Now substituting in Equation (**27**) the area of steel required to resist this stress is $A_v=\frac{V'}{16{,}000 \sin \alpha}$ or $A_v=\frac{11{,}020}{16{,}000\times .707}=0.97\square''$. And as the two 1″ $\phi$ bars furnish an effective area of $1.57\square''\times \frac{3}{4}=1.18\square''$ the bent up bars take all the shear stresses for the distance which they cover. It will, however, be necessary to provide stirrups for the distance of 22″ from the end of the bent up bar to the support and from the point of bend 32″ toward the center of the beam where the concrete can safely carry the load. So while the bent up bars can take the shear stresses, in practice stirrups usually are used throughout the beam.

If the bars had been bent up singly at, say, 18″ and 36″ from the support, Equation (**26**) would be used to determine the required steel area.

# CHAPTER XI

## T-BEAM DESIGN

When concrete beams are placed in conjunction with overlying floor slabs, the concrete for both the beams and the slabs being placed in one operation, the strength of such beams is very much greater than their strength considered merely as plain beams, even though the depth of the plain beams is equal to the total depth of the beam and the slab. An explanation of this added strength may be made as follows:

If a very wide beam is constructed as shown by the complete rectangle in Fig. 90 there is no hesitation about calculating its strength as that of a rectangular beam whose width is $b$, and whose effective depth to the reinforcement is $d$. Previous study in rectangular beams has shown that the steel in the bottom of the beam takes care of practically all the tension; that the neutral axis of the beam is somewhat above the center of its height; that the only work of the concrete below the neutral axis is to transfer the stress in the steel to the concrete in the top of the beam; and that even in this work it must be assisted somewhat by stirrups or by bending up the steel bars. If, therefore, two rectangles are cut out from the lower corners of the beam as shown by the unshaded areas, a large part of the concrete is saved with very little loss in the strength of the beam, provided certain conditions are fulfilled. The steel, instead of being distributed uniformly throughout the bottom of the wide beam, is concentrated into the comparatively narrow portion which shall hereafter be called the stem of the beam. The concentrated tension in the bottom of this stem must be transferred to the compression area at the top of the beam. The beam must also be designed so that the shearing stresses in the plane $mn$ immediately below the slab shall not exceed the allowable shearing stress in the concrete, and so that failure shall not occur on account of shearing in the vertical planes $mr$ and $ns$ between the sides of the beam and the flanges.

**Resisting Moments of T-Beams.** The resisting moments of **T**-beams will be computed in accordance with straight-line formulas. There are three possible cases, according as the neutral axis is: (1) *below* the bottom of the slab (which is the most common case, and which is illustrated in Fig. 91); (2) *at* the bottom of the slab; or (3) *above* it. All possible effect of tension in the concrete is ignored. For *Case I*, even the compression furnished by the concrete between

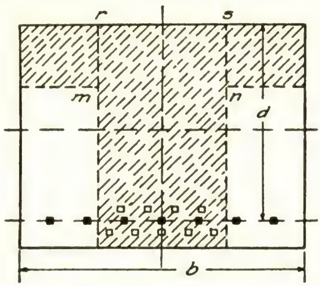



Fig. 90. Diagram of T-Beam in Cross Section

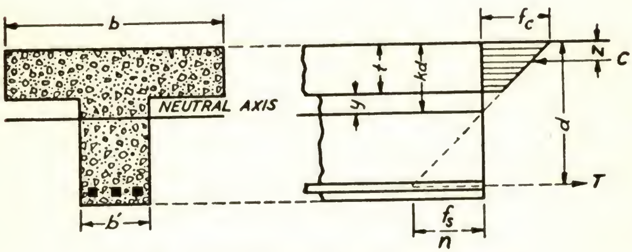



Fig. 91. Compression Stress Diagram for T-Beam

the neutral axis and the under side of the slab is ignored. Such compression is, of course, zero at the neutral axis; its maximum value at the bottom of the slab is small; the summation of its compression is small; the lever arm is not more than $\frac{2}{3}y$; therefore, the moment due to such compression is insignificant compared with the resisting moment due to the slab. The computations are much more complicated if it is included; the resulting error is a very small percentage of the true figure, and the error is on the side of safety.

*Case I*. Let $f_c$ be the maximum unit compression at the top of the slab, and, as the stress-strain diagram is rectilinear, Fig. 91, the unit

compression at the bottom of the slab is $f_c \frac{kd-t}{kd}$. The average unit compression equals $\frac{1}{2}(f_c + f_c \frac{kd-t}{kd}) = \frac{f_c}{kd}(kd - \frac{1}{2}t)$. The total compression $C$ equals the average unit compression multiplied by the area $bt$; or

$$C = bt\frac{f_c}{kd}(kd - \tfrac{1}{2}t) = A_s f_s \tag{28}$$

The center of gravity of the compressive stresses is evidently at the center of gravity of the trapezoid of pressures. The distance from the upper base of a trapezoid to its center of gravity, $z$, is given by the formula

$$z = \frac{d(b+2b_1)}{3(b+b_1)}$$

where $d$ is the height of the trapezoid, $b$ is the upper base, and $b_1$ is the lower base. Substituting the proper values in this expression and reducing to the simplest terms,

$$z = \frac{t}{3} \times \frac{3kd-2t}{2kd-t} \tag{29}$$

It has already been shown (Equations 10 and 11) that

$$\frac{\epsilon_s}{\epsilon_c} = \frac{1-k}{k}; \text{ and } \frac{\epsilon_s}{\epsilon_c} = \frac{f_s}{nf_c}$$

Equating these expressions and solving for $f_s$,

$$f_s = \frac{n f_c(1-k)}{k} \tag{30}$$

Combining this equation with Equation (28) and simplifying

$$kd = \frac{A_s nd + \frac{1}{2}bt^2}{A_s n + bt} \text{ or } k = \frac{A_s n + \frac{bt^2}{2d}}{A_s n + bt} \tag{31a}$$

or in terms of $p$

$$k = \frac{pn + \frac{1}{2}\left(\frac{t}{d}\right)^2}{pn + \frac{t}{d}} \tag{31b}$$

The arm of the resisting couple is $(d-z)$, but this is defined as $jd$.

Then

$$\left.\begin{aligned} M_c&=C(d-z)=Cjd=bt\frac{f_c}{kd}(kd-\tfrac{1}{2}t)jd \\ M_s&=T(d-z)=Tjd=A_sf_sjd=pbdf_sjd \end{aligned}\right\} \qquad (32)$$

These equations enable us to compute the moment with reference to the steel and to the concrete, and to determine which condition limits the moment.

All of the above calculations have been made neglecting the compression in the stem between the neutral axis and the under side of the flange. This is justified as the stem usually is small compared to the width of the flange, and the compression area in the stem is negligible. Occasionally the stem is large compared to the flange and it is desirable and economical to include the compression in the stem in the calculations. In this case it is simpler to calculate the position of the neutral axis, etc., by means of the transformed section. The formulas for $k$ etc., are very complicated as shown in the expression for $kd$

$$kd=\sqrt{\frac{2ndA_s+(b-b')t^2}{b'}+\left(\frac{nA_s+(b-b')t}{b'}\right)^2}-\frac{nA_s+(b-b')t}{b'}$$

*Case II.* When the neutral axis is located at the bottom of the slab, $k=\frac{t}{d}$. Substitute $\frac{t}{d}$ for $k$ in Equation (30) and solve for $d$. The equation becomes

$$d=\frac{t(f_cn+f_s)}{f_cn} \qquad (33)$$

This gives a relation between $d$, $f_c$, $f_s$, $n$, and $t$, which holds when the neutral axis is just at the bottom of the slab. A combination of dimensions and stresses which would place the neutral axis *exactly* in this position is improbable, although readily possible; but Equation (33) is useful to determine whether a given numerical problem belongs to *Case I* or *Case III.* When the stresses $f_s$ and $f_c$ in the steel and concrete, the ratio $n$ of the elasticities, and the thickness $t$ of the slab are all determined, then the solution of Equation (33) will give a value of $d$ which would bring the neutral axis at the bottom of the slab. But it should not be forgotten that the compression in the concrete and the tension in the steel will not simultaneously have certain definite values, say $f_c=800\#/\square''$ and $f_s=20{,}000$, unless the ratio of steel has been so chosen as to give those simultaneous values.

When some other ratio of steel is used, the equation is not strictly applicable, and it therefore should not be used to determine a value of $d$ which will place the neutral axis at the bottom of the slab and thus simplify somewhat the numerical calculations. For example, for $f_c=1{,}000\#/\square''$, $f_s=20{,}000\#/\square''$, $n=12$, and $t=6''$, $d$ will equal 16″. Of course this particular depth may not satisfy the requirements of the problem. If the proper value for $d$ is *less* than that indicated by Equation (33), the problem belongs to *Case III*; if it is *more*, the problem belongs to *Case I*.

*Case III.* Where the neutral axis of a **T**-beam is above the bottom of the flange, then the stress diagram is a triangle, with a base $f_c$ and a height $kd$ which is less than $t$. Thus the equations of a simple beam apply to this case with the exception that $b$ is to be taken as the width of the flange or slab, and not confined to the width of the stem. *Case II* may be considered as the limiting condition of *Case III.*

**Width of Flange.** In reinforced concrete construction the beams and slabs are monolithic and there is no distinct separation between the slabs and the beams and girders. Therefore, when the beams and girders are loaded, the slab adjacent to the beams and girders helps the beams and girders to resist the load. The question arises as to how much of the slab should be included as being effective in resisting the moments which the beams and girders have applied to them.

To guard against using too much flange in calculating the resisting moment of a **T**-beam, the effective flange width has three restrictions placed on it.

1. The effective flange width should not exceed the center-to-center distance between beams. In this case it extends from the center of one panel to the center of the next panel.

2. The effective flange width cannot exceed one-fourth of the span length of the beam.

3. The overhanging width on either side of the web shall not exceed 8 times the thickness of the slab. This limits the effective width to $16t+b'$.

The effective flange width used in calculating the resisting moment of a **T**-beam is the least value determined by these three conditions.

If the beam has the flange on one side only, the effective *over-*

*hanging* flange width should not exceed: (1) half the clear distance to the next beam; (2) nor one-twelfth of the span length of the beam; (3) nor six times the thickness of the slab. Then the effective flange width is the least of these three values plus the width of the stem.

**Width of Stem.** Since it is assumed that all of the compression occurs in the slab, the only work done by the concrete in the stem is to transfer the tension in the steel to the slab, to resist the shearing and web stresses, and to keep the bars in their proper place. The width of the stem is somewhat determined by the amount of reinforcing steel which must be placed in the stem, and whether it is desirable to use two or more rows of bars instead of only one row. As indicated in Fig. 90, the amount of steel required in the base of a **T**-beam is frequently so great that two rows of bars are necessary.

The requisite width of beam which will allow for proper spacing of the bars has been explained and tabulated in Table XXII. A glance at the table will show immediately whether it is possible to space them in one row; and, if this is not possible, the necessary arrangement can be readily designed. For example, assume that six $\frac{7}{8}$-inch round bars are to be used in a beam. The table shows that adding the common difference ($2\frac{3}{16}$) to $12\frac{5}{8}$ inches (for 5 bars), the required width of the beam will be $14\frac{13}{16}$ inches; but if, for any reason, a beam $10\frac{1}{2}$ inches wide is considered preferable, the table shows that four $\frac{7}{8}$-inch bars may be placed side by side, leaving two bars to be placed in an upper row. Following the same rule regarding the spacing of the bars in vertical rows, the distance from center to center of the two rows should be $2\frac{1}{2} \times \frac{7}{8} = 2\frac{3}{16}$ inches, showing that the rows should be, say $2\frac{1}{4}$ inches apart center to center. It should also be noted that the plane of the center of gravity of this steel is at two-sixths of the distance between the bars above the lower row, or that it is $\frac{2}{6} \times 2.25 = .75 = \frac{3}{4}$ inch above the center of the lower row.

**Depth of Beams.** In designing floors for buildings, the structural designer seldom can use the most economical depths for beams and girders. Usually architectural features control to a large extent the depth of the construction. When deep beams and girders are used, higher walls and longer columns are required to secure the same head room, compared with a more shallow depth of floor construction. When a beam that can be designed satisfactorily with a depth of

16″ is increased to 20″ or more in depth, a saving will be made in the amount of reinforcing steel required, but this saving is partly offset by additional concrete and form work. The reduction in head room may also enter the problem.

Beams of ordinary span and spacing are often designed with a depth of ¾ inch to 1 inch for each foot of span. Small continuous beams may be designed with only ½ inch in depth per foot of span, while larger beams may require a depth of over an inch. Girders should always be deeper than the beams they support. The above is intended as a guide to the beginner and not as a set of rules.

**Example 1.** A floor with a 4″ slab is subjected to a live load of 200#/□′, and is supported by beams spaced 5′ on centers which have a span of 20′. Determine the dimensions of the beams and the size and number of the reinforcing bars if the allowable fiber stresses are $f_c = 800\#/\square''$ and $f_s = 18{,}000\#/\square''$ with $n = 15$.

*Solution.* There is an economical depth for such a beam, based on the cost of the forms. This depth is probably 20″ (1″ of depth per foot of span). A beam 8″ wide with a total depth of 20″ will be assumed to be satisfactory for this problem.

A 4″ slab weighs 50#/□′. Then the weight of the slab for a 5′ panel is $50\# \times 5' \times 20' = 5{,}000\#$. With a total depth of 20″ the beam will be 16″ deep under the slab, and the weight of the beam equals $\frac{8 \times 16}{144} \times 150\# \times 20 = 2{,}670\#$. The live load carried by the beam equals $200\#/\square' \times 5 \times 20 = 20{,}000\#$. Then the total load carried by the beam equals $20{,}000 + 5{,}000 + 2{,}670 = 27{,}670\#$. Assume that the beams are simply supported. Then the moment equals $\frac{Wl}{8}$, or $M = \frac{27{,}670 \times 20 \times 12}{8} = 830{,}100''\#$. Assuming that the steel will be placed in two layers with 1½″ protection on the bottom of the lower bars, the distance from the bottom of the beam to the center of the steel is 3″, and $d$ equals $20 - 3 = 17''$. The area of steel required is found by using the lower form of Equation (32). Substituting in this equation,

$$830{,}100 = A_s \times 18{,}000 \times .87 \times 17$$

or

$$A_s = \frac{830.100}{18{,}000 \times .87 \times 17} = 3.12\square''$$

This area can be furnished by four 1″φ bars (3.14□″) and $p = \frac{3.14}{60 \times 17}$, or $p = .0031$. It is advisable to check the compressive stress in the concrete by using the upper form of Equation (32). However, before this can be done, the value of $kd$ must first be determined by using Equation (31a). Substituting in this equation,

$$kd = \frac{(3.14 \times 15 \times 17) + (\frac{1}{2} \times 60 \times (4)^2)}{(3.14 \times 15) + (60 \times 4)} = \frac{1280.7}{287.1} = 4.46''$$

Using this value in Equation (29)

$$z = \frac{4}{3} \times \frac{(3 \times 4.46) - (2 \times 4)}{[(2 \times 4.46) - 4]} = 1.46''$$

Then $jd$ equals $17.00'' - 1.46'' = 15.54''$. Now all the values in the upper form of Equation (32) are known except $f_c$. Substituting

$$830{,}100 = 60 \times 4 \times \frac{f_c}{4.46}(4.46-2)\ (15.54)$$

and
$$f_c = \frac{830{,}100 \times 4.46}{60 \times 4 \times 2.46 \times 15.54} = 404\#/\square''$$

This stress is well within the allowable stress for concrete. The beams will be 8″ wide and 20″ deep reinforced with four $1''\phi$ bars. The depth of the beam can be reduced 3″ or 4″ if desired. Of course, the steel area will be increased. The shear stresses for this beam will be discussed under the subject of "Shear Stresses."

**Example 2.** A floor is to be designed for a live load of $50\#/\square'$ using steel tile joists 25″ center to center. The joists are 5″ wide with 20″ clear between the joists. The floor finish will weigh $25\#/\square'$ and a suspended ceiling below the floor will weigh $10\#/\square'$. To provide space for electric conduits, the floor slab is made 2½″ thick. Determine the depth of the joists and the reinforcing required if the joists span 16′ and $f_c = 800\#/\square''$, $f_s = 20{,}000\#/\square''$ and $n = 15$. Conditions are such that $M = \frac{Wl}{12}$.

*Solution.* The weight of a 2½″ slab is $31\#/\square'$. Then the total load per square foot to be carried by the joist is $50+25+10+31=116\#/\square'$. The total load per joist equals $116 \times 2.08 \times 16 = 3860\#$ plus the weight of the joist itself. Assuming a joist with a total depth of 8″, the weight of the joist below the slab is $\frac{5'' \times 5\frac{1}{2}''}{144} \times 150 \times 16 = 458\#$. Then the total weight carried by the joist is $3860+458=4318\#$. The moment $M = \frac{Wl}{12} = \frac{4318 \times 16 \times 12}{12} = 69{,}088''\#$. Allowing 1¼″ from the bottom of the beam to the center of the steel, $d$ equals $8'' - 1.25'' = 6.75''$, and the area of steel required $A_s = \frac{M}{f_s jd} = \frac{69{,}088}{20{,}000 \times .875 \times 6.75} = 0.58\square''$ of steel. Then two $\frac{5}{8}''\phi$ bars ($.61\square''$) will be used. Checking for the stress in the concrete, as in Example 1, $kd$ is found equal to 1.95″. As $kd$ is less than $t$, $z = \frac{kd}{3} = 0.65''$, and $jd$ is therefore equal to 6.10″ (6.75−0.65). Substituting in the upper form of Equation (32), $f_c$ is found equal to $505\#/\square''$ which is well within the allowable limit.

Referring to Table XXII, note that two ⅝″ round bars require a beam 5¼″ in width if the bars are to have 1½″ of protection on the side. However, in this type of construction the protection allowed is one inch. Therefore, joists 5″ wide are satisfactory.

The terra cotta tile and joist construction with tile 12″, 16″ or 20″ wide used as a filler between the joists is designed in the same way.

**Calculations by Approximate Formulas.** A great deal of **T**-beam computation is done on the basis that the center of pressure of the concrete is at the middle of the slab and, therefore, that the lever

arm of the steel equals $d-\frac{1}{2}t$. From this assumption the following approximate equation can be written

$$M_s = A_s f_s (d-\tfrac{1}{2}t) \tag{34}$$

If the values of $M_s$ and $f_s$ are known or assumed, a reasonable value for either $A_s$ or $d-\frac{1}{2}t$ may be assumed and the corresponding value of the other calculated. On the assumption that the slab takes all the compression, the distance between the steel and the center of compression of the concrete varies between $d-\frac{1}{2}t$ and $d-.14t$, which is the approximate value when the beam becomes so small that it merges into the slab. The smaller value $d-\frac{1}{2}t$ is the absolute limit which is never reached. Therefore the lever arm is always larger than $d-\frac{1}{2}t$. Therefore, if Equation (34) is used to compute the area of steel $A_s$ for a definite moment $M_s$ and unit steel tension $f_s$, the resulting value of $A_s$ for an assumed depth $d$, or the resulting value of $d$ for an assumed area $A_s$ will be larger than necessary. In either case the result is safe.

As an illustration, using the values in Example 2 of $M_s = 69{,}088\#''$; $f_s = 20{,}000\#/\square''$; $(d-\frac{1}{2}t) = 6.75''-1.25''$ or $5.50''$, the resulting value of $A_s$ equals $\dfrac{69{,}088}{20{,}000\times 5.5} = .63\square''$, which is larger than the value previously calculated.

Equation (34) is particularly applicable when the neutral axis is in the stem. Under this condition, the average pressure on the concrete of the slab is always greater than $\frac{1}{2}f_c$, or at least it is never less than $\frac{1}{2}f_c$. As before explained, the average pressure just equals $\frac{1}{2}f_c$ when the neutral axis is at the bottom of the slab. We may, therefore, say that the total pressure on the slab is always greater than $\frac{1}{2}f_c bt$. Therefore the following approximate equation may be written,

$$M_c = \tfrac{1}{2}f_c bt(d-\tfrac{1}{2}t) \tag{35}$$

As before, the values obtained from this equation are safe, but are unnecessarily so. Apply the equation to Example 1 by substituting $M_c = 830{,}100''$, $b = 60''$, $t = 4''$, and $(d-\frac{1}{2}t) = 15''$, and compute $f_c$.

$$830{,}100 = \tfrac{1}{2}f_c \times 60 \times 4 \times 15$$

$$f_c = \frac{830{,}100\times 2}{60\times 4\times 15} = 461\#/\square''$$

But this approximate value of $f_c$ is greater than the true value; and

if this value is safe, then the true value is certainly safe. The more accurate value of $f_c$, computed in the example cited, is 404#/□″.

If the beam is so shallow that it is known even without the test of Equation (33), that the neutral axis is certainly within the slab, then the center of pressure is certainly less than $\frac{1}{3}t$ from the top of the slab, and the lever arm is certainly more than $d-\frac{1}{3}t$; and therefore Equation (34) may be modified to read

$$M_s = A_s f_s (d - \tfrac{1}{3}t) \tag{36}$$

The area of steel obtained by use of this equation will again be larger than the actual amount needed and is on the safe side.

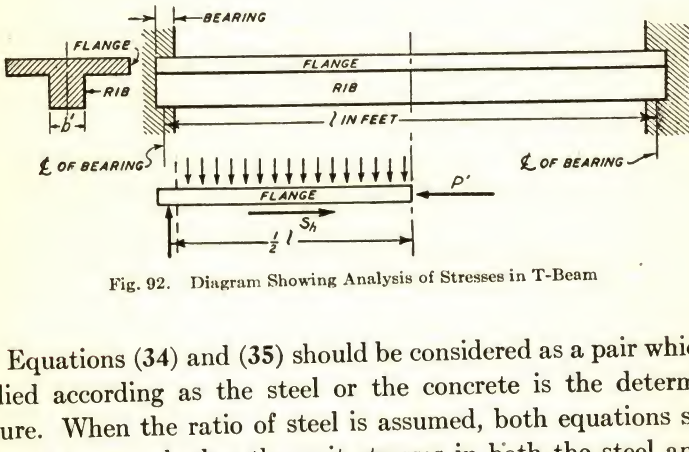



Fig. 92. Diagram Showing Analysis of Stresses in T-Beam

Equations (34) and (35) should be considered as a pair which are applied according as the steel or the concrete is the determining feature. When the ratio of steel is assumed, both equations should be used to test whether the unit stresses in both the steel and the concrete are safe. It is impracticable to form a simple approximate equation corresponding to Equation (36), which will express the moment as a function of the compression in the concrete. Fortunately it is unnecessary, since, when the neutral axis is within the slab, there is always an abundance of compressive strength.

**Shearing Stresses between Beam and Slab.** Every solution for **T**-beam construction should be tested at least to the extent of knowing that there is no danger of failure on account of the shear between the beam and the slab, either on the horizontal plane at the lower edge of the slab, or in the two vertical planes along the two sides of the beam.

Consider a **T**-beam such as is illustrated in Fig. 92. In the lower

part of the figure is represented one-half of the length of the flange, which is considered to have been separated from the stem. Following the usual method of considering this as a free body in space, acted on by external forces and by such internal forces as are necessary to produce equilibrium, it is acted on at the left end by the abutment reaction, which is a vertical force, and also by a vertical load on top. Consider $P'$ to represent the summation of all compressive forces acting on the flange at the center of the beam. In order to produce equilibrium, there must be a shearing force acting on the underside of the flange. Represent this force by $S_h$. Since these two forces are the only horizontal forces, or forces with horizontal components, which are acting on this free body in space, $P'$ must equal $S_h$. Consider $q$ to represent the shearing force per unit of area. From the laws of mechanics it is known that, with a uniformly distributed load on the beam, the shearing force is maximum at the ends of the beam, and diminishes uniformly toward the center, where it is zero. Therefore the average value of the unit shear for the half length of the beam must equal $\frac{1}{2}q$. As before, represent the width of the stem by $b'$. For convenience in future computations, consider $l$ to represent the length of the beam, measured in feet. All other dimensions are measured in inches. Therefore the total shearing force along the lower side of the flange will be

$$S_h = \tfrac{1}{2}q \times b' \times \tfrac{1}{2}l \times 12 = 3qb'l \qquad (37)$$

from which $$q = \frac{S_h}{3b'l}$$

There is also a possibility that a beam may fail in case the flange, or the slab, is too thin; but the slab is always reinforced by bars which are transverse to the beam, and the slab will be placed on both sides of the beam, giving two shearing surfaces.

It should be clearly understood that this shear is *true shear*, as distinguished from the shear previously discussed under the general heading, "Vertical Shear and Diagonal Tension," and which is frequently the only kind of shear investigated. True shear, *not* complicated with tension, and which is similar to "punching shear," shows an ultimate value of about one-half that of the compressive ultimate of the same grade of concrete. In other words, a "2000-lb." concrete would have an ultimate true shearing stress of about 1,000#/□″.

If we only used 20 per cent of this ultimate (rather than the 40 per cent of the compressive ultimate, as used for compression) we would have 200#/□″ as working stress for *true* shear.

**Numerical Illustration.** It is required to test the beam which was computed in Example 1 on page 215. The values already worked out are: $f_c=404\#/\square''$, $f_s=18{,}000\#/\square''$, $b=60''$, $b'=8''$, $d=17.0''$, $l=20'$, $n=15$, and $t=4''$, $kd=4.46''$. Then the total compressive stress on the flange, see Equation (28), equals $f_c\frac{bt}{kd}(kd-\frac{1}{2}t)=404\times\frac{60\times 4}{4.46}$ $(4.46-2)=53{,}480\#$. This compressive stress measures the shearing stress between the flange and the stem.

Then $q=S_h\div 3b'l=53{,}480\#\div(3\times 8\times 20)=111\#/\square''$. Considering that this grade of concrete can safely carry 200#/□″ in *true* shear, the beam is safe in this respect. But the beam must be tested also for its ability to withstand shear in vertical planes along the sides of the stem. Since the slab in this case is 4″ thick and as both surfaces will withstand the shear, we have a width of 8″ to withstand the shear, the same as the 8 inches on the underside of the slab. The unit shear would therefore be the same as the unit shear on the underside of the slab, or 111#/□″.

Perhaps the reason why this kind of shear is not always investigated is due to the fact that, in nearly all cases, such shear is very greatly reduced by the stirrups, and, near the ends of the beam, by the bent-up bars, which always penetrate the slab and cross the plane between the slab and the stem. The two vertical planes through the slab on each side of the beam are likewise invariably crossed by numerous slab bars. Even in the above numerical case, where the beam would have stirrups and there would be bars in the slab, the true shears above computed would *not* be fully developed on account of those bars. But the above problem has been worked out to show the *method* of investigating such shear and the designer should *know* that every case is safe in this respect.

**Shear in a T-Beam.** The shear here referred to is the shear of the beam as a whole on any vertical section. It does *not* refer to the shearing stresses between the slab and the stem. The shear in **T**-beams is considered identical with that of rectangular beams except that, when using any shear formula, the width $b'$ of the stem shall be used in place of $b$, the width of a rectangular beam. The compara-

tively small shearing resistance which would be furnished by the slab is ignored. This simplification is on the side of safety. In Example 1, page 215, the total load is 27,670# and therefore the maximum shear at the end is 13,835#; the width to be used is 8″ and $jd = 15.54''$. From Equation (**21**), $v = V \div bjd = 13{,}835 \div (8 \times 15.54) = 111\#/\square''$. For this concrete $f_c' = 2000\#/\square''$; then $111\#/\square'' = 0.056 f_c'$. This is less than the limit $0.09 f_c'$ but stirrups, or bent-up bars, or a combination of the two, would be used. Since only four longitudinal bars are used and two must be run through on the bottom, only two are left for bending up. Therefore the extra shearing strength must be provided by using stirrups. These would be designed as previously explained.

## BEAMS REINFORCED FOR TENSION AND COMPRESSION

**Beams Reinforced for Both Tension and Compression.** It sometimes happens that it is difficult, if not impossible, to have a sufficient area of concrete in compression to take up all the compression stress in a beam. This happens when the depth of a beam is limited by some architectural requirement, or when the flange of a **T**-beam cannot be made wide enough to provide the necessary area in concrete, or when there is insufficient compressive resistance in the *bottom* of a beam where there is a negative moment over a support and where there is no slab to assist in the compression. In such cases the simplest and perhaps least expensive solution is to insert one or more bars near the compression face of the beam, usually at about one-tenth the total depth of the beam from the compression face or at least sufficiently far from that face to have proper concrete protection. Note that wherever the moment of a beam is "negative," as it is where a "continuous" beam passes over a support, the *bottom* of the beam is the compression face. The depth from compression surface of beam (or slab) to the center of compression reinforcement is called $d'$. The ratio of the effective area of the compression reinforcement to the effective area of concrete in the beam is called $p'$. Note that $p'$ for compression corresponds exactly to $p$ for tension.

The *location of the neutral axis* is given by the equation

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p+p')^2} - n(p+p') \qquad (38)$$

Equation (38) is derived as follows: In the stress diagram, Fig. 93, it follows from the relation of similar triangles that

$$\frac{f_s \div n}{f_c} = \frac{d-kd}{kd} \text{ or } f_s = nf_c\frac{(1-k)}{k} \tag{38a}$$

Similarly

$$\frac{f_s' \div n}{f_c} = \frac{kd-d'}{kd} \text{ or } f_s' = nf_c\frac{k-\frac{d'}{d}}{k} \tag{38b}$$

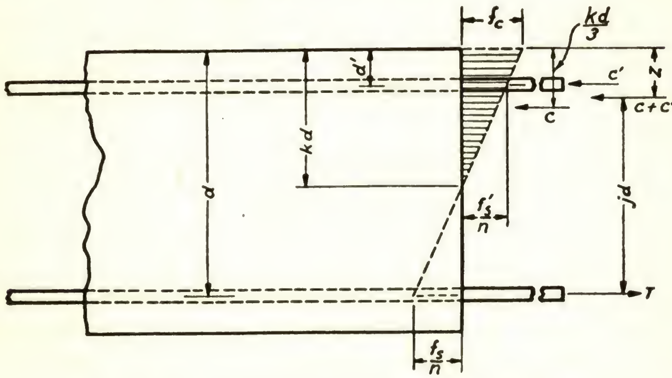



Fig. 93. Beam Reinforced for Both Tension and Compression

In simple flexure, as noted in the simple beam derivation, the total compression and total tension are equal, so

$$T = C + C' \tag{a}$$

or

$$f_sA_s = \tfrac{1}{2}f_cbkd + f_s'A_s' \tag{b}$$

where $A_s$ and $A_s'$ are the areas of the tension and compression steel respectively.

Replacing $f_s$ and $f_s'$ in Equation (b) with the values determined in Equations (38a) and (38b) and replacing $A_s$ by its equal $pbd$ and $A_s'$ by its equal $p'bd$,

$$nf_c\frac{(1-k)}{k}pbd = \tfrac{1}{2}f_cbkd + nf_c\frac{k-\frac{d'}{d}}{k}p'bd$$

Multiplying by $\frac{2k}{f_cbd}$

$$2pn(1-k) = k^2 + 2p'n\left(k - \frac{d'}{d}\right)$$

or $\qquad k^2+2kn(p+p')=2n\left(p+p'\frac{d'}{d}\right)$

Solving this for $k$,

$$k=\sqrt{2n\ (p+p'\frac{d'}{d})+n^2(p+p')^2}-n(p+p')$$

which is Equation (38).

The expression may also be solved for $p'$ giving

$$p'=\frac{p(1-k)-\frac{k^2}{2n}}{(k-\frac{d'}{d})} \tag{39}$$

It should be noted that Equation (38) corresponds closely to Equation (14) when they are compared term by term, by substituting $(p+p'\frac{d'}{d})$ for $p$ as the coefficient of $2n$, and $(p+p')$ for $p$ as the coefficient of $n^2$ and of $n$ in the second and third terms.

The *position of the resultant compression* provided by the concrete and the compressive steel is

$$z=\frac{\frac{1}{3}k^3d+2p'nd'(k-\frac{d'}{d})}{k^2+2p'n(k-\frac{d'}{d})} \tag{40}$$

In a simple beam having only tensile reinforcement $z=\frac{1}{3}kd$ as shown in Equation (8). Equation (40) may be derived by taking moments about the outer fiber of the compressive forces $C$ and $C'$. Then $z=\dfrac{C'd'+C\times\frac{kd}{3}}{C'+C}$. Substituting the values for $C$ and $C'$ given in Equation (b) and reducing will give Equation (40). As this is largely a matter of algebraic work, the detailed steps will not be shown here. To verify the equation, the student should make the substitution and solve for $z$.

The *arm of the resisting couple* is

$$jd=d-z \tag{41}$$

This is the same as for beams reinforced for tension only. See the paragraph following Equation (8), and also Fig. 78.

As for a simple beam, the resisting moment of a beam reinforced for both compression and tension can be expressed in terms of the steel stress or the concrete stress. In terms of the tension steel stress, the resisting moment is

$$M_s = f_s A_s jd = f_s pbjd^2 \tag{42a}$$

This is the same as the expression for the resisting moment for a simple beam. The resisting moment in terms of the concrete stress can be obtained by using the relation established in Equation (38a) that $f_s = nf_c\frac{(1-k)}{k}$. Then

$$M_c = nf_c\frac{(1-k)}{k}pbjd^2 \tag{42b}$$

The fiber stresses in the main steel and the outer fibers of the concrete are obtained by solving Equations (42a) and (42b) for $f_s$ and $f_c$. This gives

$$f_s = \frac{M}{A_s jd} = \frac{M}{pbjd^2} \tag{43}$$

and

$$f_c = \frac{Mk}{n(1-k)pbjd^2} \tag{44a}$$

Frequently Equation (44a) is given in the form

$$f_c = \frac{6M}{bd^2\left[3k - k^2 + \frac{6p'n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right]} \tag{44b}$$

This may be derived from Equation (44a) by replacing $j$ by its equivalent $(1 - \frac{z}{d})$, see Equation (41), and substituting for $z$ its value given in Equation (40). As this is again only a question of the proper algebraic procedure, the steps will not be shown.

It is well to note here that all the equations and expressions given above are approximate in the sense that there has been no allowance made for the reduction of the concrete compression area due to the presence of the compression steel. To take this into account, the area $A'$ should be multiplied by a factor $\left(\frac{n-1}{n}\right)$ in the equations.

There are an indefinite number of combinations of values of $p$, $p'$, $d$, and $b$, for any grade of concrete, which will fulfill the require-

TABLE XXIV-A

| $p$ | $p'$ | $k$ | $p$ | $p'$ | $k$ |
|---|---|---|---|---|---|
| .010 | .000 | .418 | .010 | .005 | .384 |
| .010 | .002 | .404 | .010 | .006 | .379 |
| .010 | .0025 | .400 | .010 | .007 | .373 |
| .010 | .003 | .397 | .010 | .008 | .368 |
| .010 | .0035 | .393 | .010 | .009 | .362 |
| .010 | .004 | .391 | .010 | .010 | .357 |

TABLE XXIV-B

**Values of $k, H,$ and $K$ for Various Values of $p$ and $p'$ When $n = 15$ and $\frac{d'}{d} = .14$**

| $p'$ \ $p$ | .010 | .011 | .012 | .015 | .020 | Values |
|---|---|---|---|---|---|---|
| .000 | .418 | .432 | .446 | .482 | .531 | Values of $k$ |
| .005 | .384 | .399 | .413 | .449 | .498 | |
| .010 | .357 | .371 | .385 | .421 | .469 | |
| .000 | 1.079 | 1.109 | 1.139 | 1.214 | 1.311 | Values of $H$ |
| .005 | 1.250 | 1.289 | 1.324 | 1.411 | 1.524 | |
| .010 | 1.414 | 1.457 | 1.499 | 1.603 | 1.730 | |
| .000 | 144 | 148 | 152 | 162 | 175 | Values of $K$ for $f_c =$ 800#/□″ |
| .005 | 167 | 172 | 177 | 188 | 203 | |
| .010 | 188 | 194 | 200 | 214 | 231 | |
| .000 | 180 | 185 | 190 | 202 | 219 | Values of $K$ for $f_c =$ 1000#/□″ |
| .005 | 208 | 215 | 221 | 235 | 254 | |
| .010 | 236 | 243 | 250 | 267 | 288 | |

ments of any definite problem. These several values may be selected almost at will within certain limitations, but when one or two elements are fixed, say $p$ and $p'$, the other elements are narrowed to very close limits.

A study of Equation (38) will show that for a given grade of concrete (e.g., 2000-lb. concrete, with $n = 15$) and for a constant ratio $d' \div d$, the values of $k$ may be tabulated for certain combinations of $p$ and $p'$. Such a fragment of the tabulation is shown in Table XXIV-A. It indicates how the value of $k$ decreases as the value of $p' \div p$ increases. The first value shown (.418) is the value of $k$

for $p'=0$, or when there is *no* compressive steel reinforcement. The student should note the almost perfect regularity in the decrease in $k$ for a difference of .001 in the value of $p'$.

Since, as before, $M \div bd^2 = K$, let the value of the bracket in the denominator of Equation (**44b**) $= H$, then rewrite Equation (**44b**) as follows:

$$f_c = \frac{6K}{H} \text{ from which } K = \frac{Hf_c}{6} \tag{45}$$

In Table XXIV-B the values of $H$ were computed and tabulated on the basis of the several values of $p'$, $p$, $n$, $d' \div d$, and $k$. Then the several values of $K$ were computed and tabulated on the basis of $f_c = 800\#/\square''$ and also for $f_c = 1000\#/\square''$.

Table XXIV-B gives a few of the values covering a wider range. As before, they include values for $p' = .000$, which is the condition when there is *no* compressive steel reinforcement, in order to show how the tabular values for the two conditions merge together. But it should be realized that the table is only a fragment of its extent if it included all values of $p'$ (varying by .002) from zero to .030, all values of $d' \div d$ (varying by .02) from .02 to .20, and also for different values of $f_s$ (18,000 and 20,000) and different values of $f_c'$ (2,000, 2,500, and more).

The choice of a combination of ratios which will give safe but economical values for the tensile and compressive stresses can be greatly facilitated by extensive tables, of which Table XXIV-B is but a sketchy fragment.

In the usual problems encountered where compression reinforcement is needed, the size of the beam has been already fixed by other factors and it is necessary to calculate only the amount of tensile and compression reinforcement necessary to keep the working stresses within their limits. There are two methods generally used to solve such problems.

In the first method the beam is considered to be a composite of two beams, one a simple beam with the proper amount of reinforcing to make a balanced design, and the other a beam of the same size as the simple beam which consists of the compression steel and the balance of the tension steel. See Fig. 94. It should be noted that the neutral axis is in the same position for both beams. The moment for the first beam $M_1$ is determined in the usual manner; that is, $M_1 =$

$f_s A_1 jd$ or $\frac{1}{2} f_c bjkd^2$. Then the excess moment $M_2$ is resisted by the couple formed by the compression steel and the rest of the tensile steel. This is equivalent to $A_2 f_s$ or $A' f'_s$ multiplied by the lever arm $(d-d')$.

Then $$M_2 = A_2 f_s (d-d')$$

and $$A_2 = \frac{M_2}{f_s(d-d')} \tag{46}$$

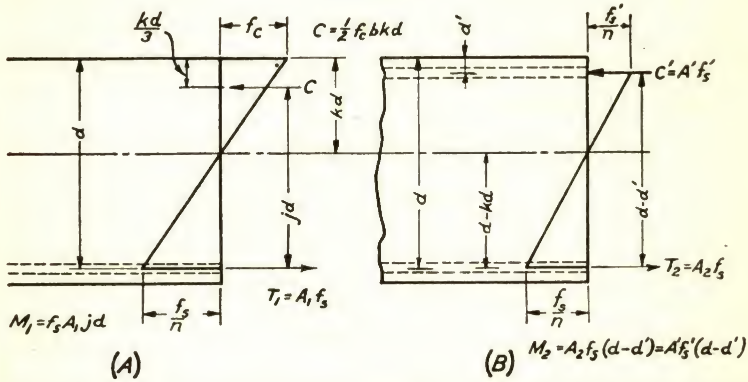



Fig. 94. Component Beams Replacing Beam Reinforced with Compressive Steel; (*A*) Simple Concrete Beam of Balanced Design; (*B*) Beam Consisting of Balanced Compression and Tensile Reinforcement

From the stress diagram of Fig. 94 at (*B*) by similar triangles

$$\frac{\frac{f_s}{n}}{\frac{f'_s}{n}} = \frac{d-kd}{kd-d'} \text{ or } \frac{f_s}{f'_s} = \frac{1-k}{k-\frac{d'}{d}} \tag{47}$$

Also $$A_2 f_s = A' f'_s \text{ or } A' = \frac{A_2 f_s}{f'_s}$$

Substituting the value of $\frac{f_s}{f'_s}$ from Equation (47) in the equation for $A'$

$$A' = \frac{A_2(1-k)}{(k-\frac{d'}{d})} \tag{48}$$

In the above equation, $k$ has the value of $k$ for the simple beam in balanced design. Then having determined the amount of tensile steel

in the simple beam $A_1$ by using the usual formulas, the amount of additional tensile steel, $A_2$, required for the unbalanced moment, is calculated by Equation (46) and the total tensile steel $A$ is the sum of the two or $A = A_1 + A_2$. The quantity $A_2$ being known, the compression steel is determined by using Equation (48) and the beam is completely determined. Equation (48) is subject to the same remarks that were made regarding the general equations first developed. No account is taken of the decrease in the area of the compression concrete. To take this into consideration, the right-hand side of Equation (48) must be multiplied by the factor $\left(\frac{n}{n-1}\right)$. For all ordinary cases it is usual to neglect this factor.

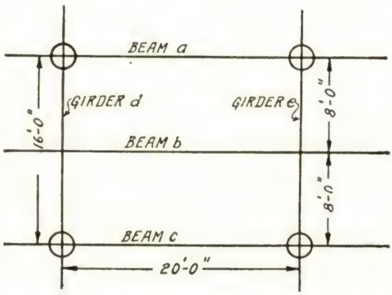



Fig. 95. Skeleton Outline of Floor Panel Showing Slab, Beam, and Girder Construction

In the second method, use is made of the formulas first developed. The dimension of the beam being fixed, an average value of ⅞ for $j$ is used and the area of tension reinforcement required is calculated. The position of the neutral axis is fixed by the values selected for $f_s$, $f_c$ and $n$, which determines the value of $k$, Equation (30). Then using Equation (39) the value of $p'$ is calculated. With the value of $p'$ known, the correct value for $j$ is easily determined and the calculations revised. The time required for this procedure can be reduced with the help of charts and tables prepared for this purpose.

## DESIGN OF A FLOOR BAY

**Numerical Illustration of Slab, Beam, and Girder Construction.** Assume a floor construction as outlined in skeleton form in Fig. 95. The columns are spaced 16 feet by 20 feet. Girders which support

the alternate rows of beams connect the columns in the 16-foot direction. The live load on the floor is 150#/□′. The concrete is to be "2000-lb." grade, with $n=15$, $f_c=800\#/□''$, and $f_s=20{,}000\#/□''$. Required: the proper dimensions for the girders, beams, and slab.

*Slab.* Assume that a 4″ slab is satisfactory. Then the total load supported by the slab is the live load (150#/□′) plus the weight of the slab (50#/□′) or 200#/□′. Taking a strip 1′ wide, the total load carried will be 8′×200#/□′ = 1600#. For interior panels this load produces a maximum moment, $M=\frac{Wl}{12}=\frac{1600\times 8\times 12}{12}=12{,}800''\#$. With $f_c=800\#/□''$, $f_s=20{,}000\#/□''$ and $n=15$, $K=131$ (Table XIX).

$$M=131bd^2 \qquad \text{Equation (B)}$$
$$12{,}800''\#=131\times 12\times d^2$$
$$d^2=8.14$$
$$d=2.85''$$

Adding 1″ for protection and 0.19″ for half the thickness of the bars, the total depth required is 2.85+1.00+0.19=4.04″. A 4″ slab will be used. The area of steel required for moment is $A_s=\frac{M}{f_sjd}$. Then $A_s=\frac{12{,}800}{20{,}000\times .87\times 2.81}=.262□''$. Use ⅜″ φ bars spaced 5″ center to center. For end spans, the bars may be hooked into the spandrel beams to form a restrained slab, or the number of bars can be increased to satisfy the equation $M=\frac{Wl}{10}$. The area of the temperature steel required is 12″×4″×.0020=.096□″. Bars ⅜″ φ spaced 14″ on centers will fulfill this requirement.

*Beams.* As shown in Fig. 95, the beams will be spaced 8′ on centers and will have a span of 20′. Assume the beam to be 10″ wide and 16″ deep. Then the beam will be 12″ deep below the slab and will weigh $\frac{10\times 12}{144}\times 150\times 20=2{,}500\#$. The load carried by a beam equals $8\times(150+50)\times 20=32{,}000\#$, plus the weight of the beam (2,500#) making a total load of 34,500#. Then for an interior panel $M=\frac{Wl}{12}=\frac{34{,}500\times 20\times 12}{12}=690{,}000''\#$. This is the positive moment at the center of the beam and the negative moment at the supports; that is,

at the girders or columns. Consider first the positive moment at the center of the beam. The total depth of the beam is 16″; allowing 2″ to the center of the steel from the bottom of the beam, $d=14''$.

As shear is quite often the determining factor in the size of a beam, the shear value will be investigated first. The end shear is $\frac{1}{2}\times 34{,}500=17{,}250\#$. Substituting in Equation (**22**), $v=\frac{8V}{7bd}=\frac{8\times 17{,}250}{7\times 10\times 14}=141\#/\Box''$. This stress is within the allowable limit of $180\#/\Box''$, but stirrups will be required since the shear stress is greater than $120\#/\Box''$, and the longitudinal steel must have anchored ends.

The area of steel required for the beam

$$A_s=\frac{M}{f_s jd}=\frac{690{,}000}{20{,}000\times .87\times 14}=2.83\Box''$$

This area will be furnished by four 1″ $\phi$ bars ($3.14\Box''$). Following the usual custom, two bars will be bent up and two bars will be straight. Investigate the compression in the flange. The width of flange is limited to the smallest of the following three quantities (See page 213): (*1*) $\frac{1}{4}\times 20\times 12=60''$, (*2*) $[(2\times 8\times 4)+10]=74''$, and (*3*) $8\times 12=96''$. Then $b$ equals 60″. Substituting in Equation (**31a**)

$$kd=\frac{A_s nd+\frac{1}{2}bt^2}{A_s n+bt}=\frac{(3.14\times 15\times 14)+\frac{1}{2}(60\times (4)^2)}{(3.14\times 15)+(60\times 4)}=\frac{1139.4}{287.1}=3.97''$$

As $kd$ is less than the thickness of the slab, $z=\frac{kd}{3}=1.32''$. Then $jd=14-1.32=12.68''$.

Substituting in the upper form of Equation (**32**) and solving for $f_c$

$$690{,}000=60\times 4\times\frac{f_c}{3.97}\left(3.97-\frac{4}{2}\right)12.68$$

$$f_c=\frac{690{,}000\times 3.97}{60\times 4(3.97-2)12.68}=457\#/\Box''$$

This value is less than the allowable $800\#/\Box''$.

Investigate the beam for moment at the girder or column. Here the moment is again $\frac{Wl}{12}$ or 690,000″#, but it is negative, which means that the bottom of the beam is in compression. Consequently, there is no flange to assist the beam and it acts as a simple beam; the

allowable stress of the concrete is increased from 800 to 900#/□″. With $f_c = 900\#/□''$, $f_s = 20{,}000$ and $n = 15$, $K = 157$, $p = .0091$, $k = .403$ and $j = .866$, the moment the beam can carry equals $M = 157bd^2 = 157 \times 10 \times (14)^2 = 307{,}720''\#$ and the tension steel required in the top equals $10 \times 14 \times .0091 = 1.27□''$. The total moment is 690,000″# which leaves an unbalanced moment of 382,280″#. This will be taken care of by steel. Allowing 2″ from the top of the beam to the center of the steel, the lever arm of the steel is $14'' - 2'' = 12''$. Using Equation (46) the additional tensile steel required for the excess moment equals $\frac{382{,}280}{20{,}000 \times 12} = 1.59□''$. Substituting in Equation (48), the amount of compressive steel required is found to be

$$A' = \frac{A_2(1-k)}{\left(k - \frac{d'}{d}\right)} = \frac{1.59(1-.403)}{\left(.403 - \frac{2}{14}\right)} = \frac{1.59 \times .597}{.260} = 3.65□''$$

The total tension steel required (in the top of the beam) equals $1.27 + 1.59□'' = 2.86□''$, and the compression steel required (in the bottom of the beam) equals 3.65□″. As two bars in each beam will be bent up at the fifth point and extended to the quarter point of the adjacent span, there will be four 1″ϕ bars or 3.14□″ of steel available in the top of the beam. This is more than enough to meet the requirement for tension steel. The straight bars in the bottom of the beam will be extended as far as needed into the adjacent span. Here again there are four 1″ϕ bars or 3.14□″ of steel furnished, but an area of 3.65□″ is needed, so an additional 0.51□″ of steel must be provided in the bottom of the beam over the girder or column. This can be supplied best by using one ⅞″ϕ bar (0.60□″). Using this bar will reduce $d$ slightly, but the excess area furnished will be more than the additional area required by the smaller value of $d$.

The fiber stress of the compression steel $f_s'$ is found by using Equation (47) and equals

$$\frac{f_s\left(k - \frac{d'}{d}\right)}{(1-k)} = 20{,}000\left[\frac{(.403 - .143)}{(1 - .403)}\right] = 8{,}710\#/□''$$

The beam always has at least two 1″ϕ bars or 1.57□″ of steel in the bottom and, at 8700#/□″, the compression they take equals 13,670#. With a lever arm of 12″ this equals a moment of 164,040″#. Then the

bottom of the beam can resist the compression equal to a moment of $307{,}720+164{,}040=471{,}760''\#$.

It is now necessary to determine how far the straight bars must be extended beyond the center of support to take care of the excess moment. The negative moment drops off quickly from the support toward the center of the span, and the point of inflection, or point of zero moment, is considered to be at $0.2l$ from the support or in this case, $0.2\times20=4'$.

Then the point where the beam can take care of the moment is determined by the point where the moment is $471{,}760''\#$. The moment curve for uniform load is a parabola. Using the properties of the parabola, the point is at a distance from the support $x$ equal to

$4\sqrt{\dfrac{690{,}000-471{,}760}{690{,}000}}$ or $2.25'$. Allowing $1.67\times\dfrac{f_s'}{f_c'}$ or $1.67\times\dfrac{8710}{2000}=$ $7.3''$ for anchoring the steel, the bars must be extended $[(2.25\times12)+7.3]=34.3''$, say $3'$ beyond the center of the support. The $\frac{7}{8}''\phi$ bar will also be extended $3'$ on each side of the center of support. Then the beam section will be $10''$ wide and $16''$ deep reinforced with four $1''\phi$ bars and one additional $\frac{7}{8}''\phi$ bar $6'$ long in the bottom of the beam over the support. Two bars in the beam will be straight and will extend $3'$ beyond the center of each support, and two bars will be bent up at the fifth point of the span at an angle of 45° and will be extended in the top of the beam to the quarter point of the adjacent span. The section will be as shown in Fig. 96.

The continuous beams ($a$, $b$, and $c$ in Fig. 95) have outer panels at each end as distinguished from the inner panel shown in Fig. 95. The moment at the center of an outer panel and at the first support is $M=\dfrac{Wl}{10}$. Since the load and span length are the same, the moment is increased over the inner panel by the ratio of 12 to 10 and therefore equals $828{,}000''\#$. Using the same dimensions as for the interior beams, the amount of steel required for this increased moment is figured in the same manner as for an interior beam. The detailed work for this will not be shown.

At the wall end the reinforcing bars must be anchored. This usually is done by means of a hook. The diameter of the hook usually is 6 bar diameters. When $1\frac{1}{8}''$ and $1\frac{1}{4}''$ square bars are used in com-

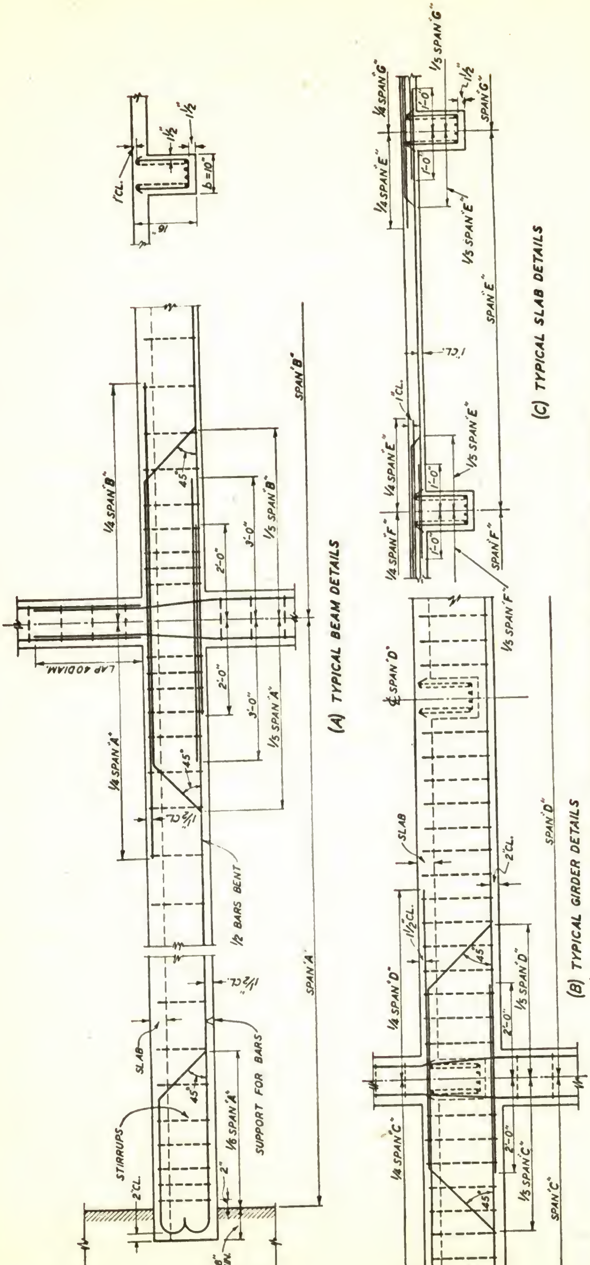



Fig. 95. Reinforcing Details for Slab, Beam, and Girder

paratively shallow beams, the diameter of the hook is reduced to 4 bar diameters.

*Girder.* The moment to be resisted by the girder shown in Fig. 95 may be computed in either one of two ways, both of which give the same result. The girder may be considered (1) as supporting a uniformly distributed load on an area of 16′ by 20′, neglecting the beams entirely, or (2) as supporting one end of two beams (equal to the total beam load) as a concentrated load in the center. The design moment in either case is the same. The shear curve, however, would not be correct for case (1). The beam load is 34,500# which is carried by the girder as a concentrated load at the center. The maximum moment at the center, due to this load, is the equivalent of a uniformly distributed load of $2\times34{,}500$ or 69,000#. To this is added the weight of the girder below the slab. In determining the depth of a girder one of the conditions to consider is the relation to the depth of the beam. The girder should be 3″ to 4″ deeper than the beams being supported by the girder, so that the reinforcing steel for the beam will be well above the steel in the girder. Accordingly the girder will be assumed to be $12''\times20''$ making a section $12''\times16''$ below the slab. The weight of the girder below the slab is $\frac{12\times16}{144}\times150\times16=3{,}200$#. Then the total equivalent load on the girder is $69{,}000+3{,}200=72{,}200$#. As in the case of the beams, the girder will be made continuous over the column. Then $M=\frac{Wl}{12}=\frac{72{,}200\times16\times12}{12}=1{,}155{,}200''$#. This is the positive moment at the center of the girder and the negative moment at the face of the column.

**Center Moment.** Allowing 3″ from the bottom of the beam to the center of the steel, $d=17''$. Then, as before, the area of steel required to resist the moment in the center of the beam $A_s=\frac{1{,}155{,}200}{20{,}000\times17\times.87}=3.91\square''$. This is furnished by four 1″ square bars (4.00). As these bars can be placed all in one row $d=(20-2)=18''$, but the same size bars will be used as steel area required with $d=18''$ is $3.69\square''$.

The allowable flange width for the girder is $\frac{1}{4}\times16\times12=48''$. Then checking the concrete stress, $kd=\frac{(4\times15\times18)+[\frac{1}{2}\times48\times(4)^2]}{(4\times15)\quad+\quad(48\times4)}=$

5.81″. $z=\frac{4}{3}\ \frac{[(3\times5.81)-2\times4]}{(2\times5.81)-4}=1.65''$; and $jd=16.35''$. Substituting in the upper form of Equation (**32**) and solving for $f_c$, $f_c=$ $\frac{1{,}155{,}200\times5.81}{48\times4\times(5.81-2)16.35}=562\#/\square''$, which is within the allowable limit.

**Negative Moment at Support.** At the support, as in the case of the beam, $f_c$ is increased to $900\#/\square''$. Then $M=Kbd^2=157\times12\times(18)^2=610{,}400''\#$, the moment which the beam can carry as a simple beam. This leaves an unbalanced moment of 1,155,200−610,400 or 544,800″# to be carried by the steel. The lever arm of the steel areas is 18″−2″ or 16″. Therefore the additional tensile steel required, using Equation (**46**), is $A_s=\frac{544{,}800}{20{,}000\times16}=1.70\square''$ and from Equation (**48**) the amount of compression steel $A'=\frac{1.70(1-.403)}{\left(.403-\frac{2}{18}\right)}=3.48\square''$.
The area of steel required for tension in balanced design equals $12\times18\times.0091=1.97\square''$ and the total tension steel equals $1.97+1.70=3.67\square''$. As four square inches of steel are furnished in both top and bottom, the girder design is adequate. Determining the length which the bottom bars must project beyond the center line of the columns as was done for the beam, it is found that 24″ or 2′0″ will meet the requirement and also provide for anchorage.

**Investigating the Shear.** The shear at the column is half the beam load plus half the weight of the girder, or $V=\frac{1}{2}(34{,}500+3{,}200)=18{,}850\#$. Substituting the known values in Equation (**21**) $v=\frac{V}{bjd}=\frac{18{,}850}{12\times16.35}=96\#/\square''$. This is satisfactory when bent up bars and stirrups are used. These would be designed as previously explained. It is well to note here, however, that the concentrated loads in the center produce considerable shearing stress throughout the beam and that the total shear at the center is only 1,600# (half the weight of the girder) less than the total end shear. In practice the spacing of stirrups required to take the end shear would be calculated and that spacing used throughout the beam.

The girder in the outer panel, like the beam in the outer panel, is subjected to a moment $M=\frac{Wl}{10}$ and the statements made with regard to the beam apply with equal weight to the girder.

SHOWING MODERN CONCRETE CONSTRUCTION IN THE GENERAL OFFICE OF JOHNSON'S WAX COMPANY, RACINE, WISCONSIN

# CHAPTER XII

## FLAT-SLAB CONSTRUCTION

**General Description.** The "flat-slab floor," Figs. 97 and 98, is one in which the floor is built of a continuous slab of uniform thickness and is supported directly on columns which usually are built with enlarged heads, called capitals. These slabs have no supporting beams. Frequently a part of the slab around the column capital is made thicker than the rest of the floor and is known as the "dropped panel."

Flat-slab floors are especially suited for heavy loads, large areas, and fairly regular column spacing. They are primarily used for factories, warehouses, garages. The advantages of the flat slab are: (1) there is a saving in the required height of the building on the basis of a given net clear height between the floors; (2) the architectural appearance is improved by having a flat ceiling surface rather than projecting beams and girders; (3) forms for flat slabs are less expensive than for beams and girders, which offsets an increase in the cost of steel and concrete; (4) it is easier to erect shafting on the ceiling; (5) there is better daytime lighting because the windows can be extended to the ceiling, and easier artificial lighting as there are no beams in the ceiling to cast shadows, and (6) ventilation is made more effective because there are no beams and girders to form pockets in the ceiling. Almost the only disadvantage is the difficulty in making definite and exact computations of the stresses as may be done for the beam and slab. Formulas, rules, and methods of computation for flat slabs have been devised which are largely empirical. However, structures designed by use of these methods have been in use for many years and have, in general, given satisfactory results.

**Analysis.** In supporting the load, the flat slab acts as a continuous flat plate and the reinforcement must be arranged accordingly.

Assume that a uniformly loaded plate of indefinite extent is supported on four columns, $A$, $B$, $C$, and $D$, Fig. 99, the extensions beyond the columns being such that planes tangent to the plate just over the

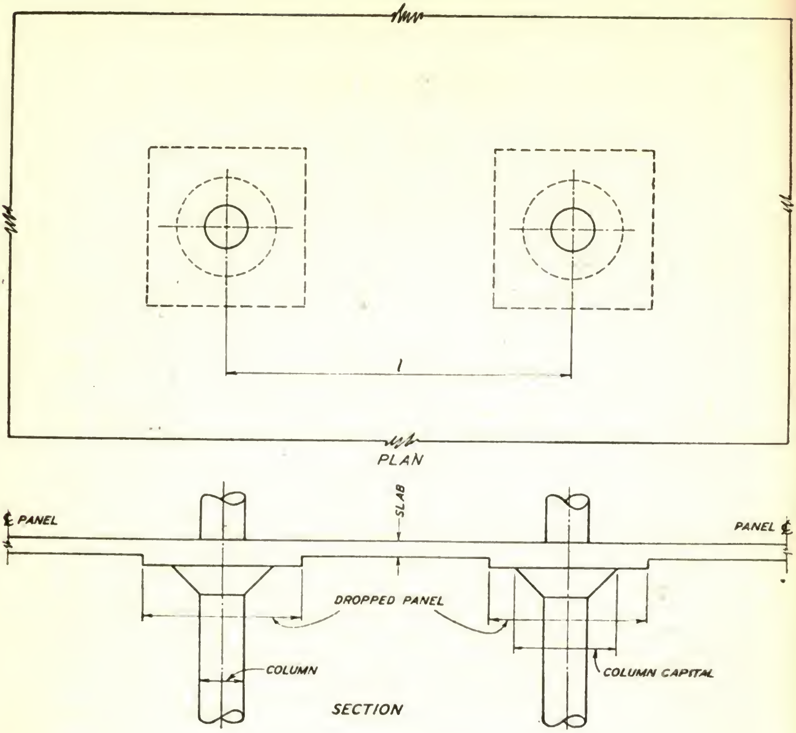



Fig. 97. Plan and Section of a Flat-Slab Floor

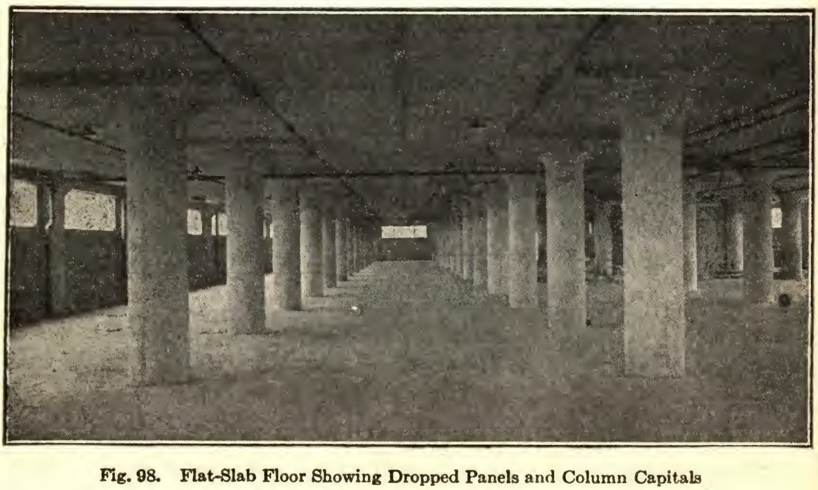


**Fig. 98. Flat-Slab Floor Showing Dropped Panels and Column Capitals**

columns will be horizontal. Then the following conditions will be observed:

1. The plate will be convex upward over the columns.
2. The plate will be concave upward at the point $O$ in the center.
3. There will be points of inflection, approximately as shown by the dotted curves sketched in around the columns. Study of the shape taken by the plate reveals where tension occurs.

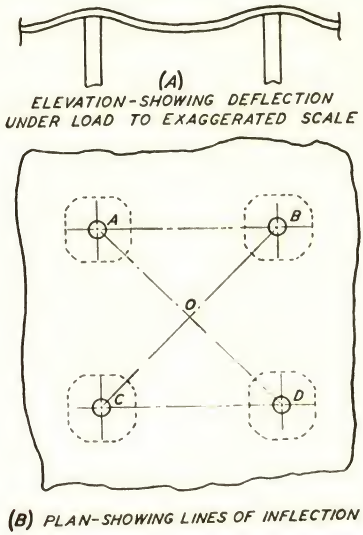



Fig. 99. Flat Plate Under Load

By use of statics it is possible to derive an expression for the total moment to which a panel of a flat plate is subjected. This has the general form of the familiar $\frac{1}{8}Wl$; specifically it is $\frac{1}{8}Wl\left(1-\frac{2c}{3l}\right)^2$ where $c$ is the diameter of the column. As will be indicated later, the expression for the total moment of a flat slab in the direction which moments are taken, is of the same general type as this equation.

**Systems.** There are four common systems of reinforcing flat-slabs: (1) the two-way; (2) the four-way; (3) the three-way; and (4) the circumferential. In addition there are several special types which are mostly a variation of the two-way system. All flat slabs were originally patented systems. However, most of the patents have

expired and flat slabs are being built generally without payment of royalty. The two-way system has reinforcing bars parallel to the column center lines both ways. The four-way system has the two lines of reinforcing parallel to the column center lines and, in addition, has two lines parallel to the diagonals of the panels. The circumferential system uses hoops and radial rods. The three-way system has the columns located at the apexes of equilateral triangles, and

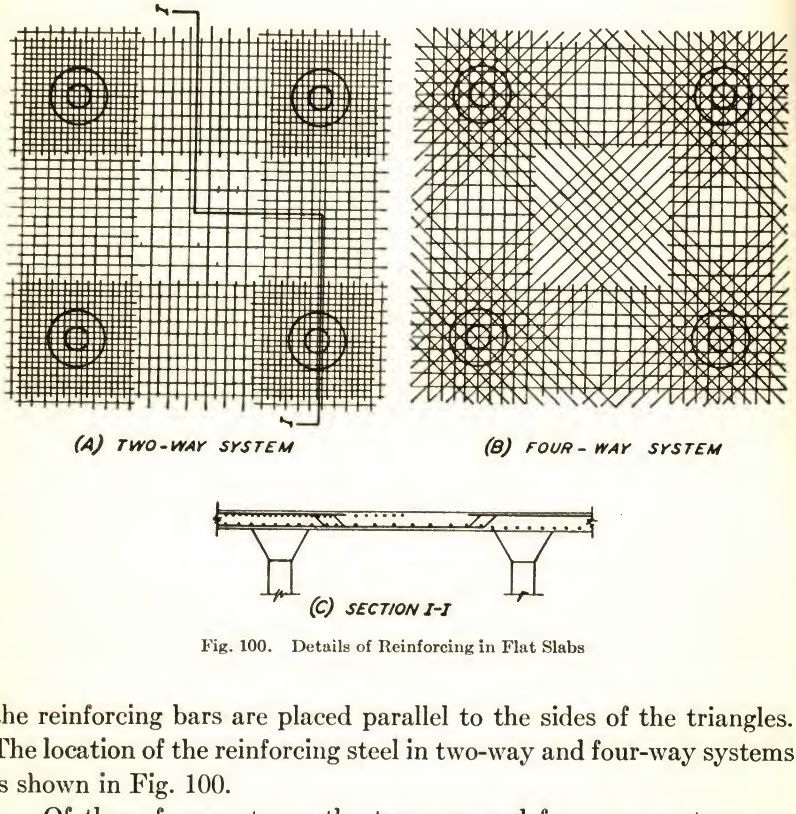



Fig. 100. Details of Reinforcing in Flat Slabs

the reinforcing bars are placed parallel to the sides of the triangles. The location of the reinforcing steel in two-way and four-way systems is shown in Fig. 100.

Of these four systems, the two-way and four-way systems are the systems most generally used today. The four-way system was the original one, but the two-way system was found to be less complicated. Therefore, it is much used at the present time.

**Notations.** The following notations, figures, and formulas are in general taken from the "1936 Joint Code of the A.C.I." The formulas as given are practical simplifications of the theoretical

values. These have been demonstrated by extensive tests to give adequate strength. The moment coefficients, moment distribution, and slab thicknesses used here are for a series of rectangular slabs of approximately uniform size arranged in three or more bays in each direction, where the ratio of length to width of bay does not exceed 1.33.

The notation previously used for beams will be used here so far as it applies. In addition, several new terms must be employed. They are as follows:

Let $c$ equal diameter in feet of column capital of a flat slab at the underside of the slab, or dropped panel (see Fig. 101). No portion of the column capital is considered for structural purposes which lies

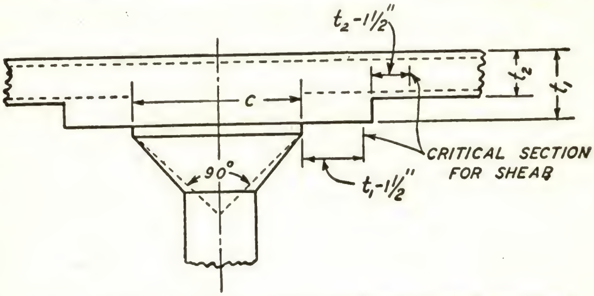



Fig. 101. Typical Column Capital and Sections of Flat Slab, with Dropped Panel

outside of the largest 90° cone that can be included within the outlines of the column capital. The value of $c$ varies ordinarily from $0.2l$ to $0.25l$, the value of $0.225l$ being most commonly used.

$l$ equals the length of a flat slab panel (usually expressed in feet) center to center of columns, in the direction in which moments are considered.

$l_1$ = span length of flat slab, center to center of columns perpendicular to the rectangular direction in which moments are considered.

$W$ = total dead and live loads uniformly distributed over a single panel area. (The weight of all raised or depressed slabs at the top of columns is included as part of the load.)

$M_o$ = sum of positive and negative bending moments at the principal design sections of a panel of a flat slab, in the direction in which the length is given by $l$. This moment is in foot-pounds when $c$ and $l$ are in feet and $W$ is in pounds.

For convenience of reference, a flat slab panel is considered as consisting of strips as follows: (1) A middle strip one-half panel in width symmetrical with respect to the panel center line and extending through the panel in the direction in which moments are being considered; (2) a column strip one-half panel in width occupying the two quarter-panel areas outside of the middle strip. The location of these strips is shown in Fig. 102. When considering moments in the direction of the width of the panel, the panel is similarly divided by strips the widths of which are each one-half the length of the panel.

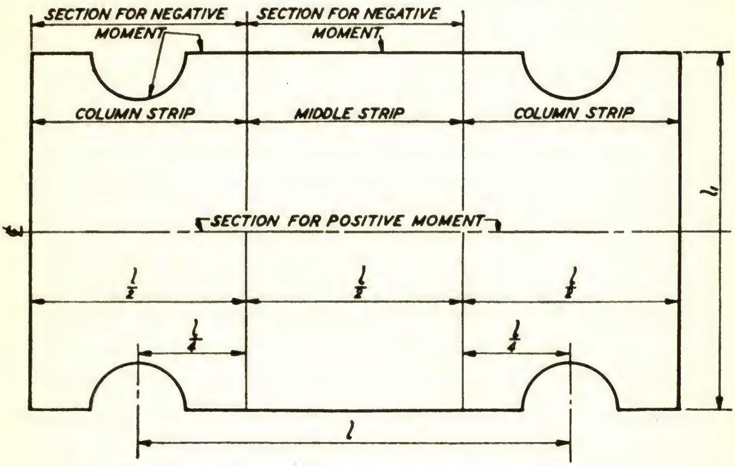



Fig. 102. Panel Strips and Principal Design Sections of a Flat Slab

**Bending Moments.** The numerical sum of the positive and negative moments in the direction of either side of a rectangular panel is given by Equation (49).

$$M_o = 0.09Wl\left(1 - \frac{2c}{3l}\right)^2 \qquad \textbf{(49)}$$

In this formula the theoretical coefficient of ⅛ has been replaced by .09, a reduction of 28%. Tests and past experience justify the use of this coefficient instead of the theoretical value.

Equation (49) gives the total bending moment in the panel in the direction of one side of the panel but does not indicate how these moments are to be distributed. The distribution of the total moment is dependent upon the relative stiffness of the different parts of the panel and is larger in the strips over the columns than between the

## TABLE XXV

### Moments to Be Used in Design of Flat Slabs (Interior Panel)

| Strip | Flat Slabs without Dropped Panels | | Flat Slabs with Dropped Panels | |
|---|---|---|---|---|
| | Negative | Positive | Negative | Positive |
| Slabs with Two-Way Reinforcement | | | | |
| Column strip | $0.46\ M_o$ | $0.22\ M_o$ | $0.50\ M_o$ | $0.20\ M_o$ |
| Middle strip | $0.16\ M_o$ | $0.16\ M_o$ | $0.15\ M_o$ | $0.15\ M_o$ |
| Slabs with Four-Way Reinforcement | | | | |
| Column strip | $0.50\ M_o$ | $0.20\ M_o$ | $0.54\ M_o$ | $0.19\ M_o$ |
| Middle strip | $0.10\ M_o$ | $0.20\ M_o$ | $0.08\ M_o$ | $0.19\ M_o$ |

## TABLE XXVI

### Moments to Be Used in Design of Flat Slabs (Exterior Panel)

| Strip | Flat Slabs without Dropped Panels | | Flat Slabs with Dropped Panels | |
|---|---|---|---|---|
| | Negative | Positive | Negative | Positive |
| Slabs with Two-Way Reinforcement | | | | |
| Column strip | $0.41\ M_o$ | $0.28\ M_o$ | $0.45\ M_o$ | $0.25\ M_o$ |
| Middle strip | $0.10\ M_o$ | $0.20\ M_o$ | $0.10\ M_o$ | $0.19\ M_o$ |
| Slabs with Four-Way Reinforcement | | | | |
| Column strip | $0.45\ M_o$ | $0.25\ M_o$ | $0.485\ M_o$ | $0.24\ M_o$ |
| Middle strip | $0.062\ M_o$ | $0.25\ M_o$ | $0.05\ M_o$ | $0.24\ M_o$ |

1. Moments in the strips perpendicular to the discontinuous edge are given above where they differ from an interior panel.
2. All negative moments given are at the discontinuous edge.
3. Negative moments in column strip and middle strip on line of first interior columns are the same as for an interior panel.
4. Moments in the strips parallel to the discontinuous edge are the same as for an interior panel.

columns. The coefficients which give the distribution of the total bending moment have been obtained as the result of experiments and tests on existing flat-slab floors. The distribution of the total moment for an interior panel is given in Table XXV and for an exterior panel in Table XXVI. About 60% of the total moment is negative, the balance is positive.

**Principal Design Sections.** The critical sections for moment calculations are referred to as principal design sections and are located as follows:

*Sections for Negative Moment.* These are taken along the edges of the panel, on lines joining the column centers, but follow the

circumference of the column capital instead of passing through it.

*Sections for Positive Moment.* These are taken on the center lines of the panel.

**Moments in Principal Design Sections.** The moments in the principal design sections for an interior panel are given in Table XXV and moments in an exterior panel are given in Table XXVI. Any of the moments in these two tables may be varied not more than 6% provided that the total moment on the principal design section is not reduced.

**Dropped Panel.** As mentioned before, in flat-slab construction a portion of the slab around the column capital generally is made thicker than the rest of the floor and is known as the dropped panel. By using the dropped panel, the section at the column head is stiffened and carries a greater portion of the total bending moment. The stress in the rest of the slab is correspondingly reduced and a thinner slab can be used in the balance of the panel, effecting a saving of concrete. This is offset, however, by an increase in the cost of the forms. Whether it is more economical to use a dropped panel or one depth of slab depends on the conditions present in each job. The dimensions of the dropped panel are limited. The thickness cannot be less than 1.25 nor more than 1.50 times the thickness of the slab outside the dropped panel, and the length of its side or diameter cannot be less than $0.35l_1$, and is represented by $b$.

**Slab Thickness.** The thickness of a flat slab is limited by two considerations, (1) deflection and (2) moment. In the first consideration, the thickness is limited to a certain proportion of the span. For floor slabs the minimum thickness is limited to 0.375 times the long side of the panel, expressed in feet, and for roof slabs the minimum thickness is limited to 0.300 times the long side, expressed in feet.

Stated in another way, the thickness of floor slabs must at least be equal to $\frac{1}{32}$ of the long side of the panel, and roof slabs at least to $\frac{1}{40}$ the long side of the panel.

When a dropped panel is used, the expression for the required thickness of slab beyond the dropped panel, $t_2$, considering moment, is

$$t_2 = .02l\sqrt{w'} + 1 \tag{50}$$

where $w'$ is the total uniformly distributed load per unit area of floor or roof. When the dropped panel is omitted, the required thickness

of slab, $t_1$, considering moment again, is given by the expression

$$t_1 = 0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{w'} + 1\frac{1}{2}'' \tag{51}$$

In determining the thickness by use of these equations, $l$ is taken as the distance center to center of the columns on the long side of the panel.

Equations (**50**) and (**51**) are based on using a concrete with an ultimate 28-day strength of 2,000#/□″. Where concrete of a higher strength is used, the thickness determined by these formulas may be reduced by multiplying by the factor $\sqrt[3]{\frac{2000}{f_c'}}$ where $f_c'$ is the 28-day ultimate strength of the concrete used.

**Reinforcement.** In designing the reinforcing for the principal sections, the areas of all bars which cross the section under consideration are used, provided they are properly anchored. If diagonal rods are used they can be considered to have an effective area equal to their total area multiplied by the cosine of the angle which the bars make with the side of the panel. The ratio of reinforcement in any strip must not exceed the value of $p$ calculated for balanced reinforcement, nor must the ratio be less than .0025. The spacing of bars should not exceed $1\frac{1}{2}$ times the thickness of the slab.

In the two-way system the bands of bars within each strip resist the moments in the strip. Each band is $0.5l_1$ in width.

In the four-way system the moments in the two strips are resisted as follows: The positive moment in the column strip is resisted by the direct band. The negative moment in the column strip is resisted by the area of the steel in the direct band plus the area of the two diagonal bands multiplied by the cosine of the angle between the direct bands and the diagonal bands. The positive moment of the middle strip is resisted by the steel in the two diagonal bands multiplied by the cosine of the angle between the axis of the middle strip and the diagonal bands. The negative moment in the middle strip is resisted by an independent top band across the middle of the direct band. Direct bands are to have an approximate width equal to $0.4l_1$, and diagonal bands are to have an approximate width of $\frac{l+l_1}{5}$.

Where reinforcement is perpendicular to a discontinuous edge,

TABLE XXVII

Length of Bars and Points of Bend

| | With Drop | Without Drop |
|---|---|---|
| **TWO-WAY FLAT SLAB (COLUMN STRIP)** | | |
| Length of straight bars (not less than .4 of total band steel) | $l-b+(2'$ or $40d)$ | $.75l$ |
| Length of bent bars (not less than .4 total band steel) | $1.5l+.6c$* | $1.44l+.66c$* |
| Length of additional straight bars over column head (if required) | $.5l+.6c$ | $.44l+.66c$ |
| Point of top bend in bent bars (from column center) | $.25l$ | $.25l$ |
| **MIDDLE STRIP** | | |
| Length of straight bars (not more than .5 total band steel) | $.65l$ | $.7l$ |
| Length of bent bars (not less than .5 total band steel) | $1.5l$* | $1.5l$* |
| Point of top bend in bent bars (from column centers) | $.175l$ | $.15l$ |
| **FOUR-WAY FLAT SLAB COLUMN STRIP** | | |
| Length of straight bars (not less than .4 total band steel) | $l-b+(2'$ or $40d)$ | $.75l$ |
| Length of bent bars (not less than .4 total band steel) | $1.5l+.6c$* | $1.44l+.66c$* |
| Length of additional straight bars over column head (if required) | $.5l+.6c$ | $.44l+.66c$ |
| Point of bend for bent bars (from column centers) | $.2l$ | $.2l$ |
| **DIAGONAL BAND** | | |
| Length of straight bars (not more than .6 total band steel area) | $l-b+(2'$ or $40d)$ | $.75l$ |
| Length of bent bars (not less than .4 total band steel area) | $2.21l$* | $2.21l$* |
| Point of bend for bent bars (from column centers) | $.33l$ | $.33l$ |
| Length of additional straight bars over column head (if required) | $.8l$ | $.8l$ |
| Top band across middle of direct band (length of straight bars) | $.5l$ | $.5l$ |

*Note: To these lengths proper allowance to be added for bends.

proper bond resistance shall be provided by means of hooks or proper embedment in spandrel beams.

**Length and Bending of Bars.** The steel in the bottom of the flat slabs is bent up, when it is no longer required for the positive moment, and is used to resist the negative moment in the top of the slab. All of the steel cannot be turned up, therefore additional steel often must be supplied for the negative moment. These bars are shown in Fig. 100. They must be of sufficient length so that they can be anchored at the ends after the moment has been served.

Table XXVII gives the length of the bars and the location of the points of bend for the two-way and four-way slabs, with and without dropped panels. As can be seen from Table XXVII the number of bars which must be kept straight and the number of bars which must be bent up varies, depending on whether two-way reinforcing or four-way reinforcing is used.

**Diagonal Tension and Shear.** In flat slabs large diagonal tension stresses develop at a section which is beyond the edge of the column capital a distance equal to the effective depth of the slab $d$ (taken as equal to the thickness of the slab $t$ minus $1\frac{1}{2}''$). Where a dropped panel is used, similar stresses develop at a similar section beyond the edge of the dropped panel. The shear on these sections is taken as a measure of the diagonal tension, and the unit shear calculated by using Equation (21) $v=\frac{V}{bjd}$ in which $d$ is replaced by $(t-1\frac{1}{2}'')$. As reinforcement to resist diagonal tension stresses would complicate the placing of the reinforcing steel, the unit shear at these critical sections must be low so that the concrete can carry the stresses safely. In general the thickness of flat slabs is determined by the requirement for moment rather than for shear.

The unit shear on the critical section outside the column capital should not exceed $0.025f_c'$ when 25% of the total negative reinforcement passes directly over the column capital, and should not exceed $0.030f_c'$ when 50% of the total negative reinforcement passes directly over the column capital. For intermediate percentages, intermediate values of the unit shear are used.

When a dropped panel is used, the unit shear on the critical section beyond the dropped panel should not exceed $0.03f_c'$ and at least 50% of the negative reinforcement in the column strip must be within the width of strip directly above the dropped panel.

**Panels with Spandrel Beams.** When a flat-slab panel has a spandrel beam or a reinforced concrete bearing wall on one edge, the moments to be used in the design of the panel are given in Table XXVIII. If the spandrel beam has a depth equal to or less than $1\frac{1}{2}$ times the thickness of the slab, it is designed for the load which is directly superimposed on it, without considering any panel load. If the depth of the spandrel beam is greater than $1\frac{1}{2}$ times the

## TABLE XXVIII

### Moments to Be Used in Design of Panels with Marginal Beams or Reinforced Bearing Walls

| | Marginal Beams with Depth Greater than 1½ Times the Slab Thickness or Reinforced Bearing Wall | | | | | Marginal Beam with Depth 1½ Times the Slab Thickness or Less | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Two-Way System | | | Four-Way System | | Two-Way System | | | Four-Way System | |
| **(a)** | | With Drop | Without Drop | With Drop | Without Drop | | With Drop | Without Drop | With Drop | Without Drop |
| Moments to be used in the design of half column strip adjacent and parallel to marginal beam or wall... | Neg. | $.125M_o$ | $.115M_o$ | $.135M_o$ | $.125M_o$ | Neg. | $.25M_o$ | $.23M_o$ | $.27M_o$ | $.25M_o$ |
| | Pos. | $.05M_o$ | $.055M_o$ | $.0475M_o$ | $.05M_o$ | Pos. | $.10M_o$ | $.11M_o$ | $.095M_o$ | $.10M_o$ |
| **(b)** Negative moment to be used in the design of middle strip continuous over beam or wall | Neg. | $.195M_o$ | $.208M_o$ | $.104M_o$ | $.13M_o$ | Neg. | $.15M_o$ | $.16M_o$ | $.08M_o$ | $.10M_o$ |

slab thickness, it is designed for a uniform load equal to one-fourth of the total panel load in addition to the load directly superimposed on it.

Slabs that are supported by spandrel beams on opposite edges are designed as solid one- or two-way slabs to carry the total panel load.

**Openings in Flat Slabs.** Openings of almost any size may be cut through a flat-slab floor in the area common to two intersecting middle strips provided the total positive and negative resisting moments are maintained as given in Table XXV and that these total positive and total negative moments are redistributed between the remaining principal design sections to meet the new conditions.

In any area common to two column strips, not more than one opening is allowed, and the greatest dimension of such an opening should not exceed $0.05l$. In any area common to one column strip and one middle strip, openings should not interrupt more than one-quarter of the bars in either strip, and the equivalent of the bars so interrupted should be provided by extra steel on both sides of the

opening. Any opening larger than those mentioned must be completely framed on all sides with beams to carry the loads to the columns.

**Example 1.** Assume a typical interior panel similar to Fig. 102, except that it is square, and columns are 20′ center to center. Live load is 200#/□′. Design a flat-slab floor using 2,500-lb. concrete, four-way reinforcement, without dropped panels, $c = 0.225l = 4\frac{1}{2}'$.

Unit stresses are $f_s = 20{,}000\#/\square''$; $f_c' = 2{,}500\#/\square''$; $f_c = 1{,}000\#/\square''$; $v = 62.5\#/\square''$, to $75\#/\square''$; $n = 12$.

*Solution.* Assume an 8″ slab, weight 100#/□′. Then the total weight per sq. ft. on the slab, $w'$, equals $200 + 100 = 300\#/\square'$.

Trial Slab Span $l = l_1 = 20'$ c.c. of columns

Slab minimum $= .375 \times l = .375 \times 20 = 7.50''$ (see page 244)

$$t_1 = [(.025l\sqrt{w'}) + 1\tfrac{1}{2}] \times \sqrt[3]{\frac{2000}{2500}}$$
$$= [(.025 \times 20\sqrt{300}) + 1\tfrac{1}{2}] \times .93$$
$$= [(8.66) + 1\tfrac{1}{2})] \times .96$$
$$= 10.16 \times .93 = 9.45''$$

A 10″ slab is required and the calculations must be revised for the additional weight. A 10″ slab weighs 125#/□′ making the total load 325#/□′. Substituting this value for $w'$ in the equation

$$t_1 = [(.025 \times 20\sqrt{325}) + 1\tfrac{1}{2}] \times .93$$
$$t_1 = 9.8''$$

A 10″ slab is satisfactory. The column capital has a diameter equal to $0.225l$, or $c = 0.225 \times 20 = 4\frac{1}{2}'$ or 54″. The critical section for diagonal tension stress is at a section $t_1 - 1\frac{1}{2}''$ or $8\frac{1}{2}''$ outside the capital, and this section has a diameter of $54'' + (2 \times 8\frac{1}{2}'') = 71''$ or 5.92′. The total shear on the section is $325\,[(20 \times 20) - \frac{\pi}{4} \times (5.92)^2] = 121{,}060\#$. This is resisted by an area equal to the circumference of the section multiplied by $\frac{7}{8}$ times the effective depth (taken as $t_1 - 1\frac{1}{2}$). The shear area is $= \frac{7}{8}(71 \times \pi) \times 8.5'' = 1659\square''$. Then the unit shear $v = \frac{121{,}060}{1{,}659} = 73\#/\square''$. This is satisfactory provided 50% or more of the total negative reinforcement passes directly over the column capital.

*Moments.* The total moment on the panel $= M_o = 0.09Wl\left(1 - \frac{2c}{3l}\right)^2$.

Since the value of $c = 0.225l$, this expression reduces to $0.065Wl$.

Then $M_o = .065Wl = .065 \times (325 \times 20' \times 20') \times 20'$
$= 169{,}000'\#$

This total moment is then distributed as follows (See Table XXV):

$$\text{Column strip}\begin{cases}\text{Negative moment} = 0.50M_o = 84{,}500'\#\\ \text{Positive moment} = 0.20M_o = 33{,}800'\#\end{cases}$$
$$\text{Middle strip}\begin{cases}\text{Negative moment} = 0.10M_o = 16{,}900'\#\\ \text{Positive moment} = 0.20M_o = 33{,}800'\#\end{cases}$$

The column strip positive moment is resisted by the direct band.

With only one layer of bars and using $\frac{1}{2}''\phi$ bars, $d=10-1.25=8.75''$ and $f_c=1000$; $f_s=20{,}000$; $j=.875$.

Then $$A_s=\frac{33{,}800\times 12}{20{,}000\times .875\times 8.75}=2.65^{\square''}$$

Use fourteen $\frac{1}{2}''$ round bars, area $=2.74^{\square''}$. Six bars straight and eight bent up. Also the middle strip positive moment is resisted by the two diagonal bands multiplied by the cosine of the angle between the middle strip and the diagonal bands. In this case, the angle between the middle strip and diagonal bands is 45° and cosine $45°=\frac{1}{2}\sqrt{2}=0.707$. Since there are two bands, the effective area of reinforcement resisting the middle strip positive moment is equal to 1.414 times the area of one diagonal band. Now the middle strip positive moment is the same as the positive moment for the column strip. However, there are two layers of bars and, again using $\frac{1}{2}''\phi$ bars, the average value of $d=10''-1.5''=8.5''$. As before, $j=.875$.

$$\text{The area of steel}=\frac{33{,}800\times 12}{20{,}000\times .875\times 8.5}=2.72^{\square''}$$

As the effective area $=1.414\times$area of 1 band $=2.72^{\square''}$, one band must furnish $\frac{2.72}{1.414}=1.92^{\square''}$.

Use ten $\frac{1}{2}''\phi$ bars in each band $=1.96^{\square''}$.

The column strip negative moment is 84,500′# and will be resisted by four layers of steel, and average $d$ will equal $10''-2''=8''$. As before $jd=.875$.

Then $$A_s=\frac{84{,}500\times 12}{20{,}000\times .875\times 8}=7.24^{\square''}$$

Furnished is $$(2\times 8\times .196)+(1.414\times 1.96)=5.91^{\square''}$$

Use 10 additional $\frac{1}{2}''\ \phi$ bars (area $1.96^{\square''}+5.91^{\square''}=7.87^{\square''}$) in top of slab over capital.

*Check the Concrete stress.*

$$p=\frac{7.87}{10\times 12\times 8}=.0082 \text{ allowable}$$

$$M=\tfrac{1}{2}f_ckjbd^2$$

$$84{,}500\times 12=\tfrac{1}{2}f_c\times .375\times .875\times 120\times 8^2$$

$$f_c=\frac{84{,}500\times 12\times 2}{.375\times .875\times 120\times 8^2}=805\#/^{\square''}$$

This is satisfactory as the allowable is $1000\#/^{\square''}$.

The middle strip negative moment is to be resisted by an independent band of steel across the middle of the direct band. As there is only one layer of steel, $d=8.75$ and $j=.875$.

Then $$A_s=\frac{16{,}900\times 12}{20{,}000\times .875\times 8.75}=1.32^{\square''}$$

Use twelve $\frac{3}{8}''\phi$ bars.

*Using Moments by Bands.* The A.C.I. Code also gives directly the moments to be resisted by each band of bars for 4-way slabs. For this slab they are as follows:

$$\text{Direct band}\begin{cases}\text{Negative moment} = 0.30M_o = 50{,}700'\# \\ \text{Positive moment} \ = 0.20M_o = 33{,}800'\#\end{cases}$$

$$\text{Diagonal band}\begin{cases}\text{Negative moment} = 0.141M_o = 23{,}900'\# \\ \text{Positive moment} \ = 0.141M_o = 23{,}900'\#\end{cases}$$

$$\text{Cross Band} \quad \text{Negative moment} = 0.10M_o = 16{,}900'\#$$

Using the values for $d$ previously used, the area of steel required for each band is:

$$A_s = \text{Direct band negative} = 4.35\square''$$
$$A_s = \text{Direct band positive} = 2.65\square''$$
$$A_s = \text{Diagonal band positive} = 1.93\square''$$
$$A_s = \text{Diagonal band negative} = 1.93\square''$$
$$A_s = \text{Cross band negative} = 1.32\square''$$

All the steel areas check exactly or very nearly exactly with the values previously calculated.

**Example 2.** To illustrate the difference resulting from the use of dropped panels, assume the same data as Example 1, but use dropped panels.

*Solution.* The minimum diameter or side of the dropped panel should be $0.35l'$ or $7'$. Use a panel $7'$ square.

The thickness of the main slab, beyond the dropped panel, $t_2 = 0.02l\sqrt{w'}+1$ and minimum $= 0.375l$. Minimum $t_2 = 7\frac{1}{2}''$. Assume the weight of the slab $= 100\#$, then $w' = 300\#$.

$$t_2 = .02\times 20\sqrt{300}+1 = 6.93+1 = 7.93'' \text{ or an } 8'' \text{ slab.}$$

*Determine the thickness of the dropped panel.* Minimum $= 1.25t_2$, maximum $= 1.50t_2$. So $t_1$ must be $10''$ or more, but less than $12''$. The $10''$ value will be used.

*Check shearing stress in slab.* This is to be taken at a section $t_2 - 1\frac{1}{2}''$ or $8'' - 1.5'' = 6.5''$ from the edge of the dropped panel.

$$\text{Load causing shear} = V = 300\left[20\times 20 - \left(7+\frac{2\times 6.5}{12}\right)^2\right]$$

$$= 300(400-65) = 100{,}500\#$$

$$\text{Shear area} \quad = \tfrac{7}{8}(8-1.5)\times 4[(7\times 12)+(2\times 6.5)]$$

$$= \tfrac{7}{8}\times 6.5\times 388 = 2207\square''$$

$$v = \frac{100{,}500}{2207} = 45.7\#/\square''$$

this is within the allowable limit.

The shear in the dropped panel critical section is located at a distance $t_1 - 1\frac{1}{2}''$ or $(10''-1\frac{1}{2}'') = 8.5''$ from the edge of the column capital.

$$\text{Load causing shear} \quad V = 300\left[20\times 20 - \frac{\pi}{4}\left(4.5+\frac{2\times 8.5}{12}\right)^2\right]$$

$$= 300(400-27.5) = 111{,}750\#$$

$$\text{Shear area} \quad = \tfrac{7}{8}\times 8.5\times \pi[(4.5\times 12)+2(8.5)]$$

$$= \tfrac{7}{8}\times 8.5\times \pi\times 71 = 1659\square''$$

$$v = \frac{111{,}750}{1659} = 67\#/\square''$$

which requires at least 35% of the negative steel to be bent up over the column capital. The total moment carried by a panel $= M_o = 0.065Wl$.

$$M_o = .065 \times (300 \times 20 \times 20) \times 20 = 156{,}000'\#$$

With the dropped panel, the moment distribution will be different from that without a dropped panel, see Table XXV.

$$\text{Column strip}\begin{cases}\text{Negative moment} = 0.54M_o = 84{,}240'\# = 1{,}010{,}880''\# \\ \text{Positive moment} \ = 0.19M_o = 29{,}640'\# = \ \ 355{,}680''\#\end{cases}$$

$$\text{Middle strip}\begin{cases}\text{Negative moment} = 0.08M_o = 12{,}480'\# = \ \ 149{,}760''\# \\ \text{Positive moment} \ = 0.19M_o = 29{,}640'\# = \ \ 355{,}680''\#\end{cases}$$

The positive moment of the middle strip $= 355{,}680''\#$. This is resisted by the two diagonal bands. Then the average $d$ will be $8 - 1\frac{1}{2} = 6.5''$ and the area of the steel required will be $= A_s = \dfrac{355{,}680}{20{,}000 \times \frac{7}{8} \times 6.5} = 3.13^{\square''}$. As this moment is resisted by the diagonal bands, the effective area of each band is only 0.707 times the area of the band. Then if $A'_s$ represents the area of one diagonal band, $1.414A'_s$ represents the effective area of the two diagonal bands. So $1.414A'_s = 3.13^{\square''}$ and $A'_s = 2.21^{\square''}$. Each diagonal band must have an area of $2.21^{\square''}$.

Use twelve $\frac{1}{2}''\ \phi$ bars $= 2.35^{\square''}$.

The positive moment in the column strip is also $355{,}680''\#$. As this is resisted by one band of steel only, therefore $d = 8 - 1\frac{1}{4} = 6\frac{3}{4}''$.

Then
$$A_s = \frac{355{,}680}{20{,}000 \times 6.75 \times \frac{7}{8}} = 3.01^{\square''}$$

Comparing the values obtained so far, it is seen that, by using a dropped panel, the main portion of the slab can be reduced in thickness from 10″ to 8″; and as the main slab covers an area of 351□′ out of 400□′ this is quite an appreciable saving. On the other hand, the forms will cost more, being more complicated. Also a larger area of steel will be required. Other details of these alternate plans should be similarly compared by the student. A comparison of the relative costs of concrete, of steel, and of forms is needed to determine which is less expensive. The relative desirability is also an element.

# CHAPTER XIII

## FLEXURE AND DIRECT STRESS

**General Principles.** In all of the previous work, the forces acting on a beam are assumed to be perpendicular to the beam; the forces acting on a column are assumed to coincide with the axis of the column. There are many cases in designing in which the resultant of the forces is oblique to the axis of the beam—or column—and, therefore, develops both flexural and direct stress. This is particularly the case in elastic arches. Usually, in concrete work the combination is that of a compressive thrust and flexure, although tension combined with flexure is not impossible. The following demonstration will be made on the basis of the direct stress being exclusively compression.

Columns have reinforcement near two (or four) faces. If the load is eccentric, and especially if it is variable in position, direction, and magnitude, the steel in either face may be alternately in tension and in compression. In the case of arches, steel is placed near the extrados, or upper surfaces of the arch, and also near the intrados, or lower surface, and variation in the live load may cause the stress in either set of bars to be alternately tension or compression. The reinforcement is, therefore, in compression as well as in tension. And since, for practical reasons, the reinforcement is made uniform throughout the length of the column (beam or arch) and usually the same on both faces, the stresses in the steel are sometimes compression, sometimes tension, sometimes zero, and in general will average far less than the possible safe working value. It is economically impracticable to vary the cross section of the steel to be everywhere at the lowest safe limit of unit stress, especially when the stresses at any section are variable for different loadings. It is, therefore, necessary to use a design which shall be safe for the worst section under the worst condition, although the strength will be excessive at all other sections.

**Moment of Inertia of Any Section.** In the perfectly general

case, the steel near one face is not the same as that near the other. If the steel were replaced by two external "wings" of concrete, each of which is as far from the center as the steel and each of which has an area $n$ times the area of the steel ($n = E_s \div E_c$), we would have a section such as is indicated in Fig. 103. In Fig. 103 the shaded areas are each assumed to be $n$ (say 15) times the area of the corresponding steel bar, so that if the steel were entirely replaced by the concrete, then the section, with the concrete wings added, has the same moment of inertia as the concrete and steel section. Note that in the perfectly general case considered, the area of steel at the top ($A'$) is *not* the

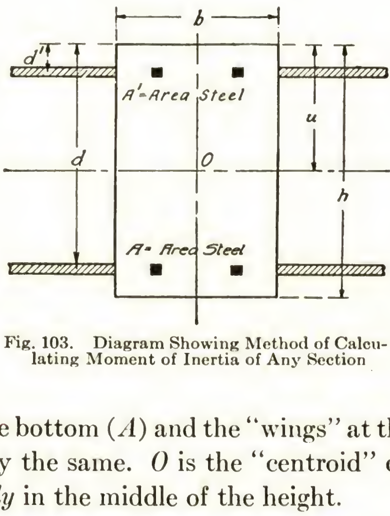



Fig. 103. Diagram Showing Method of Calculating Moment of Inertia of Any Section

same as that at the bottom ($A$) and the "wings" at the top and bottom are not necessarily the same. $O$ is the "centroid" of that figure, but it is not *necessarily* in the middle of the height.

Let $I_c$ = moment of inertia of the concrete rectangle with respect to the axis through $O$

$I_s$ = moment of inertia of the areas of steel about the same axis

Then

$nI_s$ = moment of inertia of the concrete wings about the same axis

$I$ = moment of inertia of the "transformed section"—the rectangle and wings

Then

$$I = I_c + nI_s \tag{52}$$

Let $p$ = steel ratio on tension side (assumed here as lower side) $= (A \div bh)$

$p'$ = steel ratio on compression side $= A' \div bh$

Then, taking moments about the upper edge of the concrete, the sum of the moments of the transformed section, consisting entirely of concrete, are $bh(\frac{1}{2}h)+(nA')d'+(nA)d$. But since $O$ is the centroid of the figure or the point at which the entire area might act as an equivalent unit, then the moment also equals $u(bh+nA'+nA)$. Placing these equal to each other we have

$$u = \frac{bh(\frac{1}{2}h)+nA'd'+nAd}{bh+nA'+nA}$$

But $$A' = p'bh \text{ and } A = pbh.$$

Then

$$u = \frac{bh(\frac{1}{2}h+np'd'+npd)}{bh(1+np'+np)}$$

$$u = \frac{\frac{1}{2}h+npd+np'd'}{1+np+np'} \tag{53}$$

The moment of inertia of a rectangle about its base equals $\frac{1}{3}bh^3$. Then the two rectangles above and below $O$ have the moments $\frac{1}{3}bu^3$ and $\frac{1}{3}b(h-u)^3$ and their sum is

$$\left.\begin{aligned} I_c &= \tfrac{1}{3}b[u^3+(h-u)^3] \\ I_s &= A(d-u)^2+A'(u-d')^2 \end{aligned}\right\} \tag{54}$$

When, as is frequently the case, $A$ equals $A'$, and the whole section is, therefore, symmetrical, $u$ equals $\frac{1}{2}h$, and the two equations (54) reduce to

$$\left.\begin{aligned} I_c &= \tfrac{1}{12}bh^3 \\ I_s &= 2A(\tfrac{1}{2}h-d')^2 \end{aligned}\right\} \tag{55}$$

It is a common practice to make $d' = \frac{1}{10}h$, which would make

$$I_s = 2A(.4h)^2 = .32Ah^2$$

Then. $$I = I_c+nI_s = \tfrac{1}{12}bh^3+.32nAh^2 \tag{56}$$

In all the expressions for $I_s$ here given, it should be noted that the $I$ of the wing areas about their own center of gravity has not been included. As this is a small quantity compared with the other terms, it is omitted for the sake of simplicity.

**Occurrence.** The most common cases in which a member is sub-

jected to both flexure and direct stress are (1) arches and (2) columns with eccentric loads. In both cases the load may be applied to the member in a manner which will produce only compression on the section, or the load may be applied so as to produce compression on part of the section and tension on the rest of the section. As a general rule arches and columns are designed to have a symmetrical section, so that the principal axes coincide with the axes of symmetry. The discussion of flexure and direct stress will be limited to its application to columns.

**Cases Considered.** There are two cases of direct stress and flexure to be considered: (1) when the eccentricity of the load producing flexure is relatively small and compression occurs on the entire section, and (2) when the eccentricity of the load is large and produces tension on part of the section.

A common case encountered in practice is the case of a column with an eccentric load, such as a crane girder or a beam on a bracket. In this case, the load usually is applied on the line of one of the axes of symmetry, with a definite amount of eccentricity with respect to the other axis. In this case the fiber stresses are found by the familiar expression from strength of materials,

$$f=\frac{P}{A}\pm\frac{Mc}{I} \tag{57}$$

In the section on "Retaining Walls," it was shown that for a homogeneous material the load must be applied within the middle third in order to have compression over the entire section, so that the eccentricity could not exceed one-sixth the dimension of the base. In that case, Equation (57) could be rewritten as $f=\frac{P}{A}\left(1\pm\frac{6e}{b}\right)$. This applies to sections of homogeneous materials only. In the case of reinforced concrete members, the presence of the reinforcing steel modifies somewhat the limits within which the load must act in order to have only compression over the entire section, and the modified form of Equation (57) cannot be used.

If the load is not applied on line with one of the principal axes, it will produce a moment about each axis, and the fiber stresses are then a composite of three actions, a direct load and two moments, and their intensity is given by the equation

$$f=\frac{P}{A}\pm\frac{M_x c_x}{I_x}\pm\frac{M_y c_y}{I_y} \tag{58}$$

where the subscripts $x$ and $y$ denote the axes about which the moment occurs and about which the properties of the section are calculated.

**Design of Section.** In the design of sections subjected to flexure and direct stress, it is possible to derive expressions for the location of the neutral axis $k$, intensity of the fiber stresses $f_s$ and $f_c$, moment $M$, etc., as was done for the case of simple beams and **T**-beams. However, these expressions are very complicated and cannot be

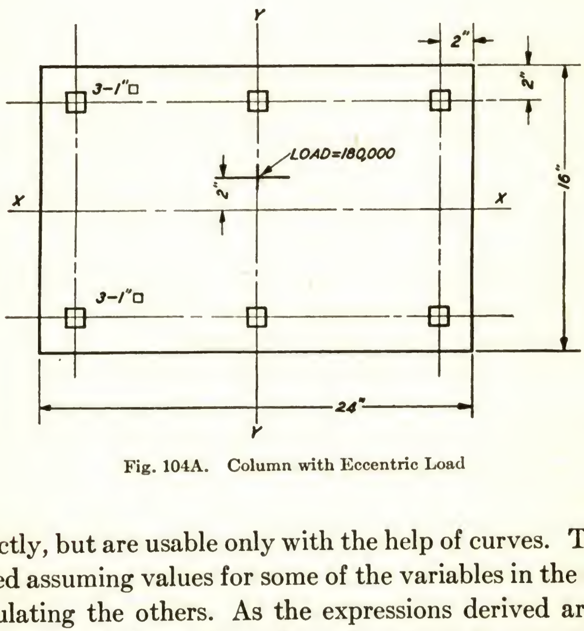



Fig. 104A. Column with Eccentric Load

used directly, but are usable only with the help of curves. The curves are plotted assuming values for some of the variables in the equations and calculating the others. As the expressions derived are complicated, the method of the "transformed section" is used here in the solution of problems of this type.

**Example 1.** The column section shown in Fig. 104A has a load of 180,000# applied in the plane of the $y-y$ axis, and 2″ from the $x-x$ axis. With $n=12$, what are the maximum and minimum values of $f_c$?

*Solution.* The "transformed section" of the column section is given in Fig. 104B. The student should note particularly the position of the wings replacing the steel area. If the wings were drawn parallel to the $y$ axis, they would have a lever arm greater than the steel they replace and the section therefore would not be equivalent to the original section. This point must be carefully watched in problems of this type.

The moment of inertia of the section about the axis $x-x$ is now calculated neglecting the moment of inertia of the wings about their own axis.

For the concrete area, $I_c = \frac{bh^3}{12} = \frac{24\times(16)^3}{12} = 8{,}192''^4$.

For the wing areas, $I_s = 2A(\tfrac{1}{2}h - d')^2 = 33\times2\times(6)^2 = 2{,}376''^4$. Then $I$ for the total area $= 10{,}568''^4$.

Substituting in Equation (**57**)

$$f_c = \frac{180{,}000}{450} \pm \frac{(180{,}000\times2)\times8}{10{,}568}$$

$$f_c = 400 \pm 273$$

$$\text{Max. } f_c = 673\#/\square''$$

$$\text{Min. } f_c = 127\#/\square''$$

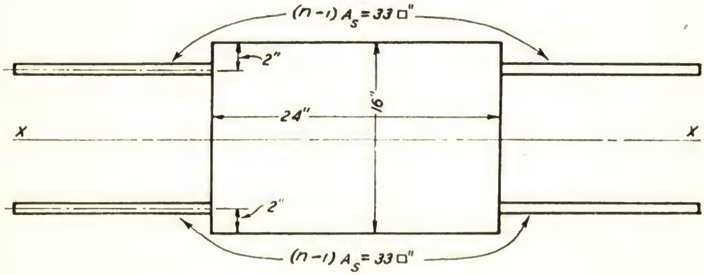



Fig. 104B. Transformed Section of Column

**Example 2.** In the column of Example 1, assume that a load of 100,000# acts at a point 5" from the $x-x$ axis. See Fig. 105 at ($A$). With $n=12$, determine the fiber stress of the concrete.

*Solution.* In this problem the load is applied so far outside the middle third that there is no doubt that there will be tension on part of this section.

Consider a small section of the column, *abdc*, shown in Fig. 105 at ($B$). This is in equilibrium under the action of the external load and the internal fiber stresses on the end *cd*. The stress diagram for this end is also shown in Fig. 105 at ($B$). The neutral axis is an unknown distance, $x$ from the compressive face. Take moments about the point of application of the load.

| Compression on *efhg* | Arm | Compression on wings | Arm | Tension | Arm |
|---|---|---|---|---|---|

$$\left[\frac{f_c}{2}\times24x\right]\times\left[\frac{x}{3}-3\right]-\left[33\times f_c\left(\frac{x-2}{x}\right)\right]\times\left[1\right]-\left[\left(\frac{14-x}{x}\right)f_c\times36\right]\times\left[11\right]=0$$

Expanding

$$\left[(12f_cx)\left(\frac{x}{3}-3\right)\right]-\left[(33f_c)\left(1-\frac{2}{x}\right)\right]-11\left[36f_c\left(\frac{14}{x}-1\right)\right]=0$$

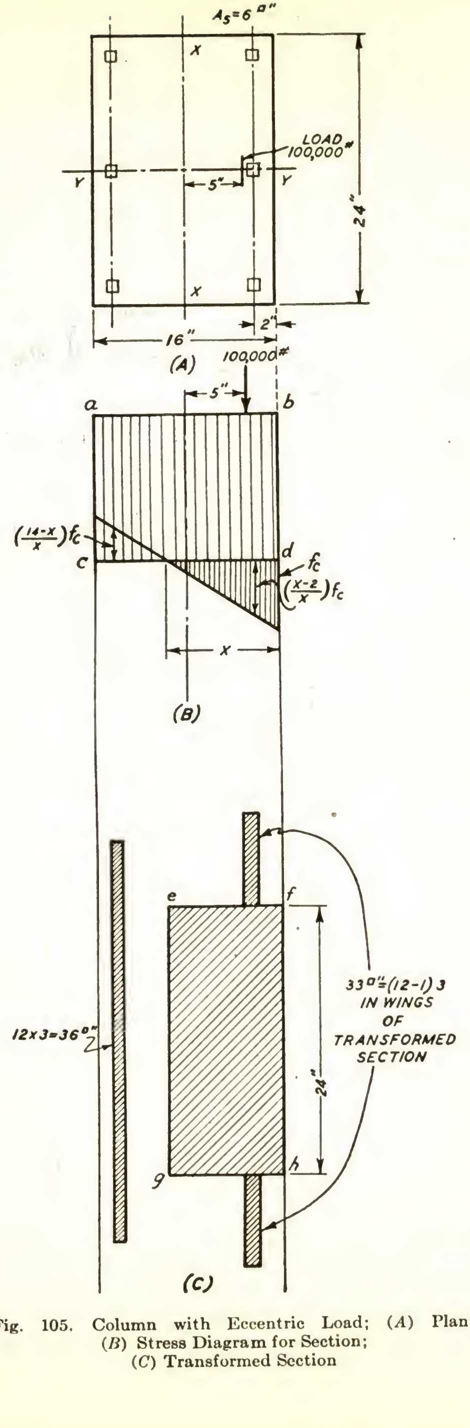



Fig. 105. Column with Eccentric Load; (*A*) Plan; (*B*) Stress Diagram for Section; (*C*) Transformed Section

$$\left[4f_cx^2-36f_cx\right]-\left[33f_c-\frac{66f_c}{x}\right]-\left[\frac{5544f_c}{x}-396f_c\right]=0$$

Multiply by $\frac{x}{f_c}$

$$4x^3-36x^2-33x+66-5544+396x=0$$

Collecting terms $$4x^3-36x^2+363x-5478=0$$

Solving by approximation $$x=11.5''$$

As the section *abdc* is in equilibrium, the resultant of all the vertical forces equals zero.

Then $$100{,}000+\left[36\left(\frac{14-x}{x}\right)f_c\right]-\left[f_c(12x)\right]-\left[33f_c\left(\frac{x-2}{x}\right)\right]=0$$

Substituting the value of $x$ just found,

$$100{,}000+\left[36\left(\frac{14-11.5}{11.5}\right)f_c\right]-\left[(12\times11.5)f_c\right]-\left[\frac{33(11.5-2)}{11.5}\right]f_c=0$$

$$100{,}000+7.8f_c-138f_c-27.3f_c=0$$

$$157.5f_c=100{,}000$$

$$f_c=635\#/\square''$$

which is permissible.

**Example 3.** In the column of Example 1, assume that a load of 120,000# acts 2″ from the axis $x$, as shown in Fig. 104A, and also 1″ to the right of the axis $y$. What are the maximum and minimum fiber stresses?

*Solution.* This column can be considered as being acted on by an axial load of 120,000#, a moment about the axis $x$ of 240,000″# and a moment about the axis $y$ of 120,000″#. Accordingly, using Equation (58)

$$f_c=\frac{P}{A}\pm\frac{M_xc_x}{I_x}\pm\frac{M_yc_y}{I_y}$$

The moment of inertia about the axis $x$ has already been determined as $10{,}568''^4$. (Example 1)

In determining the moment of inertia about the axis $y$, a new transformed section is drawn as shown in Fig. 106. The transformed section of Fig. 104B will not be satisfactory, as the wing areas would have a larger lever arm than the steel actually has.

Calculate the moment of inertia about the axis $y$ (neglecting $I$ of the wings about their own neutral axis)

$I$ of concrete area $$=\frac{16\times(24)^3}{12}=18{,}432$$

$I$ of wings $$=44\times(10)^2=\underline{4{,}400}$$

$$\text{Total } I=22{,}832$$

Substituting in Equation (58)

$$f_c=\frac{120{,}000}{450}\pm\frac{240{,}000\times8}{10{,}568}\pm\frac{120{,}000\times12}{22{,}832}$$

$$f_c=267\pm182\pm63$$

The maximum compressive stress is 512#/□″ and will occur at the upper right-hand corner; the minimum compressive stress will be 22#/□″ and will occur at the lower left-hand corner.

**Design Moments for Slabs and Beams.** The Joint Code specifies certain moment factors which are to be used in the design of continuous reinforced concrete slabs and beams of approximately equal span and supporting a uniform load. The recommended values for various conditions are shown in Fig. 107. The value of these

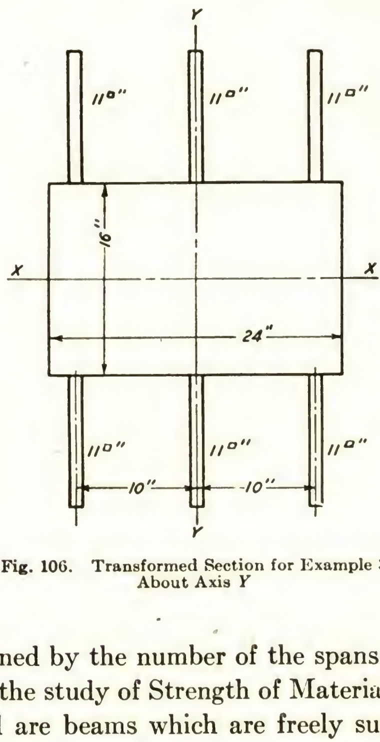



Fig. 106. Transformed Section for Example 3 About Axis $Y$

factors is determined by the number of the spans and the nature of the supports. In the study of Strength of Materials the usual conditions encountered are beams which are freely supported or rigidly fixed. A beam is *freely supported* when it merely rests on the supports and there is no restraint which can develop any resisting moment at the ends of the beam. For a uniform load on such a beam there is a positive moment at the center equal to $\frac{Wl}{8}$, $W$ being the total weight on the beam and $l$ the span. There is no moment at the supports, and the moment curve for the beam is shown by the dotted lines in Fig. 107 at ($A$). The end conditions of beams and slabs in reinforced

concrete construction usually are such that they are not considered as freely supported, but are restrained to a certain extent.

A *slightly restrained* beam or slab is one where the ends of the beam or slab are built into brick or masonry walls in a manner

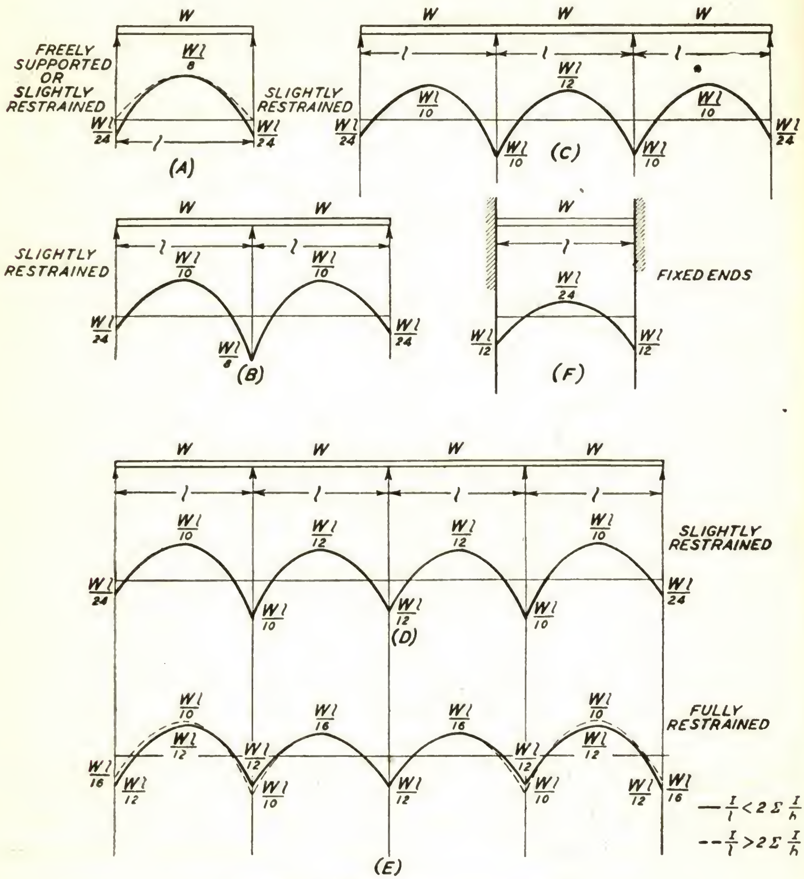



Fig. 107. Bending Moment Diagrams

which develops only partial end restraint, or where the beam or slab is built to act integrally with a wall beam. Where one of these conditions exists, the beam or slab is designed for a negative moment at the end supports of $\frac{Wl}{24}$. (The design moments at other points are given in Fig. 107.) The student should note that the values given

in the figures are greater than the theoretical values given in text-books for such conditions. These values are larger because it is realized that the conditions of being absolutely freely supported or absolutely restrained do not occur, and the specifications provide for stresses which might be developed.

A beam is said to be *fully restrained* when it is built continuously and integrally with a supporting wall or girder and is reinforced for the negative moment at the support. A beam built into a wall column so as to act integrally with it is considered restrained, but a distinction is made, depending on the relative stiffness of the beam and of the columns. The values for the moments in continuous restrained beams of approximately equal span are given in Fig. 107 at ($E$). The moment curve in the end spans, indicated by the full lines, is to be used when the stiffness factor of the beam (measured by the moment of inertia of the beam divided by the span of the beam, i.e., $\frac{I}{l}$) is less than twice the sum of the stiffness factors $\left(\frac{I}{h}\right)$ (moment of inertia of the column divided by the unsupported length of the column) for the columns above and below the beam which are built into the beam. When the stiffness factor of the beam is equal to or greater than twice the sum of the stiffness factors of the columns, the moment curve shown by the dotted lines is used.

The moment factors shown in Fig. 107 at ($A$), ($B$), ($C$), and ($D$) are the ones which usually are applicable in the ordinary concrete structures. The question of restraint should be studied thoroughly before the moment factors given in Fig. 107, at ($E$), are used. In Fig. 107, at ($F$), is shown the theoretical moment diagram for a beam of one span with the ends fixed; that is, a fully restrained beam. The end of a beam built in a brick wall is not a restrained beam as far as the center bending moment is concerned. The end support must be a column or beam designed for the restrained condition.

**Continuity in Concrete Frames.** A structure built of reinforced concrete is not an assembly of individual beams and columns, but a monolithic frame which acts as an integral unit. In a reinforced concrete frame when a load is applied to one member, the rigidity of the joints causes all the members to deform to some extent. Determination of the stresses set up in such a structure calls for the analysis of the stresses in continuous beams and open frames.

The exact calculation of the stresses in a rigid frame is rather complicated. For years the members of a reinforced concrete frame were designed to resist moments calculated by using moment coefficients as described. These coefficients were to take account of the continuity of the structure and the rigidity of the joints and were given in specifications and building codes. The 1936 Building Code of the A.C.I., however, discards these coefficients except for structures with approximately equal spans which are subjected to a live load equal to less than three times the dead load. Instead of using coefficients, the new code states that all members should be designed to resist the maximum bending moments and shears produced by dead load, live load, and wind load as determined by the *principle of continuity.* The Joint Committee on Standard Specifications for Concrete and Reinforced Concrete issued a progress report, published in January 1937, in which it has taken a similar stand on the design of a concrete frame.

The determination of the moments and shears in any member of a rigid frame or in continuous construction cannot be made by the principle of statics alone, but use must be made of the elastic deformations of the structure. The methods most used are (1) The Theorem of Three Moments, (2) The Method of Slope Deflection, (3) The Method of Least Work, and (4) the Method of Moment Distribution.

Of these four methods the most familiar one is the Theorem of Three Moments. The method in general use today is the Method of Moment Distribution. Accordingly, these two methods will be briefly explained.

**The Theorem of Three Moments.** The Theorem of Three Moments was developed by the French engineer Clapeyron. It is in the form of an equation which gives the relation between the bending moments at any three consecutive supports. The theorem is familiar to all who have studied Strength of Materials. The general equation is

$$\frac{M_1}{K_1}+2M_2\left(\frac{1}{K_1}+\frac{1}{K_2}\right)+\frac{M_3}{K_2}=-\frac{1}{4}\,\frac{w_1l_1^2}{K_1}-\frac{1}{4}\,\frac{w_2l_2^2}{K_2}-\frac{\Sigma P_1l_1(k_1-k_1^3)}{K_1}-\frac{\Sigma P_2l_2(2k_2-3k_2^2+k_2^3)}{K_2} \qquad (59)$$

in which $K$ equals the $\frac{I}{l}$ value of a member, $M_1$ = moment at first support, $M_2$ = moment at second support, etc., $w$ = uniform load in pounds per foot for each span, $l_1$ = length of span from first to second support, $l_2$ = length of span from second to third support, $P_1$ = any concentrated load on the first span, $P_2$ = any concentrated

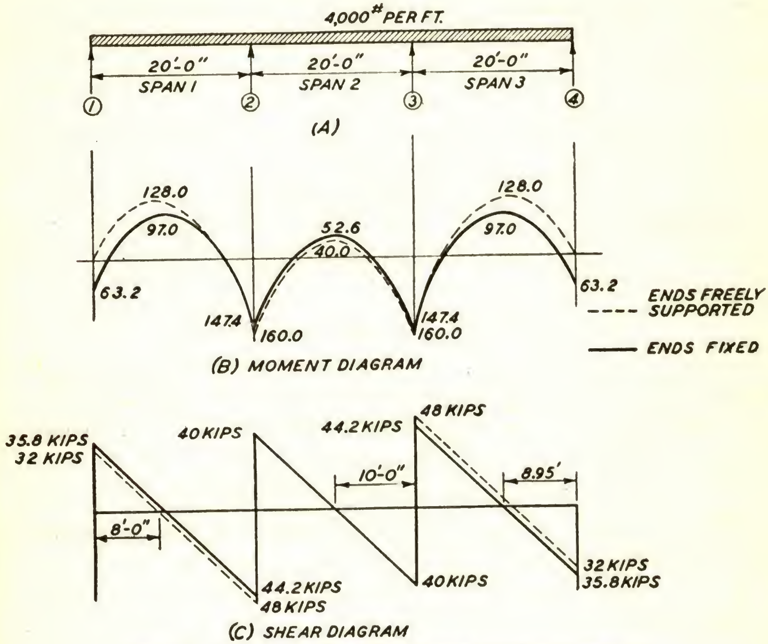



Fig. 108. Moment and Shear Diagrams for Continuous Beam

load on the second span, $k_1$ is the distance from the first support to any concentrated load on the first span divided by the length of the first span, $k_2$ is the distance from the second support to any concentrated load on the second span divided by the length of the second span. For the derivation of this formula, the student is referred to texts on Strength of Materials.

For a beam of constant cross section, i.e., uniform moment of inertia, this general expression reduces to

$$M_1l_1+2M_2(l_1+l_2)+M_3l_2=-\tfrac{1}{4}w_1l_1^3-\tfrac{1}{4}w_2l_2^3-\Sigma P_1l_1^2(k_1-k_1^3)-\Sigma P_2l_2^2(2k_2-3k_2^2+k_2^3) \qquad \textbf{(59a)}$$

This form usually is found in texts on the "Strength of Materials." By using these equations, the bending moments can be determined at all supports, and knowing these, the reaction and shears are easily obtained.

**Example.** A continuous beam of 3 equal spans of 20′, supports a total uniform load of 4,000# (4 kips) per foot of beam, Fig. 108. With the ends assumed restrained and the beam of a uniform cross section, determine the moment and the reactions at the four supports. (In this problem calculations will be made by slide rule.)

*Solution.* Since the cross section of the beam is uniform and the beam is subjected only to uniform load, Equation (**59**) reduces to

$$M_1l_1+2M_2(l_1+l_2)+M_3l_2=-\tfrac{1}{4}w_1l_1^3-\tfrac{1}{4}w_2l_2^3$$

As all spans are equal and the load is the same in each span, this equation is simplified further to

$$M_1+4M_2+M_3=-\tfrac{1}{2}wl^2$$

Considering spans *1* and *2*,

$$M_1+4M_2+M_3=-\tfrac{1}{2}[4\times(20)^2]=-800 \text{ ft. kips} \qquad (a)$$

Similarly, for spans *2* and *3*

$$M_2+4M_3+M_4=-\tfrac{1}{2}[4\times(20)^2]=-800 \text{ ft. kips} \qquad (b)$$

We have four unknowns and only two equations, so two more equations must be obtained. This is done by considering that there is a span, call it $x$, to the left of span *1*, whose length is zero. Then writing the three-moment equation for spans $x$ and *1*.

$$0+4M_1+M_2=-\tfrac{1}{4}[4\times(20)^2]=-400 \qquad (c)$$

Similarly, assume a span ($y$) to the right of span *3* with a length of zero feet. Then the three-moment equation for spans *3* and ($y$) is

$$M_3+4M_4+0=-\tfrac{1}{4}[4\times(20)^2]=-400 \qquad (d)$$

Eliminate the term $M_1$ between Equations ($a$) and ($c$)

| | | |
|---|---|---|
| Multiply ($a$) by 4 | $4M_1+16M_2+4M_3=-3200$ | ($e$) |
| | $4M_1+M_2 \qquad =-\ 400$ | |
| Subtract ($c$) from ($e$) | $15M_2+4M_3=-2800$ | ($f$) |

Now eliminate $M_4$ between equations ($b$) and ($d$)

| | | |
|---|---|---|
| Multiply ($b$) by 4 | $4M_2+16M_3+4M_4=-3200$ | ($g$) |
| | $M_3+4M_4=-\ 400$ | |
| Subtract ($d$) from ($g$) | $4M_2+15M_3 \qquad =-2800$ | ($h$) |
| Multiply ($f$) by 4 | $60M_2+16M_3=-11{,}200$ | ($j$) |
| Multiply ($h$) by 15 | $60M_2+225M_3=-42{,}000$ | ($k$) |
| Subtract ($k$) from ($j$) | $-209M_3=+30{,}800$ | |
| | $M_3=-147.4$ ft. kips | |

Substituting this value for $M_3$ in Equation ($f$)

$$15M_2-589.6=-2800$$

$$15M_2 = -2210.4$$
$$M_2 = -147.4 \text{ ft. kips}$$

Substituting the calculated value for $M_2$ in Equation ($c$)

$$4M_1 - 147.4 = -400$$
$$4M_1 = -252.6$$
$$M_1 = -63.2 \text{ ft. kips}$$

Substituting the calculated values for $M_3$ in Equation ($d$)

$$-147.4 + 4M_4 = -400$$
$$4M_4 = -252.6$$
$$M_4 = -63.2 \text{ kips}$$

As a check, substitute the calculated values in Equations ($a$) and ($b$)

$$-63.2 + 4(-147.4) + (-147.4) = -800$$
$$-800.2 = -800$$

The moments at the supports are now known and it is possible to derive the reactions at the supports by use of the relation that the moment at any point on a beam is equal to the moment at any other point in the beam plus the algebraic sum of the moments of the intervening loads and reactions. Taking span *1*

$$M_1 + R_1 \times 20 - (20 \times 4) \times 10 = M_2$$
$$-63.2 + 20R_1 - 800 = -147.4$$
$$20R_1 = 715.8$$
$$R_1 = 35.8 \text{ kips}$$

Also in span *1*, let $R_{2\,left}$ be the reaction at the left side of the second support.

$$M_2 + R_{2\,left} \times 20 - 800 = M_1$$
$$-147.4 + 20R_{2\,left} - 800 = -63.2$$
$$20\,R_{2\,left} = 884.2$$
$$R_{2\,left} = 44.2 \text{ kips}$$

In span *2*, $M_2$ and $M_3$ are equal so that the reaction on each end is equal to one-half the load on the span. Thus the reaction on each end of the span is 40 kips.

In span *3*, the moments at the supports and the loads are the same as in span *1* so the reactions will be the same as for span *1*.

Now the reaction at the left side of support *2* is 44.2 kips and the reaction at the right side of the support is 40 kips. Then the total reaction at support *2* is 84.2 kips. Similarly, the reaction at support *3* is 84.2 kips.

The maximum positive moment in the first and third span is located at the point of zero shear which is at $\frac{35.8}{4} = 8.95$ feet from the end supports and is equal to

$$M_1 + R_1 \times 8.95 - \frac{4 \times (8.95)^2}{2} = -63.2 + (35.8 \times 8.95) - 2 \times (8.95)^2$$
$$= 97.0 \text{ ft. kips}$$

The maximum positive moment in the second span occurs in the center of the span and is equal to

$$M_2 + 10R_{2\,right} - \frac{4 \times (10)^2}{2} = -147.4 + 10 \times 40 - 200 = 52.6 \text{ ft. kips}$$

The solution of this particular problem could be made more readily by noticing that because of symmetry in loading and span, $M_1$ and $M_4$ will be equal and $M_2$ and $M_3$ will be equal. Then Equation ($a$) reduces to $M_1+5M_2=-800$ ft. kips. Combining this with Equation ($c$) we can solve for $M_1$ and $M_2$ directly. Similarly, the reaction at supports *1* and *4* are equal, and the reactions at *2* and *3* are equal.

If the ends of the beam are simply supported, that means that both $M_1$ and $M_4$ are equal to zero, and it is only necessary to solve for $M_2$ and $M_3$ and, as they are equal, Equation ($a$) reduces to $5M_2=-800$ ft. kips or $M_2=-160$ ft. kips. The reaction at the first support is half the load on the span minus the moment at *2* divided by the span, or $40-\frac{160}{20}=32$ kips. The reaction at the right side of the span equals 48 kips. The reaction at the left end of span *2* is half the load on the span, or 40 kips, and the reaction at support *2* is 88 kips. Fig. 108 gives the moment and shear diagrams for a case with the ends of the beam restrained, shown as solid lines, and for a case with the ends of the beam freely supported, shown dotted.

**The Method of Moment Distribution.** The method of moment distribution, sometimes called the Cross Method, was developed by Professor Hardy Cross and was first published in 1929. The method of moment distribution utilizes three moment relations (1) the value of the end moment produced in a beam with a fixed end by any given loading condition, (2) the relation that an external moment applied to a joint is resisted by the members meeting at that joint in proportion to their $K$ values or stiffness factors (provided that the other ends of the members are fixed) and (3) the relation that a moment applied at the end of a member which is not fixed induces at the other end of the member, which is fixed, a resisting moment equal to half the applied moment. In the moment distribution method the induced moment at the fixed end of a beam (part 3 above) is known as the "carry-over moment."

In the application of this method, all the joints of a structure are first considered artificially locked; i.e., the ends are assumed as being fixed, and the fixed-end moments due to the loads on the structure are computed. The joints are then released, one at a time, and permitted to rotate under the action of the unbalanced moment at the joint. The unbalanced moment at the joint is the algebraic sum of the fixed-end moments which occur at the joint. At each release the unbalanced moment is distributed to the members meeting at the joint in proportion to the $K$ value; i.e., the stiffness factor of the members. This completes the first cycle. Then the carry-over

moments are written. As mentioned before, this carry-over moment for a fixed end is half the applied moment.

After the carry-over moments have been written, the sum of the moments at the joints usually will be unbalanced. The joints are released again, one at a time, and the unbalanced moments distributed to the members meeting at the joints. This constitutes another cycle. This process is repeated until the desired degree of accuracy is obtained. With each step, the carry-over moments decrease, thereby reducing the unbalanced moments.

In using the moment distribution method it is necessary to adopt a sign convention to express the direction in which a moment

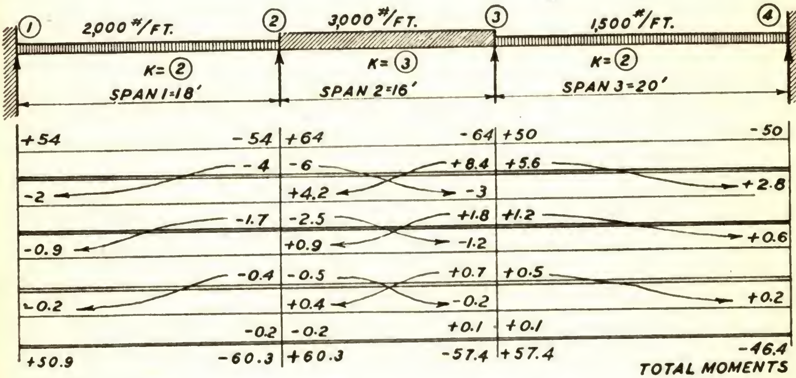



Fig. 109. Continuous Beam Solved by Moment Distribution

acts, and use that convention throughout. In the problem which follows, a clockwise moment acting on the end of a member is considered as negative, and a counterclockwise moment as positive.

The method of moment distribution lends itself to modifications for special cases. The modified methods are more quickly and easily solved, and the results, while they are not exact, do have a degree of accuracy satisfactory for designing purposes.

**Example—Moment Distribution.** Compute the bending moments at the supports for the beam shown in Fig. 109. The ends of the beam are considered fixed.

*Solution.* Following the procedure previously explained, the joints are all considered locked and the fixed-end moments calculated. As the beams are loaded with a uniform load, the fixed-end moments are $\frac{Wl}{12}$. The values for these are given in the first line below the figure. As the extreme ends of the beam are

fixed, they can take the moments. Now release joint *2*. There is an unbalanced moment of +10 ft. kips at *2*. The sum of the $K$'s of the two members meeting at the joint is 5, and the unbalanced moment is distributed—two-fifths to the member at the left and three-fifths to the member at the right. This now balances joint *2* and it again is assumed locked. Joint *3* is now released and the unbalanced moment of −14 ft. kips distributed three-fifths to the member at the left, and two-fifths to the member at the right. These values are shown in the second line. This completes one cycle and all the joints are balanced. Next the carry-over moments equal to one-half the distributed moments are written down. They are shown in the third line, and the arrows indicate the way that they are carried over. This now leaves joint *2* with an unbalanced moment of +4.2 ft. kips and joint *3* with an unbalanced moment of −3 ft. kips. These unbalanced moments are distributed as before and the results shown on the next line. This completes another cycle and the operation can be stopped here. The moments at supports *1*, *2*, *3*, and *4* would be 52, 59.7, 56.8, and 47.2 ft. kips, respectively. By comparing these figures with the final figures, it is noted that the difference is small and for practical purposes the figures for the total moments at the end of the second cycle could have been used.

It is well to point out here that the carry-over moments have the same sign as the applied moments. The distributed moments must, of course, have the opposite sign from the unbalanced moment in order to get balance at the joint.

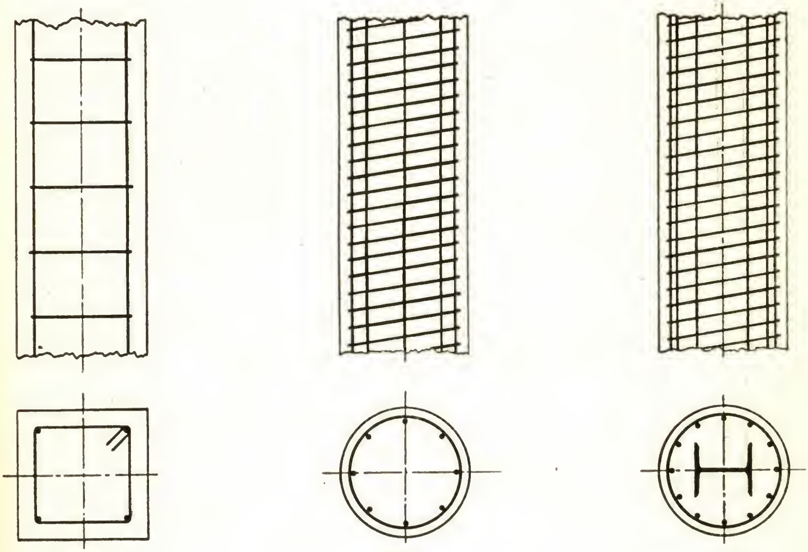
If supports *1* and *4* were hinged, the $K$ values of the members would be modified by a factor of three-fourths and there would be no carry-over moments to these supports. Carry-over moments would occur only in span *2*. The hinges cannot transmit any moment, so the moment at points *1* and *4* would be zero.

The preceding methods, briefly described, by no means include all the methods available for treatment of concrete frames. In addition to the analytical methods there is also the method of model analysis, which has been used chiefly to supplement and check analytical methods.

## CHAPTER XIV

# REINFORCED CONCRETE COLUMNS

**Classes of Columns.** There are three general classes of reinforced concrete columns: (1) tied columns, (2) spiral columns, (3) composite columns. See Fig. 110.



(A) (B) (C)

Fig. 110. At (*A*), Tied Column; (*B*) Spiral Column; (*C*) Composite Column

*Tied Columns.* Tied columns may be defined as concrete columns in which are placed vertical bars, $\frac{5}{8}'' \phi$ and larger, tied together with small bars placed 8″ to 12″ on centers. A vertical bar is placed in each corner of the column and as many additional bars are added as required for the steel area. When 6 or more vertical bars are used, ties should be made up in units with tie bars through the column, as shown in Fig. 111. For small columns $\frac{1}{4}'' \phi$ ties spaced 12″ may be used. For larger columns the ties should be $\frac{3}{8}'' \phi$ placed 8″ or 9″ on centers. The vertical bars are usually protected with 2″ of concrete;

for small columns this protection is sometimes reduced to 1½″. These are the simplest type of reinforced concrete columns; they cost less than other types and are extensively used. They may be either square or rectangular in shape. Fig. 110 at (*A*) illustrates a tied column.

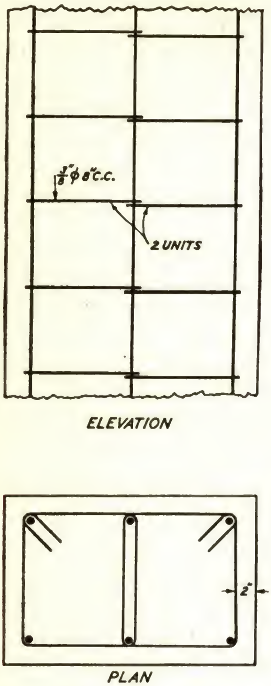



Fig. 111. Reinforced Concrete Column with Ties Made in Two Units

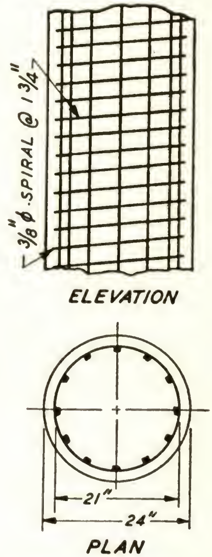



Fig. 112. Twenty-four-Inch Spiral Column

*Spiral Columns.* Spiral columns are usually made round but sometimes they are octagonal. They are reinforced with a continuous spiral and by vertical bars placed within the spiral. See Fig. 110 at (*B*) and Fig. 112. The spiral is generally made of ¼″, ⅜″ or ½″ round bars having a pitch of 1½″ to a maximum of 3″ and a diameter 3 to 4″ less than the column.

The spiral reinforcement supports the concrete laterally and prevents, to a large degree, the lateral deformation of the column under load. The spiral does not come into action until the elastic limit of

the concrete has been passed, but it does increase greatly the ultimate strength of the column, and helps to prevent sudden or complete collapse of the column. The longitudinal reinforcement carries a part of the load directly, the amount carried by the reinforcement being dependent on the ratio of the moduli of elasticity of the concrete and the reinforcement.

*Composite Columns.* Composite columns are concrete columns reinforced with bars and a structural steel section such as an **H**-column or wide-flanged **I**-beam or a cast iron section. This column is used when an ordinary reinforced column would be too large. They are usually designed so that about half the load is supported by the concrete and reinforcing steel, and half by the structural steel. This column is shown in Fig. 110 at (*C*).

**Height of Columns.** The height or unsupported length of a column must be determined before it is designed. The following rules are given as a guide to determine the unsupported length of a column: (1) in flat slab construction, the clear distance between the floor and the under side of the capital; (2) in floor construction with beams in one direction only, the distance between floor slabs; (3) in slab, beam and girder construction, the distance from the floor to the under side of the smallest beam framing into the column at the next higher floor level; (4) in columns supported laterally by beams or struts only, the clear distance between pairs or groups of beams; the beams or struts must be of sufficient size to be an adequate support, in or near the same plane, and the beams or struts must be at right angles to each other, or nearly so; (5) in construction where brackets are used at the juncture of the column and beam or girder, the distance from the floor to the bottom of the bracket of the smallest beam may be taken, provided the bracket is the same width as the beam and not less than half the width of the column.

**Design of Tied Columns.** The permissible load allowed on tied columns is:

$$P = 0.225 f'_c A_g [1 + (n - 1)p] \qquad (60)$$

in which $P$ is the permissible load, $A_g$ is the gross area of the column, and the other symbols have the same meaning as given previously. The ratio of longitudinal steel considered in the calculations should not be less than 0.005 nor more than 0.02 of the total area of the

column. This ratio should not be less than 0.01 to secure the best results.

**Example.** A column is to carry a load of 338,000# using a quality of concrete in which $f'_c = 2{,}500\#/\square''$ and $n = 12$. The height of the column is $13'6'' = 162''$. Assume the longitudinal reinforcement is equal to 0.005 of the area of the column. What is the size of the column and what is the area of reinforcing steel?

*Solution.* $P = 338{,}000\#$; $n = 12$; $f'_c = 2{,}500$ and $p = 0.005$. Substituting in Equation (**60**)

$$
\begin{aligned}
P &= 0.225 f'_c A_g[1 + (n-1)p] \\
338{,}000 &= 0.225 \times 2500 A_g \times [1 + (12-1).005] \\
&= 593 A_g \\
A_g &= 570\square''
\end{aligned}
$$

Without restrictions regarding the shape of the columns, a square one will be used. Extracting the square root of 570 it is found that a column 24″ by 24″ (576□″) is required.

The steel area equals $.005 \times 570 = 2.86\square''$ or four $1''\phi$ bars. One bar would be placed in each corner. Since the ties are not figured but selected by the designer, $\frac{3}{8}''\phi$ bars spaced 8″ on centers will be used. In practical designs the steel ratio should be higher, which would increase the amount of steel and decrease the size of the column.

In this problem, if the steel area is increased to 0.01 but all other factors are kept the same, the concrete area will be 541□″ and the steel area will be 5.41□″. If one dimension of the column is made 18″, then the other dimension $(541 \div 18 = 30)$ will be 30″. A column of this shape should be reinforced with six bars. Six 1″ square bars will give an area in excess of the required area, but it is the nearest that can be secured if all bars are made the same size. In Fig. 111 such a column is illustrated.

**Design of Spiral Columns.** An equation for designing spiral columns is as follows:

$$P = A_c[1 + (n-1)p]f_c \qquad \textbf{(61)}$$

in which

$P$ = the permissible axial load on column where $\frac{h}{R}$ is less than 50

$A_c$ = area of concrete within the outer circumference of the spiral hooping

$p$ = ratio of effective area of longitudinal reinforcement to the area of the concrete core in the column; that is, area of steel divided by the area of the concrete in the core

$f_c$ = permissible axial compression in the concrete of the column

$$f_c = 300 + [(0.10 + 4p)f_c'] \qquad \textbf{(62)}$$

$n$ = ratio of moduli $E_s \div E_c$
$h$ = the unsupported length of the column
$R$ = the radius of gyration of the column

The longitudinal reinforcement should consist of six or more bars of a minimum diameter of $\frac{5}{8}''$, and of an effective cross-sectional area of not less than 0.01, nor more than 0.06 of the area of the concrete within the spiral. The ratio of the spiral reinforcement should be not less than $\frac{1}{4}$ the ratio of the longitudinal reinforcement. Spiral reinforcement consists of an evenly spaced continuous bar held firmly in place and true to line by at least three vertical spacer bars. The spacing of the spirals should be not greater than one-sixth of the diameter of the core, and in no case more than three inches.

For example, applied to a 24″ diameter column, with a 21″ core, spiral of $\frac{3}{8}''$ round, $1\frac{3}{4}''$ pitch; twelve $1\frac{1}{8}''$ sq. bars (longitudinal steel), $A_s = 12 \times 1.27 = 15.24^{\square''}$, using 2,500-lb. concrete with $n = 12$; $A_c = \pi(10.5)^2 = 346^{\square''}$; $p = \frac{15.24}{346} = .044$. Substituting these values in Equation (**62**), $f_c = 300 + [0.10 + (4 \times 0.044)]2500 = 990\#/^{\square''}$.

Then from Equation (**61**)

$$
\begin{aligned}
P &= 346[1 + (12-1).044]990 \\
&= 508{,}316\#
\end{aligned}
$$

the permissible load on a column of the given dimensions provided that the spiral is adequate.

The area of the vertical steel in this demonstration is $15.24^{\square''}$; then $p = \frac{15.24}{346} = .044$. One-fourth of $.044 = .011$ which is the minimum ratio of spiral steel required.

A $1\frac{3}{4}''$ pitch for a spiral made of $\frac{3}{8}''\phi$ bar has been assumed. In one foot of height of the column there would be $\frac{12}{1\frac{3}{4}} = 6.86$ complete turns. The length of this bar would be $6.86 \times 3.1416 \times 21 = 453''$. The area of a $\frac{3}{8}''\phi$ bar is $0.11^{\square''}$. Then the volume of this bar would be $453 \times .11 = 49.83$ cu. in. per foot of column. The volume of concrete for a length of one foot of column equals $346^{\square''} \times 12'' = 4152$ cu. in., and the ratio of this spiral equals 49.83 cu. in. $\div$ 4152 cu. in. $= .012$. Therefore, a $\frac{3}{8}''\phi$ bar with $1\frac{3}{4}''$ pitch spiral will supply the required area of steel and is satisfactory for the given column.

The anchorage of the spiral at each end of the column should be made by 1½ extra turns of the spiral unit. If splices are necessary they should be made by 1½ extra turns or by welding. In Fig. 112 this column is illustrated.

**Design of Composite Columns.** When composite columns are used, the structural steel or cast-iron column is thoroughly encased in a concrete core and this concrete core is reinforced with both longitudinal and spiral reinforcement. The longitudinal reinforcement is located on the periphery of the core. The ratio of the longitudinal reinforcement used should be not less than 0.02 nor more than 0.04 that of the total core area enclosed within the circumference of the spiral hooping. The ratio of the spiral reinforcement should be not less than 0.01 of the total core area.

The permissible load on a composite column is the sum of the separate loads carried by the structural steel section and the concrete core. The unit stress on the steel column should not exceed 15,000#/□″; and when it must carry construction and other loads prior to its encasement, the unit stress should not exceed that given by the formula

$$f_r=\frac{18,000}{1+\dfrac{h^2}{18,000R^2}} \tag{63}$$

When a cast-iron column is used, the unit stress on the section should not exceed 9,000#/□″; and when the column must carry loads prior to encasement, the unit stress should not exceed that given by the formula

$$f_r=12,000-60\frac{h}{R}. \tag{63a}$$

The load carried by the concrete core equals $0.25f_c'$ on the net area of the concrete within the circumference of the spiral. The area of the steel or cast-iron column should not exceed 12 per cent of the core area.

**Example.** A 10″ H 49# steel column is encased in a 24″ diameter column having a 20″ core. The longitudinal reinforcement consists of eight 1″ square bars, and the spiral is made from ½″ round bars with a 3″ pitch. Using 3000-lb. concrete, what is the total load which the column can carry when the unsupported length of the column is 15′?

*Solution.* The core area is $\pi(10)^2=314$□″; $A_s=(8\times1)=8$□″. The volume of the spiral steel per foot of column, $\frac{12}{3}\times62.8\times.196=49.2$ cu. in.; in which $\frac{12}{3}$

is the number of turns in a section of column 12″ long; 62.8 is the circumference of the column; .196 is the area of ½″ $\phi$ bar. The area of a 10″ H 49# column = 14.4□″. Ratio of longitudinal reinforcement $=\frac{8}{314}=.0254$. Ratio of spiral steel $=\frac{49.2}{314\times 12}=.013$. Ratio of steel column to core area $=\frac{14.4}{314}=.046$ which equals 4.6 per cent. The ratios of .0254, .013 and 4.6 per cent are within the limits specified under the heading, "Design of Composite Columns." Least radius of gyration of 10″ H 49# = 2.54″. Assuming that the steel column carries construction loads, the allowable fiber stress on the steel section is found by substituting in Equation (63)

$$f_r=\frac{18{,}000}{1+\frac{(180)^2}{18{,}000\times(2.54)^2}}$$
$$=14{,}070\#/\square''$$

A 10″ H 49# will support a load of 14,070 × 14.4 = 202,600#. The net area of the concrete equals 314 − (14.4 + 8.00) = 291.6□″. The load carried by the concrete = 0.25 × 3000 × 291.6 = 218,700#. The total load which the composite column can carry is 202,600 + 218,700 = 421,300#.

**Short and Long Columns.** Columns also are classified as short columns and long columns according to their length. Short columns are those that are not longer than ten times the least lateral dimension. Long columns are those that are longer than ten times the least lateral dimension. Column Equations (60), (61) and (62) are all based on short columns.

When the slenderness ratio of columns becomes too large, the working stresses must be reduced by using a formula that takes the slenderness of the column into consideration.

**Design of Long Columns.** For long, axially loaded reinforced concrete or composite columns, the greatest permissible working load may be determined by Equation (64) (following the present progress report of the Joint Committee).

$$P'=P\left(1.3-.03\frac{h}{d}\right) \qquad (64)$$

in which

$P'$ = total safe axial load on a long column

$P$ = total safe axial load on a similar column whose length does not exceed 10 times its least cross-sectional dimension

$h$ = clear height of column expressed in inches

$d$ = the least lateral dimension

For example, consider the column designed under the heading

"Design of Spiral Columns". In that case a column 24″ in diameter would support a load of 508,300# as a short column. Assume that it is 30′ long and make the necessary substitutions in Equation (64).

$$P' = 508{,}300\#\left(1.3 - .03\frac{30' \times 12''}{24''}\right)$$
$$= 432{,}055\#$$

The permissible load on this column under the conditions stated is 432,055#, but as a short column the maximum load was 508,300#. A column 30′ in height is a truly long one and is not often found in

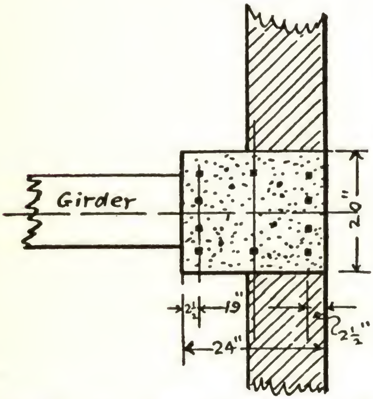


Fig. 113. Plan of Girder and Column, the Girder Developing Moment in Column

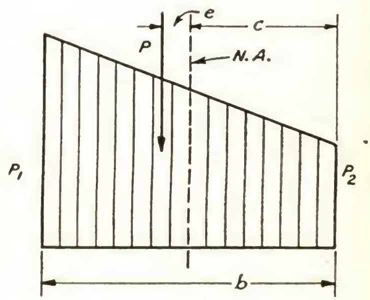


Fig. 114. Section Showing Variation of Fiber Stress

ordinary work. Therefore, such a large difference in carrying capacity of short and long columns should not be anticipated.

**Bending Stresses in Columns.** When columns are subjected to bending stresses in addition to compression, the limiting combined unit stresses should be as follows:

1. Columns with spiral reinforcement $[300+(0.10+4p)f_c']+0.15f_c'$

2. Columns with lateral ties, $0.3f_c'$. The total amount of reinforcement considered in the computations should not be more than 4 per cent of the total area of the column.

3. Tension in longitudinal reinforcement due to bending of the column shall not exceed 16,000#/□″. In no case, however, shall the column section be less than that required when axial load is considered alone.

Bending moments in columns may be caused by (1) an eccentric load such as the load on a bracket or a cantilever, or by a beam framing on only one side of a column, Fig. 113; (2) unevenly loaded floor panels; and (3) wind loads. In this text we shall consider only the question of eccentric loads.

*Eccentric Loading on Columns.* When an eccentric load is applied to a column, the amount of eccentricity is usually known, and the moment produced by this eccentric load can be calculated. The load may be applied so that (1) it is eccentric with only one of the lines of symmetry of the column, as shown in Fig. 114, or (2) it may be eccentric with respect to both axes of symmetry. In both of these

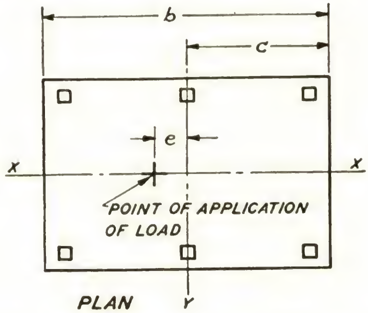



Fig. 115. Eccentric Load on Reinforced Concrete Column

instances, the eccentricity may be such as to produce (1) only compression over the entire section or (2) both tension and compression over the section.

The most common case encountered is that in which the load is applied along one axis of a column with a known eccentricity with respect to the other axis as shown in Fig. 115. The effect of the load is equivalent to that produced by an axial load of the same amount and a moment equal to $e$ times the intensity of the load. The fiber stresses are found by use of Equation (57)

$$f=\frac{P}{A}\pm\frac{Mc}{I}$$

Substituting $Pe$ for $M$,

$$f=\frac{P}{A}\pm\frac{Pec}{I}$$

The solution of problems involving this type of eccentricity may be made with the help of diagrams or by the use of the "trans-

and also because of the simple type of construction with low-cost materials.

There are two general types of footings for buildings, (1) wall footings and (2) column footings. In Fig. 116 is shown a section through a wall footing. Generally, wall footings do not support a large load; but if a large load is to be supported or if poor soil is found, they can be increased in width easily. Crosswise, these footings are reinforced when the width of the projection becomes too

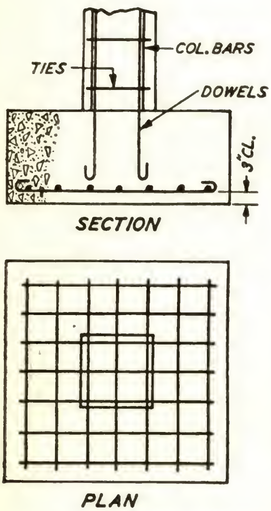



Fig. 117. Column Footing

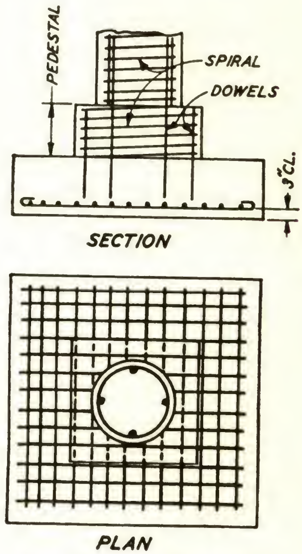



Fig. 118. Column Footing with Pedestal

reat for the depth. Longitudinally, the footings should be rein- orced at least for temperature stresses. If sufficient steel is added o develop the strength of the concrete, a better distribution of the oad over the bearing area will be secured.

In Fig. 117 is shown a reinforced concrete column footing in hich the column rests directly on the footing. The reinforcing eel is placed in two directions, although it can be placed in four irections if the designer prefers. (See Fig. 122.) Placing the steel four directions is not in favor at the present time because of the crease in the thickness of the footings for the two additional rows bars and the extra work and expense caused by cutting and placing

and also because of the simple type of construction with low-cost materials.

There are two general types of footings for buildings, (1) wall footings and (2) column footings. In Fig. 116 is shown a section through a wall footing. Generally, wall footings do not support a large load; but if a large load is to be supported or if poor soil is found, they can be increased in width easily. Crosswise, these footings are reinforced when the width of the projection becomes too

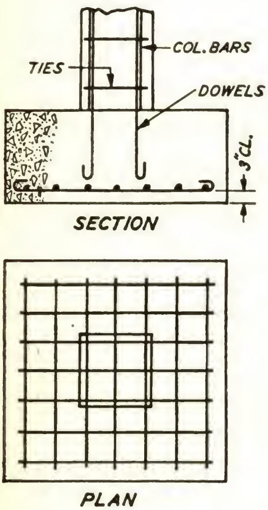



Fig. 117. Column Footing

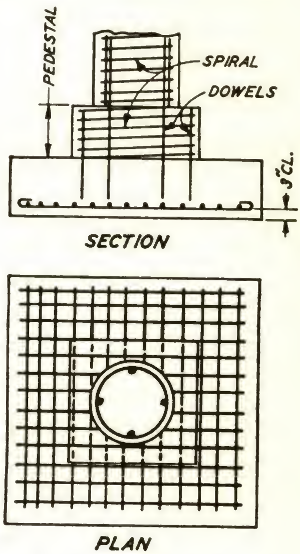



Fig. 118. Column Footing with Pedestal

great for the depth. Longitudinally, the footings should be reinforced at least for temperature stresses. If sufficient steel is added to develop the strength of the concrete, a better distribution of the load over the bearing area will be secured.

In Fig. 117 is shown a reinforced concrete column footing in which the column rests directly on the footing. The reinforcing steel is placed in two directions, although it can be placed in four directions if the designer prefers. (See Fig. 122.) Placing the steel in four directions is not in favor at the present time because of the increase in the thickness of the footings for the two additional rows of bars and the extra work and expense caused by cutting and placing

the diagonal bars. A better distribution of the load through the steel is secured by using the four-way system.

In Fig. 118 is shown a column footing in which the load of the column is transferred to the footing through a pedestal. The function of the pedestal may be (1) to distribute the load from a highly stressed concrete in the column to a lower stressed concrete in the footing; (2) to effect a saving in materials and cost which can be made by the use of the pedestal for large footings.

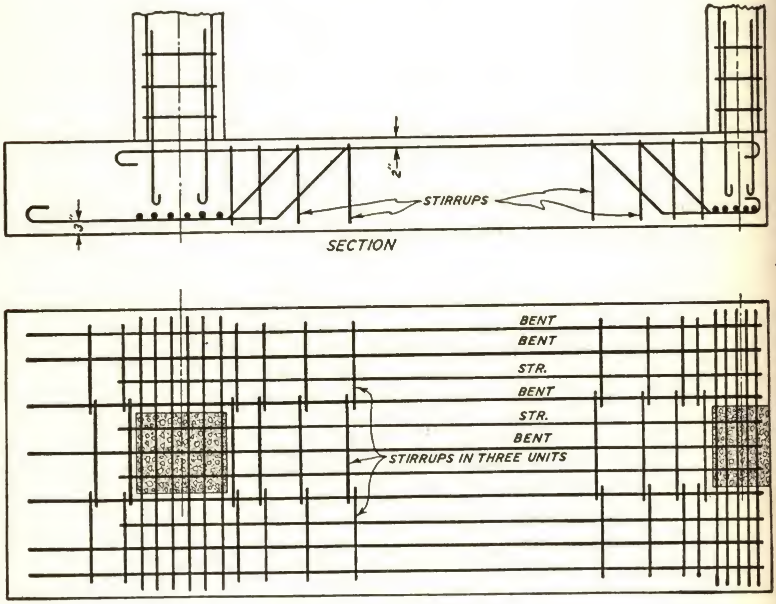



Fig. 119. Combined Footing

In Fig. 119 is shown a combination footing for a wall column and an interior column. The footing is proportioned to suit the loads.

**Wall Footing.** A wall footing is designed as a double cantilever beam to distribute the load uniformly over the supporting soil. The bending moment at any section of the footing at a distance $X$ from the edge of the footing is $M = \frac{1}{2}WX^2$, in which $W$ is the uniform bearing pressure of the soil per lineal foot of footing for a given width of section. For a section at the face of the wall, the bending moment will be $M = \frac{1}{8}W(l-a)^2$ where $l$ is the width of the footing and $a$ is

the thickness of the wall. The maximum moment is at the center of the wall and the equation for the moment is

$$M = \tfrac{1}{8}W(l^2 - la) \tag{65}$$

Tests made by Professor A. N. Talbot at the University of Illinois in 1908-1912 on both wall and column footings indicate that the critical section is at the face of the wall or column. The results of these tests form the basis of present-day practice in footing design. These tests also showed that the critical section for diagonal tension is located at a distance $d$, the effective depth of the footing, from the face of the column or wall. So the shear, used as a measure of the diagonal tension, is calculated at this point and the stress is kept low so as not to require any diagonal tension reinforcement. The critical section for bond was found to be at the face of the wall or column, so the shear at this point is used in checking bond stresses in the reinforcement. Bond stresses usually are very high in footings and always should be investigated, as they frequently limit the size of reinforcing that can be used. The design of a wall footing is best illustrated by a numerical example.

*Illustrative Example.* Design the footing for a 24″ wall with a total load of 42,000 pounds per running foot, which is to rest on a soil that can safely bear a load of 7,000#/□′.

*Solution.* See Fig. 120. The required width of footing will be $\frac{42{,}000}{7{,}000} = 6'$. The footing will project 2′ on either side of the wall. Then $l = 6'$, $a = 2'$, and $W = 7{,}000\#/\Box'$.

$$\begin{aligned} M &= \tfrac{1}{8} \times 7000 \times (6-2)^2 \\ &= 14{,}000'\# \\ &= 14{,}000'\# \times 12 = 168{,}000''\# \end{aligned}$$

Using 2,000-lb. concrete, $f_c = 800\#/\Box''$ and with $f_s = 20{,}000\#/\Box''$, $n = 15$; $K = 131$; $p = .0075$; and $b = 12$. Then

$$\begin{aligned} M = 131bd^2 = 131 \times 12 \times d^2 &= 168{,}000 \\ d^2 &= 106.8 \\ d &= 10.3 \end{aligned}$$

The steel area required is $12 \times 10.3 \times .0075 = .927\Box''$ per lineal foot. This can be furnished by ¾″ round bars at 5½″ center to center.

The depth of 10.3″ to center of steel and ¾″ $\phi$ bars spaced 5½″ will satisfy the requirement for moment.

The critical section for diagonal tension occurs at a distance equal to $d$, the effective depth, from the face of the wall. The value

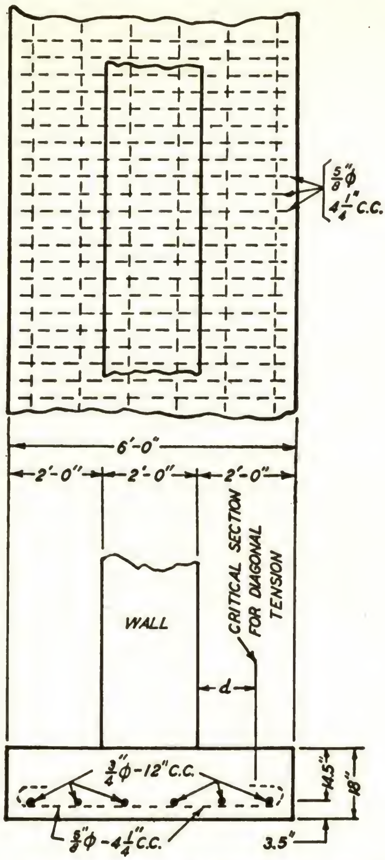



Fig. 120. Continuous Footing under Wall

for $d$ has just been calculated equal to 10.3″. The total shear at the critical section is $\frac{24''-10.3''}{12}\times 7000=8{,}000\#$. Substituting in Equation **(22)**, $v=\frac{8V}{7bd}$; $v=\frac{8\times 8000}{7\times 12\times 10.3}=74\#/\square''$. As this value for $v$ exceeds the allowable stress of $60\#/\square''$, the depth $d$ must be increased. As $d$ is increased, $v$ is decreased. Assume that the total depth of the

footing is 18″. Allowing 3½″ for cover, $d=14.5''$. The total shear is $\frac{24''-14.5''}{12''}\times 7000=5{,}540\#$ and $v=\frac{8\times 5540}{7\times 12\times 14.5}=36\#/\square''$, which is satisfactory. To satisfy the requirements for moment, 0.927□″ of steel is needed per foot of footing, with $d=10.3''$. With $d=14.5''$ the amount of steel required equals $A_s=\frac{168{,}000}{20{,}000\times .87\times 14.5}=0.666\square''$ per foot. This is furnished by ⅝″ ϕ bars spaced 5½″ center to center.

The bond stress will be investigated. The critical section for bond is located at the face of the wall. The total shear at the face of the wall is $2\times 7{,}000=14{,}000\#$. Equation (23) gives the unit bond stress, $u=\frac{8V}{7d\Sigma o}$, where $\Sigma o$ is the sum of the perimeters of all the bars in the section. In the width of 12″ there are $\frac{12}{5.5}=2.18$ bars with a circumference of 1.96″, therefore $\Sigma o=2.18\times 1.96=4.28''$.

Substituting in Equation (23), $u=\frac{8V}{7\Sigma od}$

$$u=\frac{8\times 14{,}000}{7\times 14.5\times 4.28}=258\#$$

Deformed bars with hook will be used with a maximum allowable bond stress of 200#/□″. Since the actual stress is greater than this value, the value of $\Sigma o$ must be increased. Using 200#/□″ as the value for $u$ and solving Equation (23) for $\Sigma o$ it is found that

$$\Sigma o=\frac{8\times 14{,}000}{7\times 14.5\times 200}=5.52''$$

This means that the ⅝″ bars must be spaced 4¼″ ($5.52\div 1.96=2.81$ bars per foot, then $12''\div 2.81=4\frac{1}{4}''$) center to center instead of 5½″ center to center. While a footing 18″ deep with ⅝″ ϕ bars spaced 5½″ center to center satisfies the shear and moment requirements, in order to keep the bond stress within the allowable limits it is necessary to reduce the spacing of the bars to 4¼″. The ¾″ ϕ longitudinal bars are placed in the footing to provide for the temperature stresses.

**Column Footings.** There are two general methods of reinforcing square or rectangular column footings. The most common one employs two bands of bars at right angles to each other, as shown

in Fig. 121. The other has four bands of bars, two of which are at right angles to each other and the other two run diagonally as shown in Fig. 122. In Fig. 122 stirrups are also shown. As it is difficult to place stirrups and keep them in position while pouring the footing, and as their use is expensive, square and rectangular footings are

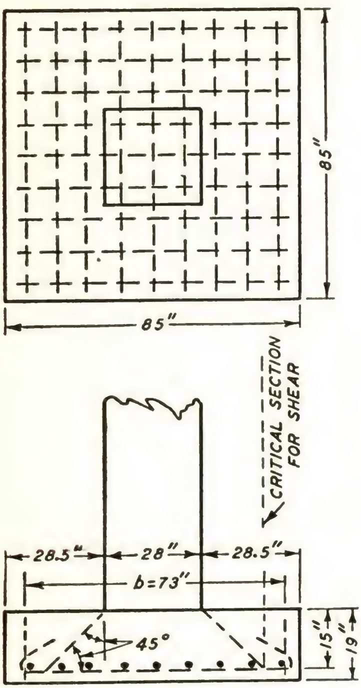



Fig. 121. Square Footing under Column

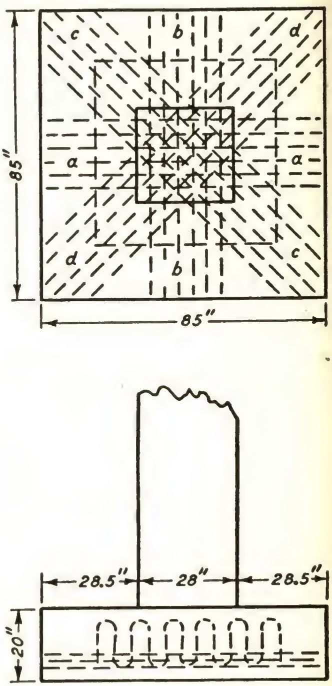



Fig. 122. Diagram of Footing for a Column Using Diagonal Bands

almost always designed without stirrups by keeping the shear and bond stress within the allowable limits. Shear and bond stresses control the thickness of footings of this type, not the depth required for the bending moment.

In a footing the critical section for bending is at the face of the column or pier, except when steel plates or cast-steel column bases are used, in which case the theoretical maximum moment is taken as acting at a point half way between the center of the column and the edge of the steel plate or base. The bending moment at the

critical section of an isolated column footing is resisted by the full vertical section of the footing at the critical section extending from the top of the footing to the center of the reinforcing steel. The critical section is shown by line $AB$ in Fig. 123. The projection of the footing beyond the face of the column is considered as a simple cantilever beam.

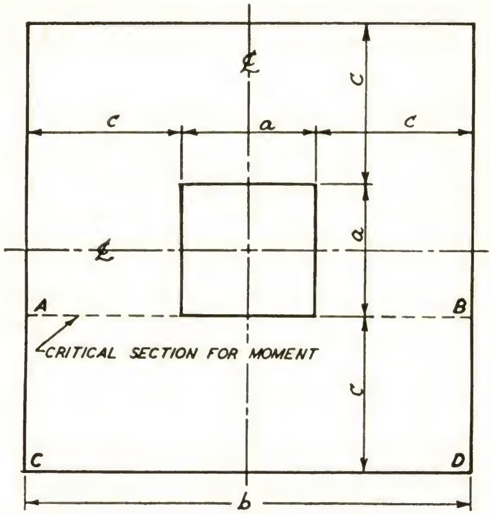



Fig. 123. Plan Showing Projection Producing Moment in Footing

The value of the bending moment is given by the equation

$$M = w\left(\frac{bc^2}{2}\right) \tag{66}$$

in which

$M$ = bending moment at the critical section of the footing

$a$ = width of the face of the column or pedestal

$b$ = width of the footing

$c$ = projection of the footing from the face of the column, $= \left(\frac{b-a}{2}\right)$ (See Fig. 123.)

$w$ = upward reaction per unit of area of the base of the footing

In examining Fig. 123, it is readily seen that Equation (66) is derived by multiplying the area $ABDC$ by half the distance from the face of the column to the edge of the footing and that product by the allowable pressure on the ground.

**Shear and Diagonal Tension in Footings.** The tests of Professor Talbot, previously referred to, showed that the critical section for diagonal tension in footings was located at a distance $d$, the effective depth of the footing, from the face of the column, and parallel with it. If straight bars are used in a footing, the unit shearing stress as calculated by Equation (**22**) should not exceed $0.02f'_c$. Almost all footings designed today have the bars anchored by means of adequate hooks, and for these footings the unit shearing stress shall not exceed $0.03f'_c$.

**Bond Stresses in Footings.** Bond stresses usually are very high in footings. In determining the bond stress, the shear at the face of the column is used. In two-way footings, the bond stress is limited to $0.03f'_c$ for plain bars and $0.0375f_c'$ for deformed bars. Where special anchorage is provided by means of hooks at the ends of the bars, these stresses may be doubled. In placing the bars in the footings, the hook should be protected by not less than 2 inches nor more than 3 inches of concrete.

**Transfer of Stress at the Base of a Column.** The compressive stress in the column reinforcement at the base of a column is transferred to the pedestal or pier by means of dowels. There must be at least one dowel for each column bar, and the dowel should be the same size as the column bar. The dowels should extend into the column and the footing a distance sufficient to develop the bar stress in bond. A minimum of 30 diameters for plain bars and 24 diameters for deformed bars generally is used.

The permissible unit compressive stress on top of the pedestal or footing directly under the column should not exceed that determined by Equation (**67**).

$$r_a = p_a \sqrt[3]{\frac{A}{A'}} \qquad (67)$$

in which

$r_a$ = permissible unit stress in concrete over the loaded area of a pedestal, pier, or footing

$p_a$ = permissible unit stress on pedestal, pier, or footing when the full area is loaded

$A$ = total area of the top of the pedestal, pier, or footing

$A'$ = loaded area of the pedestal, pier, or footing at the column base

The value of $p_a$ should not exceed $0.25f_c'$ for plain concrete. When spirals or hoops are provided, the value for $p_a$ may be increased to $0.25f_c'(1+2.5np')$, but no area outside the outer face of the spiral is considered.

In sloped or stepped footings, $A$ may be taken as the area of the top horizontal surface of the footing or as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base the loaded area $A'$, and having side slopes of 1 vertical to 2 horizontal.

*Illustrative Example.* A column 28 inches square carries a load of 300,000#. If the soil will carry safely a load of 6,000 pounds per square foot, determine the size of the footing which will be required and the size and spacing of the reinforcing bars, using $f_c=2500\#/\square''$, $f_s=20{,}000\#/\square''$ and $n=12$.

*Solution.* The load of 300,000 pounds requires a footing with an area of $300{,}000\div6{,}000=50\square'$. The sides of the footing will be $\sqrt{50}=7.07'$. Using a footing 85″ square, the offset $c$ is 28.5 inches.

Applying Equation (**66**), $b=85$, $a=28''$, $c=28.5''$, and $w=6000\#\div144\square''=42\#/\square''$.

$$M=w\left(\frac{bc^2}{2}\right)=\frac{42}{2}\Big[85\times(28.5)^2\Big]$$
$$=1{,}449{,}870''\#$$

As previously stated, in order to keep the shear stress low and eliminate stirrups, the footing must be made deeper than would be required to satisfy the moment. To satisfy the moment requirement, a depth $d$ of 11″ would be satisfactory; but the shear (diagonal tension) would be much too high.

Assume $d=15''$, and investigate the shear. The critical section is located at a distance $d=15''$ from the face of the column. The total shear acting on this section is the upward pressure on the footing outside a square $28+(2\times15)$ or 58″ on a side. The total area of the footing $=(85)^2=7225\square''$; and $(58)^2=3364\square''$. Then, $7225-3364=3861\square''$, the area producing shear on the critical section. The total shear equals $3861\square''\times42''=162{,}162\#$. The area resisting this shear equals the perimeter of the critical section, 232″, multiplied by the effective depth. Substituting in Equation (**22**)

$$v=\frac{8V}{7bd}\text{ or }v=\frac{8\times162{,}162}{7\times232\times15}=53\#/\square''$$

This is satisfactory.

With $d=15$ inches, the area of steel required for moment equals

$$A_s=\frac{1,449,870}{20,000\times 15\times .87}=5.56\square''$$

This area can be furnished by twenty-eight ½″ $\phi$ bars.

Investigate the bond stress. The critical section for bond is at the face of the column. The total shear on that section equals $(42\times 85\times 28.5)=101,750\#$.

Twenty-eight ½″ $\phi$ bars have an $\Sigma o$ value of $28\times 1.57=44.0$. Substituting in Equation (23)

$$u=\frac{8V}{7d\Sigma o}=\frac{8\times 101,750}{7\times 15\times 44.0}=176\#/\square''$$

which is satisfactory, as allowable $u=.0375\times 2500\times 2=188\#/\square''$.

The footing will be 85″×85″ with an effective depth of 15″, or a total depth of 19″, allowing 3″ cover below the lowest bars, and the footing is to be reinforced with twenty-eight ½″ $\phi$ bars each way, or a total of fifty-six ½″ $\phi$ bars.

**Footing Reinforced with Diagonal Bands.** Fig. 122 shows an alternative method of placing the reinforcement. Four bands are used, two being diagonal.

It is evident that the amount of steel required would be the same as computed by the previous method. Since there are four bands of bars, instead of only two, the reinforcement occupies a greater vertical height, which adds that much to the volume of the footing.

To design such a footing, compute the moment as before; determine the breadth $b$ as before; also the depth $d$ which will be required for shear. Compute the area of steel required, which may be considered as the area of steel in a quarter section formed by two lines from two corners to the center, the steel including that in one straight band (say $b$) and two halves (½$d$ and ½$c$) of the two diagonal bands. The combined area of all these bars should at least equal the area as computed by the previous method. The shear and bond stresses should be computed as before.

Fig. 122 shows a convenient and effective form of stirrup to resist shear in a square footing, when some such reinforcement is required. The stirrup being continuous, looping both at top and bottom, is completely anchored and is effective.

It should be noted from the solution of this and the previous problem that, on account of the combination of heavy load and small cantilever projection, the bond stress of footings is always a critical matter and its investigation should never be neglected.

Since smaller bars have a greater surface and a greater adhesion per unit both of area and of strength than larger bars, the requisite adhesion may sometimes be obtained by using a proportionately larger number of smaller bars. The ends of all bars in footings always should be bent into a hook, which should have a radius not less than six times the diameter of the bar.

**Compound Footing.** When a simple footing supports a single column, the center of pressure of the column must pass vertically through the center of gravity of the footing, or there will be dangerous transverse stresses in the column; but it is sometimes necessary to support a column on the edge of a property when it is not permissible to extend the foundations beyond the property line. In such a case, a simple footing is impracticable. The method of supporting such a column is indicated in Fig. 124. The nearest interior column (or even a column on the opposite side of the building, if the building be not too wide) is selected, and a combined footing is constructed under both columns. The weight on both columns is computed. If the weights are equal, the center of gravity is halfway between them; if unequal, the center of gravity is on the line joining their centers, and at a distance from them such that $x:y::W_2:W_1$, Fig. 124. In this case, evidently $W_2$ is the greater weight. The area *abdc* must fulfill two conditions:

(1) The area must equal the total loading $(W_1+W_2)$ divided by the allowable loading per square foot; and,

(2) The center of gravity of the footing must coincide with the center of gravity of the loads. In Fig. 124 this point is located at $O$.

As a solution for this problem, there are an indefinite number of trapezoids that can fulfill the two conditions just mentioned. However, for the sake of simplicity, a compound footing with a rectangular shape as shown in Fig. 119 is used whenever practicable. The trapezoidal shape is only used where it is not possible to extend the footing beyond either column and both columns must be placed at the edge of the footing, and the columns have unequal loads.

*Illustrative Example.* The following example is solved for a

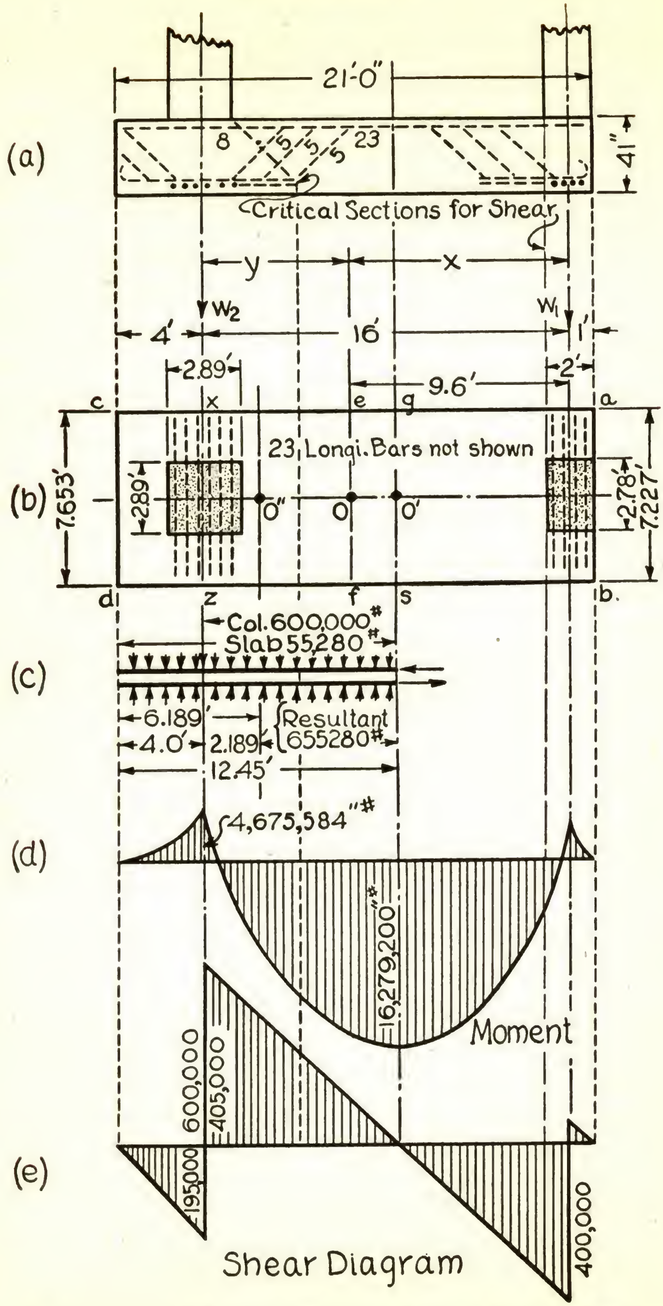



Fig. 124. Combined Footing for Two Columns, One on Edge of Property

trapezoidal section to indicate the procedure used. In actual practice, this particular footing would be designed as a rectangular section.

A column $W_1$, carrying 400,000#, is to be located on the edge of a property and another column $W_2$, carrying 600,000#, is located 16′ from it. Assume that the subsoil can sustain safely 7,000#/□′. Required, the shape and design of the footing.

*Solution.* Assume that the footing slab weighs 600#/□′ of surface; then the *net* effective upward pressure of the subsoil which will support the column equals $7{,}000-600=6{,}400$#/□′. For simplicity of calculation in the computations involving soil pressures and slab areas, feet and decimals will generally be used. The change to feet and inches can be made when the final dimensions have been computed.

The total column load is 1,000,000#; at 6,400#/□′ the area must be 156.25□′. The strength of columns has already been discussed. Assume that the columns will carry 500#/□″. Then $600{,}000 \div 500 = 1{,}200$□″ $=(34.64'')^2=2.89'$ square as the size of the $W_2$ square column. Assume that the $W_1$ column is 2′ wide and as long as is necessary to give it the requisite area. In Fig. 124, let *ab* equal $n$, and *cd* equal $m$, both still unknown. The smaller column is on the edge of the property, and the *ab* line is made 1.0′ from the column center. As a trial solution, assume that the *cd* line is 4.0′ beyond the other column center. Then the total length of the trapezoid is 21.0′; then ½ $(m+n)\ 21.0=156.25$; solving this

$$(m+n)=14.88$$

The center of gravity of the two loads is at $\dfrac{600{,}000}{1{,}000{,}000}$ of 16 feet, or at 9.6′ from the smaller column center. This locates $O$. To fulfill condition (*2*), the dimensions $m$ and $n$ must be such that the center of gravity of the trapezoid shall be at $O$. In general, the distance $z$ of the center of gravity of a trapezoid from its larger base equals one-third of the height, $h$, times the quotient of the larger base, plus twice the smaller base divided by the sum of the bases; or, as an equation

$$z=\tfrac{1}{3}h\frac{m+2n}{m+n}$$

but
$$z=21.0-(9.6+1.0)=10.4$$

Substituting $z$ equals 10.4, $h$ equals 21.0, $m$ and $n$ still unknown,

$$10.4=\frac{21.0}{3}\times\frac{m+2n}{m+n}$$

Multiply by $(m+n)$

$$10.4m+10.4n=7m+14n$$
$$n=.944m$$

Substitute this value of $n$ in the equation $(m+n)=14.88$

$$m+.944m=14.88$$
$$m=7.653$$

then $$n=14.88-7.653=7.227$$

The section $ef$, through the center of gravity $O$, has been previously computed to be 9.6′ from the right column center, or 10.6′ from the end $ab$. Since the footing widens from 7.227 to 7.653 in the total distance of 21.0′, it will widen $\frac{10.6}{21.0}\times(7.653-7.227)=0.215$ in the distance of 10.6′. $7.227+0.215=7.442^{1}$, which is the dimension $ef$ through $O$.

*Moment.* The maximum moment is found where the shear is zero, and this must be at the right-hand end of a portion of the slab on which the *net* upward pressure equals 600,000#. That portion must have an area of $(600{,}000\div6{,}400)=93.75^{\square\prime}$. Similarly, the remaining area is computed to be $62.50^{\square\prime}$. Let $p$ equal the length of this section ($gs$ in the figure) and $h$ equal its distance from $cd$. We may write the two equations

$$\tfrac{1}{2}(7.653+p)h=93.75$$

and

$$\tfrac{1}{2}(p+7.227)\ (21-h)=62.50$$

Solving these two equations for $p$ and $h$, we have $p=7.401$ and $h=12.455$. The solution of these equations involves biquadratics and is given as follows: From the first of the two above equations,

$$7.653h+ph=187.50.$$

From the second,

$$21p+151.767-ph-7.227h=125.0$$
$$21p+.426h+151.767=312.50$$
$$21p+.426h=160.733$$
$$p=7.654-.0203h$$

Substituting in the first equation above,

$$7.653h+7.654h-.0203h^2=187.50$$
$$h^2-754h=-9236$$
$$h^2-754h+142{,}129=132{,}893$$
$$h-377=\pm 364.545$$

Using the negative value,

$$h=12.455, \text{ and then,}$$
$$p=7.654-.0203h=7.654-.253=7.401$$

It should be noted that this section of maximum moment (on the line *gs*) is *not* on the line of center of gravity of the whole footing, but is in this case about two feet to the right. The center of gravity ($0''$) of the trapezoid *cdsg*, calculated as above, is at a point 6.189 feet from *gs* and the *net* upward pressure on this section is 600,000 pounds. Therefore, taking moments about *gs*, we have

$$M=600{,}000\ [(12.45-4.00)-6.189]$$
$$=1{,}356{,}600'\#=16{,}279{,}200''\#$$

In this case, $b=7.40'=88.8''$. Then, using 2000-lb. concrete, $M=138.6bd^2=12{,}308d^2=16{,}279{,}200$, from which $d=36.4''$. Adding 5″ for steel and protective concrete, we have a total depth of 41″.

$A_s=.00889\times 88.8\times 36.4=28.70^{\square''}$ which may be provided by 23 bars $1\frac{1}{8}''$ sq.

The slab tends to bend upward on the line *gs*, Fig. 124, where the moment is a negative maximum. On the column line *xz*, it tends to bend downward. Between *gs* and *xz* there is a point of flexure, where the moment is zero. The moment on the line *xz* is approximately measured by the *net* pressure on the bottom of the area *xzdc* times its lever arm ($\frac{1}{2}cx=24''$). The *net* pressure is $6{,}400\#/^{\square'}$ and the area is $4\times 7.61=30.44^{\square'}$. $30.44\times 6{,}400=194{,}816\#$. $194{,}816\times 24''=4{,}675{,}584$. $d=36.4''$, as above. $4{,}675{,}584=138.6\times b\times (36.4)^2$, from which $b=25.4''$. Of course $b$ is actually over $7'$ wide, but the calculation gives the minimum width and also permits the computation of the necessary steel. $A_s=.00889\times 25.4\times 36.4=8.22^{\square''}$, which is more than supplied by seven $1\frac{1}{8}''$ sq. bars. This shows that $(23-7)=16$ bars are available for bending to resist the diagonal tension and shear. As shown in Fig. 124, the 23 required bars run through the critical moment section *gs*, eight being run straight through. Fifteen bars, in three groups of five, are bent down as shown, and some of them are bent up again and hooked.

There is a tendency for the slab to bend up in the space between the column face and $x$, and also at $z$. The distance from the column to $x=2.35'$ and the *net* upward pressure is approximately $\frac{2.35}{7.58}\times 600{,}000=186{,}000\#$. Considering the lever arm as about $.6\times2.35=1.41'=17''$, the moment is $186{,}000\times17=3{,}162{,}000''\#$. Considering that bars can be placed across the slab (as shown) at a depth $d=36.4''$, then $3{,}162{,}000=138.6\times b\times36.4^2$. Then $b$, the required width to develop such a stress in the steel, $=17.2''$. $A_s=.00889\times17.2\times36.4=5.56^{\square''}$, which is more than supplied by six 1″ sq. bars. A similar band of bars should be placed across the slab under the other column. The upward pressure on the off-set slab is about two-thirds of that at the other end, and the lever arm is even smaller. So, without precise calculations, we can see that four 1″ sq. bars (instead of six) will furnish the required moment resistance.

**Shear.** The critical section for shear at or near both columns is shown in Fig. 124 at (*a*). For the interior column, the critical section is at a distance $d$, the effective depth, from the face of the column. As shown in the shear diagram (*e*) vertically below this point the shear at the critical section scales about 200,000#. From Equation (**22**), $v=\frac{8}{7}\times200{,}000\div(89\times36.4)=70\#/^{\square''}$. This being greater than $.02\times2{,}000=40\#/^{\square''}$, shows the necessity for bent-up bars, but, being less than $.06\times2{,}000=120\#/^{\square''}$, is tolerable. The bent-up bars shown are more than sufficient for the extra shearing strength required. The maximum shearing stress at the exterior column occurs at the inside face of the column. Since the top of the beam is in tension and a diagonal crack will start at the top, the critical section for diagonal tension is at the inside column face instead of at a distance $d$ from the face. The shear at this point is about 307,000#. Using Equation (**22**) $v=\frac{8}{7}\times307{,}000\div(87\times36.4)=111\#/^{\square''}$. This again is tolerable but requires bent-up bars or stirrups. These are designed as previously shown. The shear on the other pairs of column faces (toward the long sides of the footing) can be figured from the data already given and by the methods already explained, and will not be detailed here.

The bond stress should also be investigated to show whether, and to what extent, the longitudinal bars, and even the cross bars under the columns, should be anchored.

Although not definitely required by any computable stresses,

some light bars (say ½″ round, spaced say 18″) should be laid across the top near the upper surface. They would be even more effective by having them bend down about 6″ at the ends. As already explained under Columns, dowels should be placed vertically in the footings, under the column seats, and protruding upward the required amount, to be later encased by the columns.

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*Courtesy of Portland Cement Association*

## CHAPTER XV

# REINFORCED CONCRETE RETAINING WALLS

**Forms of Walls.** Reinforced concrete walls are usually made in such shape that advantage is taken of the weight of part of the material supported to increase the stability of the wall against overturning. Fig. 125 shows the outline of such a wall. It consists of a vertical wall *CD,* attached to a floor plate *AB*. To prevent the wall

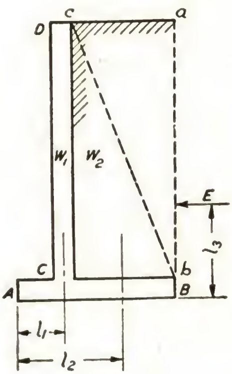



Fig. 125. Outline of Reinforced Concrete Wall

from overturning, the moment of downward forces about the outer edge of the base $M_1 = W_1l_1 + W_2l_2$ must be greater than that of the overturning moment $M_2 = El_3$. $M_1$ should have a value of from one and one-half to twice $M_2$, which would be the factor of safety. The shearing of the earth along the line *ab* would add to the factor of safety, but since the fill would be new much dependence should not be put on it. If additional security is needed against sliding, over the frictional resistance, it should be secured by pouring a projection below the base as shown at the right in Fig. 126.

Owing to the skeleton form of these walls they are usually more economical to construct than solid walls of masonry. The cost per cubic yard of reinforced concrete in the wall will be more than the

cost per cubic yard of plain concrete or stone, in a gravity retaining wall, but the quantity of material required will be reduced by 30 to 50 per cent in most cases. There are two forms of these walls. The outline in Fig. 125 shown in solid lines is the simplest to construct and is the more economical of the two types of reinforced concrete walls, up to a height of 20 feet. For higher walls the form shown by the solid lines and heavy dotted line *bc* is used. Examples of both types will be worked out in detail.

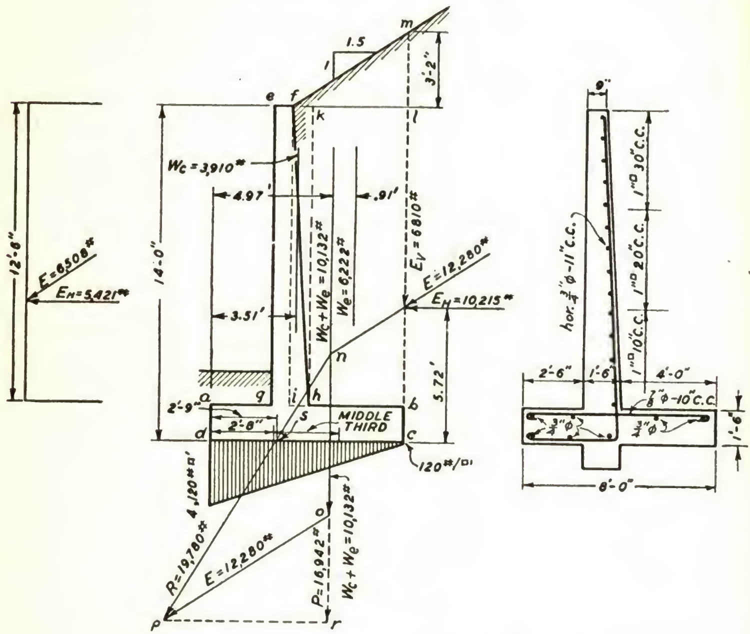


Fig. 126. Design Diagrams for Retaining Wall

*Illustrative Example.* Design a retaining wall 14′ high to support an earth face with a surcharge at a slope of 1½ to 1, with $f_s = 18{,}000\#/\square''$, $f_c = 800\#/\square''$ and $n = 15$.

*Solution.* The width of the base for reinforced concrete walls is usually made from 4/10 to 6/10 of the height. For this wall, with a surcharge the base will be made 8′, about 6/10 of the height, Fig. 126. Assume the weight of the earth at 100 pounds per cubic foot and the reinforced concrete at 150 pounds per cubic foot. Then substituting in Equation **(6C)**

$$E = .833\frac{Wh^2}{2}$$
$$= .833 \times \frac{100 \times (17.17)^2}{2}$$
$$= 12{,}280\# \text{ per linear foot of wall}$$

This force is applied on the plane *cm*, Fig. 126, parallel to the surface, at a point one-third of the height above the base, or 5.72′.

It will be necessary to determine the thickness of the vertical wall and the base plate before the stability of the wall can be determined. Assume the base plate to be 18″ thick; then the vertical slab will be 12′6″ high and the pressure against this slab will be

$$E = .833\frac{(100 \times 12.5^2)}{2} = 6{,}508\#$$

The horizontal component of this pressure is 6,508×cos 33°42′, or 5,421#, as shown diagrammatically in Fig. 126.

The bending moment at the bottom of the vertical wall will be $M = 5{,}421 \times \frac{12.5}{3} \times 12 = 271{,}050''\#$. Place this equal to $M = 138.6bd^2$ (Table XIX). With $b$ equal to 12, $138.6 \times 12d_2 = 271.050$, $d_2$ equals 163.1, and $d$ equals 12.8″. Adding 3.6″ for protecting this steel, the total thickness will be 16.4″. The thickness of the vertical wall at the bottom will be made 18″. With $d = 12.8''$ the area of reinforcing steel required would be .0089×12.8 or .114□″ of steel per inch of length of wall. With the thickness equal to 18″, $d$ equals 14.4″, and the area of steel required is $\frac{12.8}{14.4} \times .114 = .101$□″ per inch of length of wall. Bars 1 inch square spaced 10″ center to center will furnish this area.

The bending moment rapidly decreases from the bottom of the vertical wall upwards, and, therefore, it will not be necessary to keep the thickness of 18″ to the top of the wall or to have all the bars the full length. Make the top 9″ thick, drop off one-third of the bars at one-third of the height of the slab and one-third at two-thirds of the height. The total shear at the bottom of the wall is the horizontal component of the thrust, $E$, or 5,421#. Substituting in Equation (**22**), $v = \frac{8V}{7bd}$, $v = \frac{8 \times 5421}{7 \times 12 \times 14.4} = 35.8\#/$□″. As this does not exceed the working stress ($.02 \times 2{,}000 = 40$), stirrups are not required.

It is very important in a wall of this type not to exceed the bond stress. The bars must be well anchored in the base plate or they will be of no great value. The vertical bars are generally anchored as shown in Fig. 126.

To calculate the bond stress, take a section of wall one foot long. The bars are 1″□ spaced 10″ center to center, and will have a $\Sigma O$ value of $4.00\times\frac{12}{10}=4.80''$ per foot of wall. The unit shear has just been calculated at 35.8#/□″. Substitute these values in Equation **(23a)**

$$u=\frac{vb}{\Sigma O}$$

$$u=\frac{35.8\times 12}{4.8}$$

$$u=89.5\#/\square''$$

This value of *u* is within the limit allowed for bond.

In designing the footing of a reinforced concrete retaining wall the resultant force should intersect the base within the middle third, as in a masonry wall. The forces acting on the footing are the earth pressure on the plane *mc*, the weight of the earth fill, and the weight of the concrete. The distance from the toe *d* to the point where the resultant acts is obtained as follows: the centers of gravity of the concrete and the earth are found, also the weight of each. Each weight is multiplied by its distance from *d* to its center of gravity, which gives the static moment. The sum of the static moments divided by the sum of the weights equals the distance from the toe *d* to the line at which the combined weight of the fill and concrete acts. The detail figures for the problem follow.

**Center of Gravity of Wall**

| Section | Volume, Cu. Ft. | Moment Arm, Feet | Volume Moment |
|---|---|---|---|
| *abcd* | $1\times 8\times 1.5 \quad =12.00$ | $8\div 2=4$ | 48.00 |
| *efig* | $1\times 0.75\times 12.5= \ 9.38$ | $2.50+\left(\frac{0.75}{2}\right)=2.88$ | 27.01 |
| *fih* | $1\times\frac{0.75\times 12.5}{2}= \ 4.69$ | $2.50+0.75+\frac{0.75}{3}=3.50$ | 16.42 |
| | 26.07 | | 91.43 |

Distance from *d* to center of gravity is $\frac{91.43}{26.07}=3.51'$.

Weight per lineal foot is $26.07 \times 150 = 3910\# = W_c$

Static moment about $d = 3910 \times 3.51 = 13{,}724'\#$.

**Center of Gravity of Earth**

| Section | Volume, Cu. Ft. | Moment Arm, Feet | Volume Moment |
|---|---|---|---|
| *fkh* | $1 \times \frac{1}{2} \times 0.75 \times 12.5 = 4.69$ | $2.50 + 0.75 + \left(\frac{2 \times 0.75}{3}\right) = 3.75$ | 17.59 |
| *hblk* | $1 \times 4.0 \times 12.5 = 50.00$ | $2.50 + 1.50 + \frac{4.00}{2} = 6.00$ | 300.00 |
| *flm* | $1 \times \frac{1}{2} \times 4.75 \times 3.17 = 7.53$ | $2.50 + 0.75 + \left(\frac{2 \times 4.75}{3}\right) = 6.42$ | 48.34 |
| | 62.22 | | 365.93 |

Distance from $d$ to center of gravity of earth $= \frac{365.93}{62.22} = 5.88'$.

Weight of earth per lineal foot $= 62.22 \times 100 = 6{,}222\# = W_e$

Static moment about $d = 6222 \times 5.88 = 36{,}585'\#$.

The distance from $d$ to the combined center of gravity of the concrete and the earth fill is:

$$\frac{13{,}724 + 36{,}585}{3910 + 6222} = \frac{50{,}309}{10{,}132} = 4.97'$$

Where the resultant, $R$, cuts the base can be found graphically or analytically. To find it graphically, produce $E$ to meet the combined center of gravity of the concrete and the earth. From their intersection lay off on the vertical line, *nr*, at any convenient scale, *no*, the combined weight 10,132#. At the end of this distance, point *o*, draw a line *po* parallel to the line $E$ and lay off the value of $E$ which is 12,280#. Draw *pn*, which is $R$, the resultant, and in this case has a value of 19,780#. The resultant cuts the base just inside the middle third, so that the wall will not fail by overturning.

To determine analytically where the resultant, $R$, cuts the base, resolve the thrust $E$ into its vertical and horizontal components, $E_v$ and $E_h$, equal to 6810# and 10,215# respectively. Replace $E$ by these components at its point of application on the line *cm*.

Take moments about the toe, point $d$, and find the net moment acting on the wall. Divide this moment by $P$, the sum of all the vertical forces (i.e., the sum of the weight of wall and fill and the vertical component of $E$), and the result will be the distance *ds* which locates the point where the resultant cuts the base. The value of

$P$ is 16,942#. The distance *ds* is found to be 2′9″ by scaling, or 2.74′ by calculation. The edge of the middle third is 2′8″ from *d* so that the resultant is 1″ inside the middle third.

The pressure produced on the foundation is next to be investigated. Since the resultant comes inside of the middle third, Equations (**6d**) and (**6e**) are used.

$$\text{Pressure at the toe} = (4B-6Q)\frac{P}{B^2}$$

$$= [(4\times 8)-(6\times 2.74)]\frac{16{,}942}{(8)^2}$$

$$= 4{,}120\#/\square'$$

The pressure on the foundation of 4,120# at the toe is permissible on most soils.

$$\text{Pressure at the heel} = (6Q-2B)\frac{P}{B^2}$$

$$= [(6\times 2.74)-(2\times 8)]\frac{16{,}942}{(8)^2}$$

$$= 120\#/\square'$$

The stability of this wall must be fully investigated for sliding. Suppose this wall is to be located on a wet clay soil. The coefficient of friction between concrete and wet clay is .33; the horizontal force is 10,215#; and the total of the forces acting in a downward direction is 16,942#. With a coefficient of .33, or ⅓, the resistance to sliding is 16,942×⅓, or 5,647#, which is about half of the horizontal pressure 10,215#. The resistance should be about twice the pressure in order to make the wall safe against sliding, which would require that the weight should be about four times as much in order that mere friction should surely prevent sliding. This shows that it will be necessary to construct a projection in the base, as shown in Fig. 126, unless, as is usual, the base of the wall is sunk somewhat below the surface of the ground, as is indicated by the crosshatching above *a* and *g* in Fig. 126, and it is found that the soil at *ad* is considered to have enough firmness and lateral resistance to withstand the excess thrust.

The thickness of the base is always made greater than the moment requirements just behind the vertical wall (or at *h*) would demand. If the wall were actually on the point of tipping over, there would cease to be any upward pressure on the base. But there would be a downward pressure on the right cantilever equal to the weight

of the earth above it, and the moment in the base at the point $h$ would be that produced by that weight of earth and by the weight of the concrete from $h$ to $b$. Since the above calculations for the stability of the wall show that the computed lateral pressure cannot produce actual tipping about the toe, no such moment can actually be developed, but the calculation of the required thickness to resist such a moment gives a dimension which is certainly more than safe and which, for other reasons, is sometimes made still greater. The weight of the earth is 6,222# and the weight of the concrete is $4\times1\frac{1}{2}\times150=$ 900#. Then

$$M=[(6222\times1.88)+(900\times2)]\times12=161{,}968''\#$$

On the basis of 2000-lb. concrete, $M=138.6bd^2$. With $b=12''$, solving for $d$, $d^2=\dfrac{161{,}968}{138.6\times12}=97.38$ and $d=9.87''$. Adding 2.5″ for protecting the steel, the total thickness would be 12.37″. To properly anchor the bars of the vertical wall in the base plate, the base plate should be made at least as thick as the vertical wall just above the base. This has been made 18″ and $d=14.4''$. On this basis, the value of $b$ to develop the full stress in the steel is $b=161{,}968\div[138.6\times(14.4)^2]=5.64''$, and $.0089\times5.64\times14.4=.723^{\square''}$ is the area of steel actually needed for each foot of base along the length of the wall. Then $0.725''\div12=.0603^{\square''}$ per inch of width. Using ⅞″ round bars, the spacing would be $.60\div.0603=10''$, the same spacing as the vertical bars.

The left cantilever or toe has an upward pressure. At the extreme end it is 4,120# and at the face of the vertical wall it is $4120-2.5\left(\dfrac{4120-120}{8}\right)=2870\#/^{\square'}$. The average pressure is $(4120+2870)\div2=3{,}495\#/^{\square'}$. The moment at the face of the vertical wall is, therefore,

$$M=3495\times\frac{(2.5)^2}{2}\times12=131{,}060''\#$$

With $d=14.4$, $b=12$, and $M=131{,}060=Kbd^2$, we find $K=52.7$. Since this is so small compared with the resisting modulus, $K=138.6$, and since the bars in the vertical slab are bent around, as shown in Fig. 126, so as to resist all such stresses, no further consideration of this stress is necessary.

The total upward pressure on the toe is $3495 \times 2.5 = 8{,}738\#$. The shear on the vertical plane through the front face of the wall would be less than this on account of the weight of the concrete toe and some superimposed earth. Deducting the weight of the concrete $(1\frac{1}{2} \times 2\frac{1}{2} \times 150)$ or 563#, the total shear at the front face of the wall is $8738 - 563$ or 8175#. Substituting in Equation (22), $v = \frac{8V}{7bd}$, $v = \frac{8 \times 8175}{7 \times 12 \times 14.4} = 54.1\#/\square''$. In the design of a retaining wall this is permissible when the bars are anchored.

Some dimensions are necessarily assumed at the beginning, and these must be tested. For example, the thickness of the vertical slab at the bottom, and also the thickness of the base slab, was assumed to be 18″. The calculations showed these figures to be slightly excessive. If the difference were large, or if the wall were to be so long that a very slight saving in thickness was economically important, the problem should be recalculated with assumed figures which would prove more consistent. But the above figures, with slightly increased thickness of concrete, are on the side of greater safety and durability and a saving in reinforcing steel.

Some longitudinal bars must be placed in the wall to prevent temperature cracks, and also to tie the concrete together. It has been previously computed that an area of steel equal to about 0.3 per cent of the area of the concrete is adequate for this purpose. The average thickness of the wall is $\frac{1}{2}(9+18) = 13.5''$. Then $.003 \times 13.5 = .0405\square''$, the amount of temperature steel for each inch of height. Using $\frac{3}{4}''$ round bars ($A = .44$) the bars should be placed about 11″ apart.

**Reinforced Concrete Retaining Walls with Counterforts.** In this type of wall the vertical slab is held in place by the counterforts, the principal steel being horizontal. The counterforts act as cantilever beams, being held in place by the footing.

*Illustrative Example.* Design a reinforced-concrete wall with counterforts, the wall to be 20′ high, and the fill to be level with the top of the wall, $f_c = 800\#/\square''$, $f_s = 18{,}000\#/\square''$, $n = 15$, and $M = 138.6bd^2$.

*Solution.* The spacing of the counterforts is first determined. The economical spacing will vary from 8′ to 12′ or more, depending on the height of the wall. A spacing of 9′ on centers will be used for the counterforts in this case, Fig. 127. The maximum load on the vertical slab is on the bottom unit and decreases uniformly to

zero at the top, when the earth is horizontal with the top of the wall, as in this case. Assume that the base plate will be 18″ in thickness, then the center of the bottom foot of the vertical slab will be 18′ from the top of the wall. The pressure to be sustained by the lower foot of the slab will be

$$\begin{aligned} P &= \tfrac{1}{3}wh \\ &= \tfrac{1}{3}\times 100 \times 18 \\ &= 600\#/\square' \end{aligned}$$

This value of $P$ is really excessive, since it considers earth as a free-

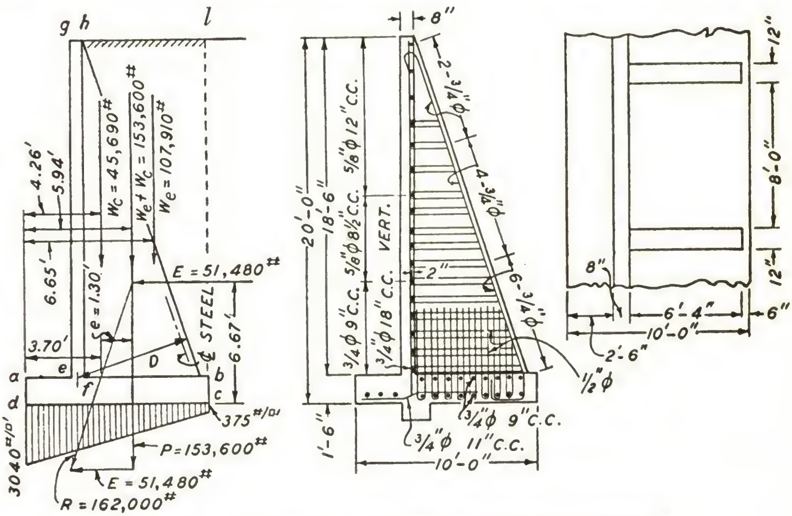



Fig. 127. Design Diagrams for Retaining Wall with Counterforts

motion liquid, having a unit weight of 100# per cubic foot. But on account of simplicity of calculation and because such a method of computation is always safe, it is used. Multiplying this value of $P$ by the distance between the centers of the counterforts—$600\times 9 = 5{,}400$—the full load is obtained.

$$M = \frac{5400\times 9\times 12}{12} = 48{,}600''\#$$

Place this value of $M$ equal to $138.6bd_2$ in which $b = 12$, and solve for $d$.

$$\begin{aligned} 138.6\times 12d^2 &= 48{,}600 \\ d^2 &= 29.2 \\ d &= 5.4 \end{aligned}$$

Adding 2.38 to this for protecting the steel, the total thickness of the wall will be 7.78, say 8″. For convenience of construction, the slab will be made uniform in thickness. The steel for the bottom foot will be $.0089 \times 5.4 \times 12 = .577\square''$. This will be furnished by ¾″ round, 9″ on centers. Use this size of bars and spacing for one-fourth the height of the wall. The next quarter will be reduced 25 per cent and $.433\square''$ will be required. Here use ⅝″ round, at 8½″ center to center. For the third quarter the required area will be half of that required in the first quarter, or $0.288\square''$ per foot, furnished by ⅝″ round, 12″ center to center. In the upper part of the wall use ⅝″$\phi$ bars 12″ on centers. This amount of steel is required for temperature stresses.

To determine the requirements of the counterforts it will be necessary to determine the horizontal pressure against the section of wall above the base 9′ long. Referring to Equation (**6a**)

$$E = .286\frac{Wh^2}{2}$$

Substituting in this equation and multiplying by 9, the distance between counterforts,

$$E = \frac{.286 \times 100 \times (18\frac{1}{2})^2}{2} \times 9$$
$$= 44{,}050\#$$

This load is applied at one-third of the height of the wall above the base, or 6⅙ feet above the base. The moment to be resisted by the counterfort is

$$M = 44{,}050 \times 6\tfrac{1}{6} \times 12$$
$$= 3{,}259{,}700''\#$$

The width of counterfort must be sufficient to insure rigidity, to resist any unequal pressures, and to thoroughly embed the reinforcing steel. The width is made by judgment and in this case will be made 12 inches wide. The counterfort and vertical slab together form a **T**-beam. It is assumed that the lever arm of the steel (the equivalent to $jd$ of **T**-beam formulas), is equal to the perpendicular distance from the center of the steel in the counterfort to the center of the front slab at the top of the base slab. This is shown on Fig. 127 and is marked $D$. By scaling, this is found to be 72″. Then

$$M = A_s \times jd \times 18{,}000$$
$$A_s = \frac{3{,}259{,}700}{18{,}000 \times 72} = 2.51\square''$$

Six $\frac{3}{4}''$ round bars will give this area. Two of these bars will extend to the top of the wall, two will be stopped at one-third and two at two-thirds the height.

Now that these dimensions have been determined, the wall will be investigated for stability against overturning. The earth pressure tending to overturn the wall is that due to a height of earth above point $c$; i.e., the full height of the wall, or 20 feet.

Substituting in Equation (6a)

$$E = .286 \times \frac{100 \times 20^2}{2}$$

$$= 5{,}720\# \text{ per foot of wall.}$$

For a section of wall 9′ long, $E = 5720 \times 9 = 51{,}480\#$.

To find the center of gravity of the wall, a section 9′ long, center to center of counterforts, will be taken.

**Center of Gravity of Concrete**
Moments taken about $a$

| Section | Volume, Cu. Ft. | Moment Arm | Volume Moment |
|---|---|---|---|
| *abcd* | 135.0 | 5.0 | 675.0 |
| *efhg* | 111.0 | 2.83 | 314.1 |
| *hfb* | 58.6 | 5.28 | 309.4 |
| | 304.6 | | 1298.5 |

Distance from $a$ to center of gravity, $\frac{1298.5}{304.6} = 4.26'$.

Weight of 9′ of wall $= 304.6 \times 150 = 45{,}690\#$.

Static moment about $a$ for section 9′ long $= 45{,}690 \times 4.26 = 194{,}640'\#$.

**Center of Gravity of Earth**
Moments taken about $a$

| Section | Volume, Cu. Ft. | Moment Arm | Volume Moment |
|---|---|---|---|
| *fblh* | 1011.3 | 6.58 | 6654.4 |
| *blh* | 67.8 | 7.72 | 523.4 |
| | 1079.1 | | 7177.8 |

Distance from $a$ to center of gravity $\frac{7177.8}{1079.1} = 6.65'$.

Weight of earth per 9 feet of wall, $1079.1 \times 100 = 107{,}910\#$.

Static moment about $a$ for section 9 feet long equals $107{,}910 \times 6.65 = 717{,}600'\#$.

Distance from $a$ to the resultant of the concrete and earth

$$\frac{194{,}640+717{,}600}{45{,}690+107{,}910}=\frac{912{,}240}{153{,}600}=5.94'$$

Draw the line $W_c+W_e$ at a distance 5.94 feet from $a$ and produce the line $E$ to meet it. From the intersection of these two lines lay off the sum of the weight of the concrete plus the weight of the earth at any convenient scale. At the end of this distance draw a line parallel to $E$ and lay off on it the value found for $E$. Draw the resultant $R$. The point of intersection of this line with the base falls within the middle third, and therefore the wall should be safe against overturning.

The position of the resultant can be located by taking moments about the point $d$, whence $x=\frac{(153{,}600\times5.94)-(51{,}480\times6.67)}{153{,}600}=\frac{569{,}010}{153{,}600}=3.70'$. The resultant cuts the middle third. Therefore, the wall should not fail by overturning.

If the distance from the center line of the base to the point of application of the resultant is denoted by $e$, the pressure at the toe is $\left(1+\frac{6e}{b}\right)$ times the average pressure on the base. This is derived by substituting $\left(\frac{B}{2}-e\right)$ for $Q$ in Equation (**6d**). The pressure at the toe is $\left(1+\frac{6(5.0-3.7)}{10}\right)\times\frac{153{,}600}{10\times9}$ or $3040\#/\square'$. Similarly, the pressure at the heel equals $\left(1-\frac{6e}{b}\right)$ times the average compression, equals $\frac{2.2}{10}\times\frac{153{,}600}{10\times9}=375\#/\square'$.

The maximum vertical upward pressure per lineal foot is $3040\#$. At a point under the front face of the wall it is $3040-\left[\frac{2.5}{10.0}(3040-375)\right]=2375\#/\square'$. The mean pressure is $\frac{1}{2}(3040+2375)=2710\#/\square'$. The total pressure per linear foot of wall equals $2710\times2.5'=6775\#$ and the moment at the face of the wall is $6775\#\times15''=101{,}625''\#$.

The total depth of the base slab has been taken as 18″. Then $d$ can be made at least 14″.

The steel required for the toe is found by the use of formula $A_s = \frac{M}{f_s \times d \times .87}$. Substituting the known quantities, $A_s = \frac{101,625}{18,000 \times 14 \times .87}$ and $A_s = 0.46□″$. Bars ¾″ in diameter spaced 11″ on center will satisfy this condition.

There is a pressure of 5400# on the lowest foot of depth of the vertical wall in each 9-foot panel. This tends to separate the vertical wall from the counterfort. A ½″ round bar, stressed to 16,000#/□″ will carry 3,140#. A loop, carried around the horizontal bars in the vertical wall, will carry 6,280#. A loop of ½″ round bar around each ¾″ round horizontal bar, spaced 9″ in the vertical wall, would be unnecessarily strong, but, as a similar loop of ⅜″ round bar would not be strong enough, the ½″ round bars must be used near the bottom. Since this pressure against the vertical wall decreases uniformly from base to top, some height will be reached where this loop can be made of ⅜″ round bars, and even of ¼″ bars. These bars are extended through the counterfort and hooked over the bars running up the rear slope of the counterfort.

In a similar manner the overturning wall pressure tends to tear the counterforts loose from the base plate. The bars in the back of the counterforts are designed for this stress, but their hold depends on their anchorage in the base plate. The stress in the six ¾″ round bars in the rear lower end of the counterforts is $18,000 \times 2.51 = 45,180\#$. This is the maximum stress between the counterfort and the base plate, and the stress decreases uniformly toward the wall end of the counterfort. Although these bars are bent twice and pass under the bars in the base plate, as shown in Fig. 127, there is a concentrated stress of 45,180# at the rear end. To relieve this concentration, vertical stirrups are placed in the counterforts, the stirrups being looped under the bars in the base plate and hooked at the upper end so that they are fully anchored. Since the division of stress between these stirrups and the six ¾″ round bars is uncertain, no precise rules for these stirrups can be made.

The portion of the base slab back of the vertical wall is designed as a continuous beam supported by the counterforts. It is figured as a beam carrying the load due to the weight of the earth and the weight

of the base slab, minus the upward soil pressure on the base slab. A section at the heel of the base slab is used for the design of the base, as the load is a maximum at this point.

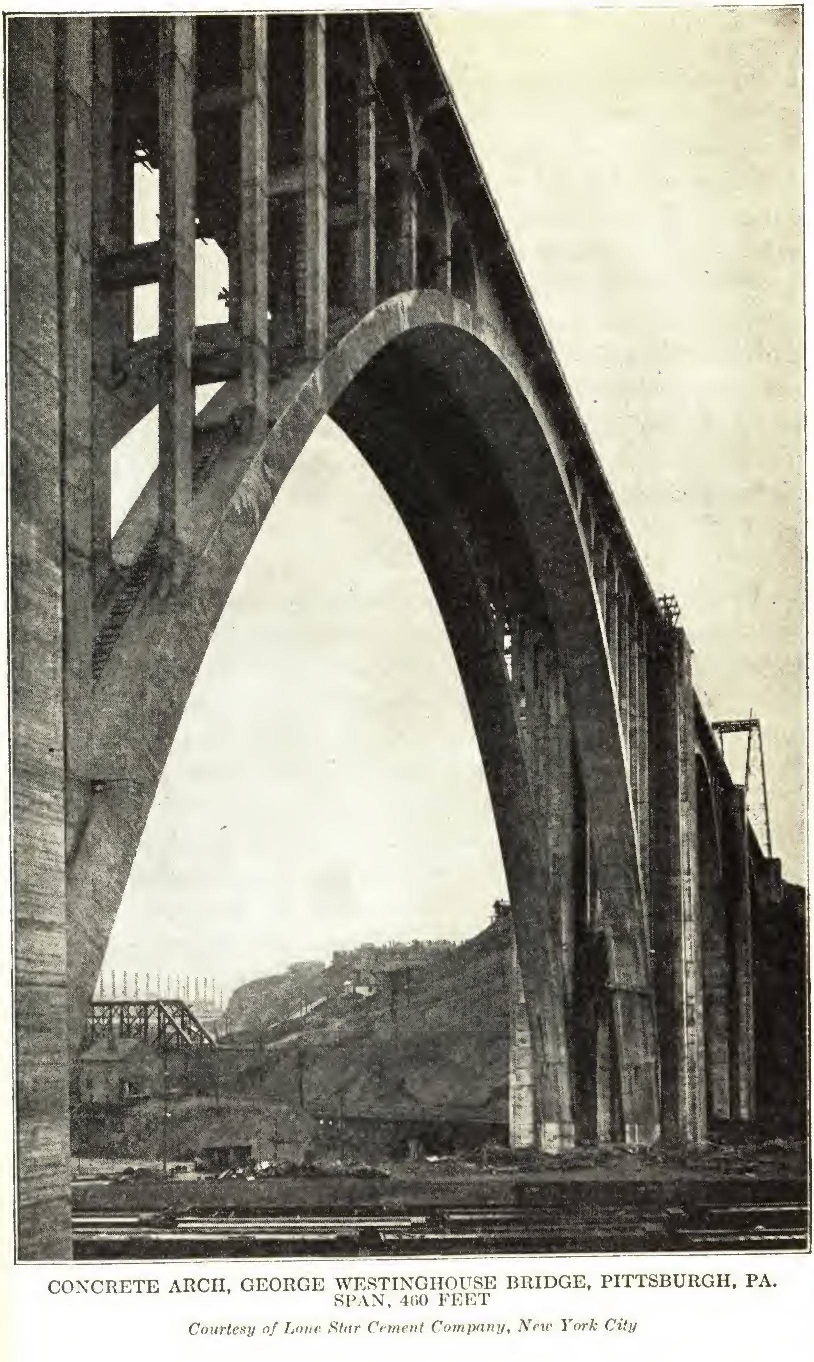
As in the design of continuous beams, half of these bars are bent up at the fifth point of the span and extend to the quarter point of the adjacent span, to take care of the negative moment at the counterforts.

Of course the usual proportion of temperature or transverse bars are used. As a general rule, transverse bars should be placed at proper intervals perpendicular to any plane which is not already crossed by some reinforcement which is required for moment or for shear.

**Coping and Anchorages.** Retaining walls generally have a coping at the top. This can be made to suit the conditions or the designer. When reinforced concrete walls are not stable against sliding, they can be anchored by making a projection of the bottom into the foundation. This is shown in Fig. 126.

## VERTICAL WALLS

**Curtain Walls.** Vertical walls which are not intended to carry any weight are sometimes made of reinforced concrete. They are then called curtain walls, and are designed merely to fill in the panels between the posts and girders which form the skeleton frame of the building. When these walls are interior walls, there is no definite stress which can be assigned to them, except by making assumptions that may be more or less unwarranted. When such walls are used for exterior walls of buildings, they must be designed to withstand wind pressure. This wind pressure will usually be exerted as a pressure from the outside, tending to force the wall inward; but if the wind is in the contrary direction, it may cause a lower atmospheric pressure on the outside, while the higher pressure of the air within the building will tend to force the wall outward. It is improbable, however, that such a pressure would ever be as great as that tending to force the wall inward. Such walls may be designed as slabs carrying a uniformly distributed load and supported on all four sides. If the panels are approximately square, they should have bars in both directions and should be designed by the same method as "slabs reinforced in both directions," as has previously been explained. If the vertical



CONCRETE ARCH, GEORGE WESTINGHOUSE BRIDGE, PITTSBURGH, PA.
SPAN, 460 FEET

*Courtesy of Lone Star Cement Company, New York City*

posts are much closer together than the height of the floor, as sometimes occurs, the principal reinforcing bars should be horizontal, and the walls should be designed as slabs having a span equal to the distance between the posts. Some small bars spaced about 1 foot apart should be placed vertically to prevent shrinkage. The pressure of the wind, corresponding to loading of the slab, is usually considered to be 30#/□′, although the actual wind pressure will very largely depend on local conditions, such as the protection which the building receives from surrounding buildings. A pressure of 30#/□′ is usually sufficient; and a slab designed on this basis will usually be so thin, perhaps only 4 inches, that it is not desirable to make it any thinner. Since designing such walls is such an obvious application of the equations and problems already solved in detail, no numerical illustration will be given here.

## TANKS

**Design.** The durability of reinforced-concrete tanks and their immunity from deterioration by rust, which so quickly destroys steel tanks, have resulted in the construction of a large and increasing number of tanks in reinforced concrete. Such tanks must be designed to withstand the bursting pressure of the water. If they are very high compared with their diameter, it is even possible that failure might result from excessive wind pressure. The method of designing one of these tanks may best be considered from an example.

*Illustrative Example.* Design a reinforced-concrete tank with an inside diameter of 18′ and with a capacity of 50,000 gallons. At 7.48 gallons per cubic foot, a capacity of 50,000 gallons will require 6,684 cubic feet. If the inside diameter of the tank is to be 18′, then the 18-foot circle will contain an area of 254.5□′. The depth of the water in the tank will, therefore, be 26.26′. The lowest foot of the tank will therefore be subjected to a bursting pressure due to 25.76 vertical feet of water. Since the water pressure per square foot increases 62½# for each foot of depth, we shall have a total pressure of 1,610#/□′ on the lowest foot of the tank. Since the diameter is 18′, the bursting pressure it must resist on each side is ½(18×1,610), or 14,490#. If we allow a working stress of 15,000#/□″, this will require .966□″ of metal in the lower foot. Since the bursting pressure is strictly proportional to the depth of the water, we need only divide this number

proportionally to the depth to obtain the bursting pressure at other depths. For example, the ring one foot high, at one-half the depth of the tank, should have .483□″ of metal; and that at one-third of the depth should have .322□″ of metal. The actual bars required for the lowest foot may be figured as follows: .966□″ per foot equals .0805□″ per inch; ¾″ round bars, having an area .4418□″, will furnish the required strength when spaced 5½″ apart. At half the height, the required metal per lineal inch of height is half of the above, or .040□″. This could be provided by using ¾″ round bars spaced 11″ apart; but this is not so good a distribution of metal as to use ⅝″ round bars having an area of .3068□″, and spaced 7½″ apart. It would give a still better distribution of metal to use ½″ bars spaced 6″ apart at this point, although the ½″ square bars are a little more expensive per pound, and, if they are spaced very closely, will add slightly to the cost of placing the steel. The size and spacing of bars for other points in the height can be similarly determined.

A circle 18′ in diameter has a circumference of somewhat over 56′. Assuming, as a preliminary figure, that the tank is to be 10″ thick at the bottom, the mean diameter of the base ring would be 18.83′, which would give a circumference of over 59′. Allowing a lap of 3′ on the bars, this would require that the bars should be about 62′ long. Although it is possible to have bars rolled of this length, they are very difficult to handle, and require to be transported on the railroads on two flat cars. It is therefore preferable to use bars of somewhat more than half this length, say 32′6″, and to make two joints in each band.

The bands which are used for ordinary wooden tanks are usually fastened at the ends by screw bolts. Some form of joint which is as strong as the bar should be used. According to Table XXIII, if deformed bars are used with 2000-lb. concrete, the bond adhesion per inch of length is as given in the second column in the following tabular form; the actual stress in the bars when the unit stress is 15,000#/□″ is as shown in the third column, and the length of bar re-

| Size of Bar | Bond Adhesion | Stress | Length, Inches |
|---|---|---|---|
| ¾″ round | 236 | 6,627 | 28 |
| ⅝″ round | 196 | 4,602 | 23½ |
| ½″ square | 200 | 3,750 | 18¾ |

quired to develop this stress by surface adhesion is as shown in the last column.

Then by lapping the bars by *at least* the amounts shown in the last column, and by staggering the lap joints so that no two joints are contiguous, the bars will be anchored. But an actual mechanical joint, as strong as the bar itself, is preferable.

**Test of Overturning.** Since the computed depth of the water is over 26′, we must calculate that the tank will be, say 28′ high. Its outer diameter will be approximately 20′. The total area exposed to the surface of the wind will be 560□′. We may assume that the wind has an average pressure of 50#/□′; but, owing to the circular form of the tank, we shall assume that its effective pressure is only one-half of this; and therefore, we may figure that the total overturning pressure of the wind equals 560×25, or 14,000#. If this is considered to be applied at a point 14′ above the ground, we have an overturning moment of 196,000 foot-pounds, or 2,352,000 inch-pounds.

It is not strictly accurate to consider the moment of inertia of this circular section of the tank as it would be done if it were a strictly homogeneous material, since the neutral axis, instead of being at the center of the section will be nearer to the compression side of the section. The simplest method of making such a calculation is to assume that the simple theory applies, and then to use a generous factor of safety. The effect of shifting the neutral axis from the center toward the compression side will be to increase the unit compression on the concrete and reduce the unit tension in the steel; but, as will be seen, it is generally necessary to make the concrete so thick that its unit compressive stress is at a very safe figure, while the reduction of the unit tension in the steel is merely on the side of safety.

Applying the usual theory, the moment of inertia of a ring section is .049 $(d_1^4-d^4)$. Assume as a preliminary figure, that the wall of the tank is 10″ thick at the bottom. Its outside diameter is, therefore, 18′(=216″)+(2×10″), or 236″. The moment of inertia $I$ equals .049 $[(236)^4-(216)^4]$=45,337,842 biquadratic inches. Calling $c$ the unit compression the ultimate moment due to wind pressure is

$$M=\frac{f_c I}{c}=\frac{f_c\times 45{,}337{,}842}{\frac{1}{2}d_1}=2{,}352{,}000''\#$$

in which $\frac{1}{2}d_1=\frac{1}{2}(236'')=118''$.

Solving the equation for $f_c$, $f_c$ equals a fraction more than 6 pounds per square inch. This pressure is so utterly insignificant, that, even if doubled or trebled to allow for the shifting of the neutral axis from the center, and if the allowance made for wind pressure also is doubled or trebled, (although the pressure chosen is usually considered ample) there is still practically no danger that the tank will fail owing to a crushing of the concrete due to wind pressure.

The above method of computation has its value in estimating the amount of steel required for vertical reinforcement. On the basis of 6#/□″, a sector with an average width of 1″ and a diametral thickness of 10″ would sustain a compression of about 60#.

Since working stresses have been figured, assume a working tension of, say 16,000#/□″ in the steel. This tension would therefore require $\frac{60}{16{,}000}$, or .0037□″ of metal per inch of width. Even if ¼″ bars were used for the vertical reinforcement, they would need to be spaced only about 17″ apart. This, however, is on the basis that the neutral axis is at the center of the section, which is known to be inaccurate.

A theoretical demonstration of the position of the neutral axis for such a section is so exceedingly complicated that it will not be considered here. The theoretical amount of steel required is always less than that computed by the above approximate method; but the necessity for preventing cracks, which would cause leakage, would demand more vertical reinforcement than would be required by wind pressure alone.

**Practical Details of the Design.** It was assumed as an approximate figure, that the thickness of the concrete side wall at the base of the tank should be 10″. The calculations have shown that, so far as wind pressure is concerned, such a thickness is very much greater than is required for this purpose; but it will not do to reduce the thickness in accordance with the apparent requirements for wind pressure. Although the thickness at the bottom might be reduced below 10″, it probably would not be wise to make such reduction. It may, however, be tapered slightly towards the top, so that at the top the thickness will not be greater than 6″, or perhaps even 5″. The vertical bars in the lower part of the side wall must be bent so as to run into the base slab of the tank. This will bind the side wall to the bottom.

The necessity for reinforcement in the bottom of the tank depends very largely upon the nature of the foundation, and also, to some extent, on the necessity for providing against temperature cracks, as has been discussed in preceding pages. Even if the tank is placed on a firm and absolutely unyielding foundation, some reinforcement should be used in the bottom in order to prevent cracks which might produce leakage. These bars should run from a point near the center and be bent upward at least 2 or 3 feet into the vertical wall. Sometimes a gridiron of bars running in both directions is used for this purpose. This method is really preferable to the radial method.

One vital necessity in the construction of tanks is that the concrete shall be made as dense and impervious as it is possible to make it, both to avoid any possible cracks which might develop into appreciable leakage, and also to avoid all possible deterioration of the steel by rust wherever cracks had formed. A low water-cement ratio, as low as 6 gallons of water per sack of cement, which means a volume ratio of $\frac{6\times 231}{1{,}728}=80$ per cent, is justified by the results.

# CHAPTER XVI

## SCHOOL BUILDINGS

**Reinforced Concrete Frames.** Reinforced concrete is used extensively in the construction of school buildings. There are several types of construction in use and the type selected is dependent usually on the amount of money to be spent for the building. If beams are permitted to project below the ceilings, a cheaper type of construction can be secured, but if it is required that the rooms have level ceilings, without the projection of beams in class or other large rooms, the construction work becomes more expensive. Three types of construction will be outlined. In these examples, the live load for classrooms will be considered as 50 pounds per square foot; for auditoriums, corridors and stairways, 100 pounds per square foot.

In all three problems the following unit stresses will be allowed: tension in steel, 20,000 pounds; direct compression in concrete, 500 pounds; extreme fiber stresses in compression, 800 pounds per square inch; and a maximum shear of 180 pounds per square inch.

**Plan 1.** In this plan, it is assumed that the beams are exposed in the ceilings and that the exposed concrete will have special forms and will be finished smooth for painting. The floor construction throughout will be supported by reinforced concrete columns as shown in Fig. 129. The exterior columns and spandrel beams will be of finished concrete, the floor covering will be linoleum glued to the top of the structural slabs. The classrooms are to be 23′ by 30′.

Assuming a live load of 50#, a finish of 5# (linoleum) and a slab 4″ in thickness which would weigh 50#, there will be a total live and dead load of 105#/□′ for the slabs. On referring to Table XX (division 2), it is observed that a 4″ slab is stronger than necessary for a span of 10′ for a live load of 50# plus 5# for linoleum. Since floor slabs for such spans are not made less than 4″ in thickness, this thickness of the slab will be maintained and the amount of reinforcing steel reduced.

$$\text{Load per foot of slab} = 1' \times 10' \times 105\# = 1050\#$$

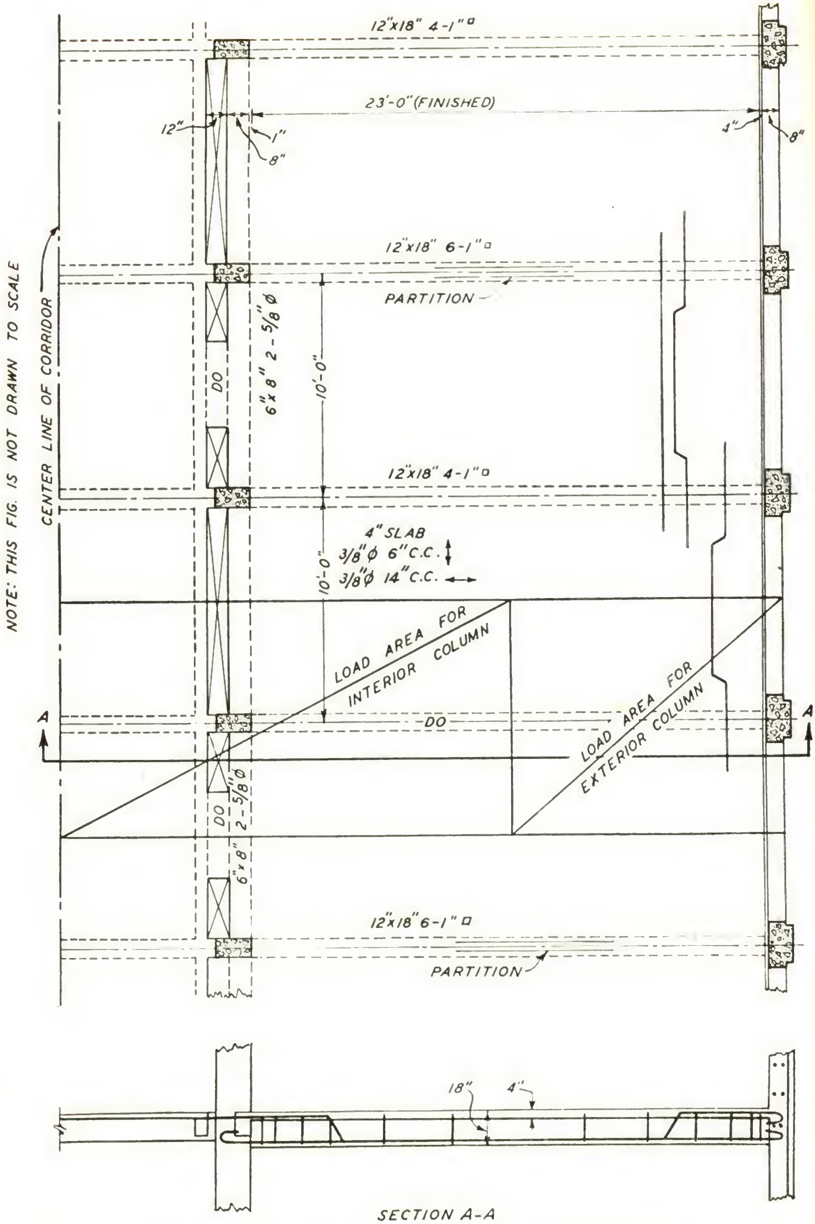



Fig. 129. Framing Plan for School Building—Beams and Slabs

$$M=\frac{1050\times10\times12}{12}=10{,}500''\#$$

$$A_s=\frac{10500}{20{,}000\times2.75\times.87}=.22\square''$$

$\frac{3}{8}''$ $\phi$ bars spaced 6″ on centers are required.

Half of the bars will be straight and half turned up at the fifth point and extended to the quarter point of the adjoining bay. Temperature bars $\frac{3}{8}''$ $\phi$ spaced 14″ on centers should be used.

The beams between the columns, allowing for bearing on the columns, will have a span of 24′. Since the beams are placed 10′ on centers and the dead load per square foot for the beam will be about 20#/□′ which, added to the slab load of 105#, equals 125#/□′ for the beam, then the load will be

$$10'\times24'\times125\#=30{,}000\#$$

$$M=\frac{30{,}000\times24\times12}{8}=1{,}080{,}000''\#$$

Assuming a total depth of 18″, and that probably only one row of bars is required, the center of the steel from the top of the beam is $18-2\frac{1}{2}=15\frac{1}{2}''$. Then the area of steel required will be

$$A_s=\frac{1{,}080{,}000}{20{,}000\times15\frac{1}{2}\times.87}=4.0\square''$$

Four 1″ square bars will be required, two of them to be turned up at the one-fifth point and extended to the quarter point of the adjacent span. For the exterior columns, the ends of the bars in the beams are hooked.

The shear in these beams is small and not many stirrups are required.

$$v=\frac{15{,}000}{12\times15\frac{1}{2}\times.87}=92\#/\square''$$

At the ends of the room the beams must support a partition, and it will be necessary to design the beams for this additional load. More steel for the additional tension and closer stirrup spacing will be required. Small beams will be needed between the interior columns as shown in Fig. 130 and a spandrel beam also will be required between the exterior columns in order to support the walls.

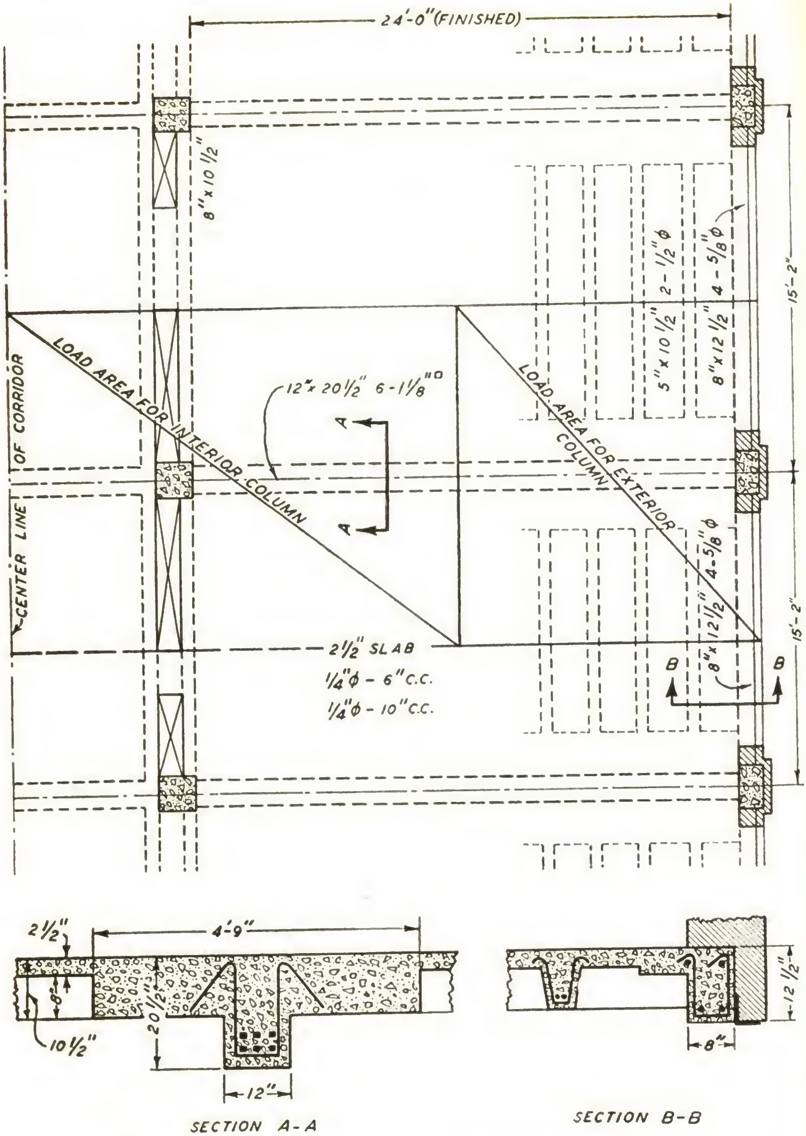



Fig. 130. Framing Plan for School Building—Beams and Joists

**Plan 2.** In this plan a complete frame of reinforced concrete will be considered. The beams are spaced 15′2″ and concrete joists and steel tile are used between the beams. Also, a finished floor with

a 2″ fill will be required. Allowing a live load of 50#, floor finish (2″ cinder concrete = 16# + 4# wood flooring) 20#, slab (2½″ thick) 30#, joist 22#, plaster 10#, there is a total load of 132#/□′. The reinforced concrete joists will be 5″ in width, and the steel tile, 20″. Therefore, the joists will be 25″ on centers. The load on each joist will be as follows:

$$2\tfrac{1}{12}' \times 15\tfrac{1}{6}' \times 132\# = 4172\#$$

$$M = \frac{4172 \times 15\tfrac{1}{6} \times 12}{12} = 63{,}275''\#$$

Assuming a depth of 10½″ and allowing 1½″ from the center of the steel to the bottom of the joist, the steel area will be

$$A_s = \frac{63{,}275}{20{,}000 \times 9 \times .87} = .4\square''$$

The area of two ½″ round bars is .39□″ and will be satisfactory for this load. The shear is very small in the joists, being only $\frac{2086}{5 \times 9 \times .87} =$ 53#/□″. A few stirrups should be used.

In designing the beam, the same load is required as for the slab plus the weight of the beam, and the compression area required must be considered (see Fig. 130, Section A-A) which amounts to about 20#/□′. Therefore, the total load for the beams will be 152#/□′ and the load on the beam will be:

$$15\tfrac{1}{6}' \times 25' \times 152\# = 57{,}633\#$$

The bending moment for this beam is

$$M = \frac{57{,}633 \times 25 \times 12}{8} = 2{,}161{,}238''\#$$

Assuming a total depth of 20½″ and allowing 3½″ to the center of the steel from the bottom (20½″ − 3½″ = 17″) the area of steel equals:

$$A_s = \frac{2{,}161{,}238}{20{,}000 \times 17 \times .87} = 7.3\square''$$

Six 1⅛″□ bars will satisfy this steel area and the width of the beam will be 12″. The shear is 57,633 ÷ 2 = 28,817#.

$$v = \frac{28{,}817}{12 \times 17 \times .87} = 162\#/\square'$$

which is within the limit allowed, but many stirrups will be required. For the compression side of the beam, the section shown in Fig. 130 will be required. That is, the forms for the joists must be stopped off at such a distance from the beam as to provide the compression area required. When compression for a beam must be provided for in this type of construction, the compression slab usually is made the total depth of the joists (unless the joists are very deep) in order to simplify construction. When joists over 8″ or 10″ are used, usually the thickness and width of the slab necessary for compression are figured, and steel tiles of less depth are used under the slab.

Since the total depth of the beam is 20½″, Fig. 130, and the compression slab is 10½″, the neutral axis is in the slab and the width of the flange may be figured as a plain beam.

In Equation (B), page 176

$$M_c = 131bd^2$$

Substituting $$2{,}161{,}238 = 131 \times b \times 17^2$$

$$b = 57''$$

which is the width of the flange.

The beams at the end of the 30′ room must be designed for the weight of the partition in addition to the floor load, and a deeper beam probably will be required; or, if it is desirable to have all beams the same depth, then the beam under the partition should be designed and the others made the same size with the proper amount of steel.

**Plan 3.** In this plan, Fig. 131, classrooms will be designed with a flat ceiling. The rooms will have finished dimensions of 23′ by 30′. Shallow beams will be used in the corridors, as these ceilings are usually furred down for the sake of appearance. Also, space between the bottom of the floor construction and a suspended ceiling is often needed for air ducts. A flat ceiling can be constructed for the corridor floors, but if the beams connecting the interior columns are to be supported directly on the columns, it will be necessary to make the columns wide enough to catch these beams. The beams are sometimes supported on brackets, but this economy is not appreciable and the brackets will cost nearly as much as the increased size of the column. If the brackets are used, the columns must be designed for the eccentric load.

The joists will be 10″ deep under the slab and will be connected by a 3″ slab, making a total depth of 13″. The joists will be made

5½″ wide. The joists supporting the partitions at each end of the room must be made much wider and have a thicker flange to provide

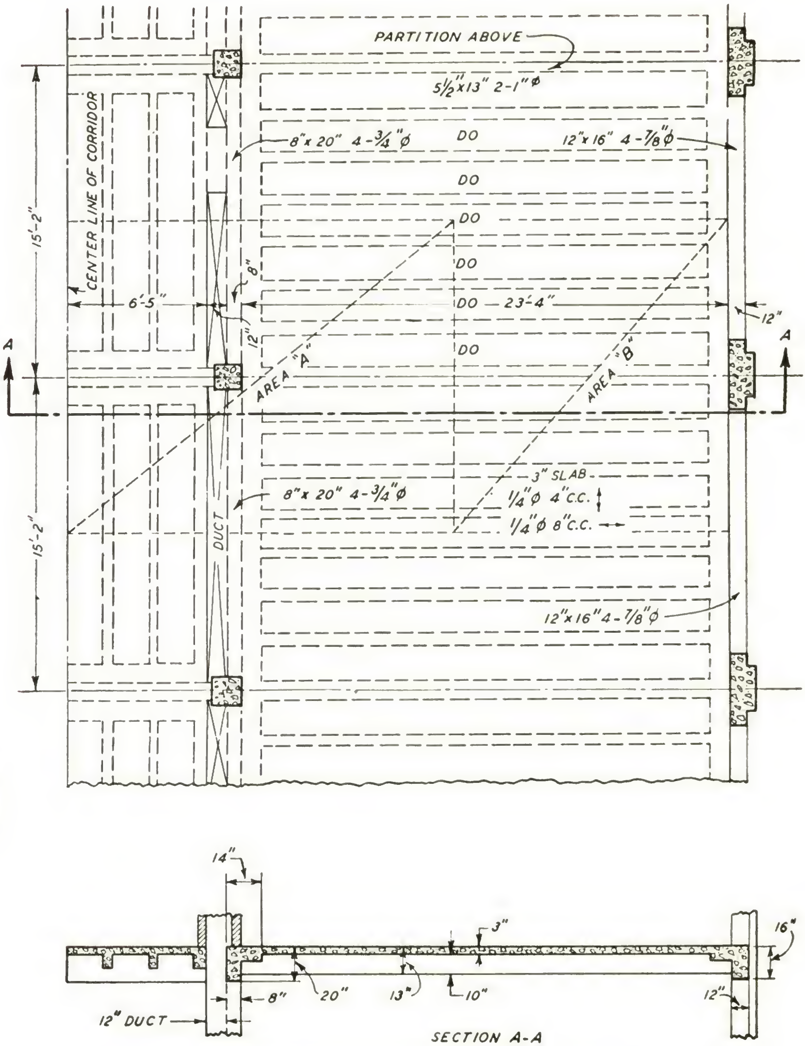



Fig. 131. Framing Plan for School Building—Beams, Joists, and Steel Tile

for the additional compression. A calculation of the typical joists will be the same as shown for Plan 2, but with an increased span and

dead load. Also, they must be designed for a simple span, that is, $\frac{Wl}{8}$.

The beam between the interior columns, which supports the ends of the joists, will have a live and dead load of 32,000#. The calculation then will be as follows:

$$M=\frac{32{,}000\times 15\tfrac{1}{6}\times 12}{12}=485{,}333''\#$$

Assuming a total depth of 20″ and allowing 3″ from the bottom of the beam to the center of the steel, the area of the steel required is

$$A_s=\frac{485{,}333}{20{,}000\times 17\times .87}=1.64\square''$$

Four ¾″ round bars will be required. The end span, on account of being semicontinuous, will require additional steel.

The width of the beam will be 8″. The flange for this beam must be determined, as the adjoining slab is only 3″ in thickness. Therefore, assuming the slab is 6″ thick,

$$M_c=485{,}333''\#=\frac{800}{2}\times 6''\times b\times 16''\times .87$$

$$b=14.5''$$

Then the beam will be L-shaped with a depth of 20″ and the width of the flange will be 14.5″.

A spandrel beam between the exterior columns will be designed in the same way. In designing these spandrel beams, if sufficient depth cannot be secured below the bottom of the joists, a rectangular beam which projects above the floor must be used.

The design of the columns for such a building is a simple problem and will be limited in this text to one interior column. The clear room height for public school buildings is about 12′, except the basement, which is usually a foot or so less in height. Generally a space of 4′ or more is required between the ceiling and the top of the structural slab of the roof. The ceiling is suspended from the roof structure. The height of the columns for the upper floor will be 12′ plus 4′ = 16′, less the thickness of the construction for the roof and the ceiling. Live load for roof, 35#; roofing, 4#; slab, 36#; joist, 22#; beams, 15#; suspended ceiling, 18#; total load, 130#/□′. Floor areas supported by interior and exterior columns are noted on Figs. 129–131.

**Interior Column.** Each interior column will support an area (marked *area A*, Fig. 131) extending from the center line of the corridor to the center line of the rooms, 19¾′ and from center to center of columns, 15⅙′.

The interior columns in the second floor will support a roof load of 15⅙′×19¾′×130#=38,940#.

Assume that a column is 12″×12″ and reinforced with 4 bars ¾″ in diameter, and investigate its strength by substituting in Equation (**60**).

$$P=0.225f_c'\ Ag[1+(n-1)p]$$

Substituting

$$=0.225\times2000\times144[1+(15-1).012]$$
$$=75{,}816\#$$

A column 12″×12″ reinforced with 4 bars ¾″ $\phi$ will support a load of 75,816# provided that the height of the column is not greater than 10 times the least side of the column. Since the column is 15′ in height, this condition is not satisfactory. It will be necessary to substitute in the equation for long columns to ascertain if this column is satisfactory to support the load. Using Equation (**64**)

$$P'=P\left(1.3-.03\frac{h}{d}\right)$$
$$=75{,}816\left(1.3-.03\frac{15'\times12''}{12}\right)$$
$$=64{,}444\#$$

Since the load on the column is 38,940# and the value of $P'$ is 64,444#, the 12″×12″ column reinforced with four ¾″ $\phi$ bars is stronger than necessary, but reinforced concrete columns are not made smaller than 12″×12″.

The interior columns in the first floor must support the second floor and the roof. The live load for corridor floor, 100#; classroom floor, 50#; floor finish, 25#; 3″ slab, 36#; joist, 22#; beam, 15#; plaster, 10#. The corridor floor load will extend 15′2″, and from center line of corridor to the beam supporting the joist for the rooms, 7′5″. In addition to the above load there will be a 3″ (terra cotta, cinder block, or gypsum) partition, plastered on one side, on both sides of the openings for ducts. The partition between the rooms will be constructed of blocks 4″ or more in thickness and plastered on both sides.

The load for the columns in the first floor will be as follows:

| | |
|---|---|
| Roof | 38,940# |
| Corridor floor, $7\frac{5}{12}' \times 15\frac{1}{6}' \times 208\#$ | 23,397 |
| Room floor, $11\frac{2}{3}' \times 15\frac{1}{6}' \times 158\#$ | 27,957 |
| Corridor partitions, $2 \times 15\frac{1}{6}' \times 12' \times 25\#$ | 9,100 |
| Room partitions, $1 \times \frac{23\frac{1}{3}'}{2} \times 12' \times 35\#$ | 4,900 |
| Second-floor column, $12'' \times 12'' \times 15' = 150\# \times 15'$ | 2,250 |
| Total load on first-floor columns | 106,544# |

The supporting capacity of a column $14'' \times 14''$ reinforced with four $1''$ $\phi$ bars will be investigated. Substituting in Equation (**60**)

$$
\begin{aligned}
P &= 0.225 f_c' Ag[1+(n-1)p] \\
&= 0.225 \times 2000 \times 196[1+(15-1).015] \\
&= 107{,}740\#
\end{aligned}
$$

This column is satisfactory to support the load.

The load to be supported by the basement columns will be found by adding the load of the first floor, second floor and roof. In this case, the weight of the first floor is the same as that of the second floor.

| | |
|---|---|
| Roof | 38,940# |
| Second floor (corridor, room, partitions) | 65,354 |
| First floor (corridor, room, partitions) | 65,354 |
| Second-floor column | 2,250 |
| First-floor column, $14'' \times 14'' \times 14' = 204\# \times 14' =$ | 2,856 |
| Total load on basement columns | 174,754# |

Assume the dimensions of a column to be $18''$ by $18''$, reinforced with four $1\frac{1}{8}''\square$ bars and substitute in Equation (**60**).

$$
\begin{aligned}
P &= 0.225 \times 2000 \times (18)^2[1+(15-1).015] \\
&= 176{,}418\#
\end{aligned}
$$

This column is satisfactory. If found desirable, the shape of the column can be changed. For example, it could be $14'' \times 23''$, but in that case six $1''$ $\phi$ bars with double ties would supply a better distribution of steel. To make up for the small deficiency of steel, the concrete should be made $14'' \times 24''$.

**Wall Column.** The floor area for the wall columns is indicated in Fig. 131 as *area B*. To the floor load must be added the weight of the wall and column.

**Footing.** The load for the footing will be the same as the basement column load plus the weight of basement column and the foot-

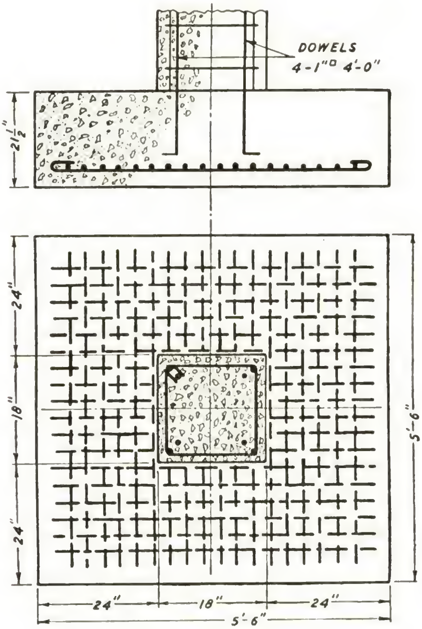



Fig. 132. Column Footing

ing. The weight of a column 18″×18″×12′ long is $\left(\frac{18'' \times 18''}{144} \times 12' \times 150\#\right) = 4050\#$. Assume the weight of the footing is 8000#. The total load on the footing 174,754+4050+8000=186,804#. Allowing a bearing on the soil of 6000#/□′, the area of the footing would be 186,804#÷6000#=31.12□′. Extracting the square root of 31.12, the dimensions of the footing are found to be 5.57′×5.57′, say 5′6″×5′6″, shown in Fig. 132. Assuming the depth of 15″ from the top of the footing to the center of the steel, the critical section for shear is 15″ from the face of the column. The area producing shear is $(66'')^2 - [18+(2\times15)]^2 = 4356 - 2304 = 2052$□″ or 14.25□′. The total shear on

the critical section is $14.25 \times 6000 = 85{,}500\#$. Substituting in Equation (22)

$$v = \frac{8V}{7bd}$$

$$= \frac{8 \times 85{,}500\#}{7 \times 4 \times 48 \times 15}$$

$$= 34\#/\square''$$

This stress of $34\#/\square''$ is within the allowable limit of $60\#/\square''$.

Knowing the depth $d$ (15″) the steel required in the bottom of the footing may be found by finding the bending moment and substituting in Equation (20): the area of footing to be considered is $24'' \times 5'6''$ and the moment is taken at the edge of the column. Reducing these dimensions to feet and substituting in Equation (66)

$$M = \frac{wb \times c^2}{2} = \frac{6000 \times 5.5 \times (2)^2}{2}$$

$$= 66{,}000'\# \text{ or } 792{,}000''\#$$

$$A_s = \frac{M}{f_s \times jd}$$

$$= \frac{792{,}000}{20{,}000 \times 15 \times .875}$$

$$= 3.02\square''$$

Sixteen ½″ $\phi$ bars are satisfactory, placed in two directions, or a total of thirty-two ½″ $\phi$ bars.

Investigate the bond stress. The shear at the face of the column is $2 \times 5.5 \times 6000 = 66{,}000\#$. Sixteen ½″ $\phi$ bars have an $\Sigma O$ value of $25.12\square''$. Substituting in Equation (23)

$$u = \frac{V}{\Sigma ojd} = \frac{66{,}000}{25.12 \times .875 \times 15}$$

$$= 202\#/\square''.$$

As this is more than the allowable, it will be necessary to increase the depth of the footing or increase the number of bars or perhaps both. Increase the depth to the center of the steel in the footing to 18″ and use eighteen ½″ $\phi$ bars. The $\Sigma O$ value for eighteen ½″ $\phi$ bars is 28.26″ and $u = \dfrac{66{,}000}{28.26 \times .875 \times 18} = 148\#/\square''$ which is satisfactory.

The footing will be $18'' + 3\frac{1}{2} = 21\frac{1}{2}''$ deep, 5′6″ square, and is reinforced with thirty-six ½″ $\phi$ bars.

## DESIGN OF A FACTORY BUILDING

Fig. 133 shows the structural plan of a typical floor of a six-story factory or warehouse. The floors are to be designed for a live load of 200 pounds per square foot. The floor finish is to be four inches thick, consisting of three inches of cinder concrete fill and one-inch maple flooring.

The following stresses are to be used in making the design except where otherwise noted:

| | |
|---|---|
| Concrete, ultimate strength at 28 days compression | 2000#/□″ |
| Extreme fiber stress in concrete in compression | 40% of the ultimate strength of the concrete |
| Ratio of moduli of elasticity | 15 |
| Extreme fiber stress in compression adjacent to supports of continuous beams | 45% of the ultimate strength |
| Shear in beams with web reinforcement and special anchorage of longitudinal bars | 9% of the ultimate strength |
| Shear carried by concrete, in beams with web reinforcement, and longitudinal bars having special anchorage | 3% of the ultimate strength of the concrete |
| Tensile stress in steel | 20,000#/□″ |

**Slab.** The live load is 200#/□′ and the finish will weigh 25#/□′. A cinder concrete slab one inch thick will weigh about 7#/□′, therefore, $3\times7+4$ (maple flooring) $=25$#.

Floor slabs are never made less than 4″ thick; and since this slab has a short span and is continuous, we will assume that it will be 4″ thick.

The total weight will be

$$200+25+50=275\#/\square'$$

The weight on a strip one foot wide will be

$$1'\times6'\times275\#=1650\#$$

$$M=\frac{1650\times6\times12}{12}=9900''\#$$

In Table XIX we find $K=\dfrac{M}{bd^2}$

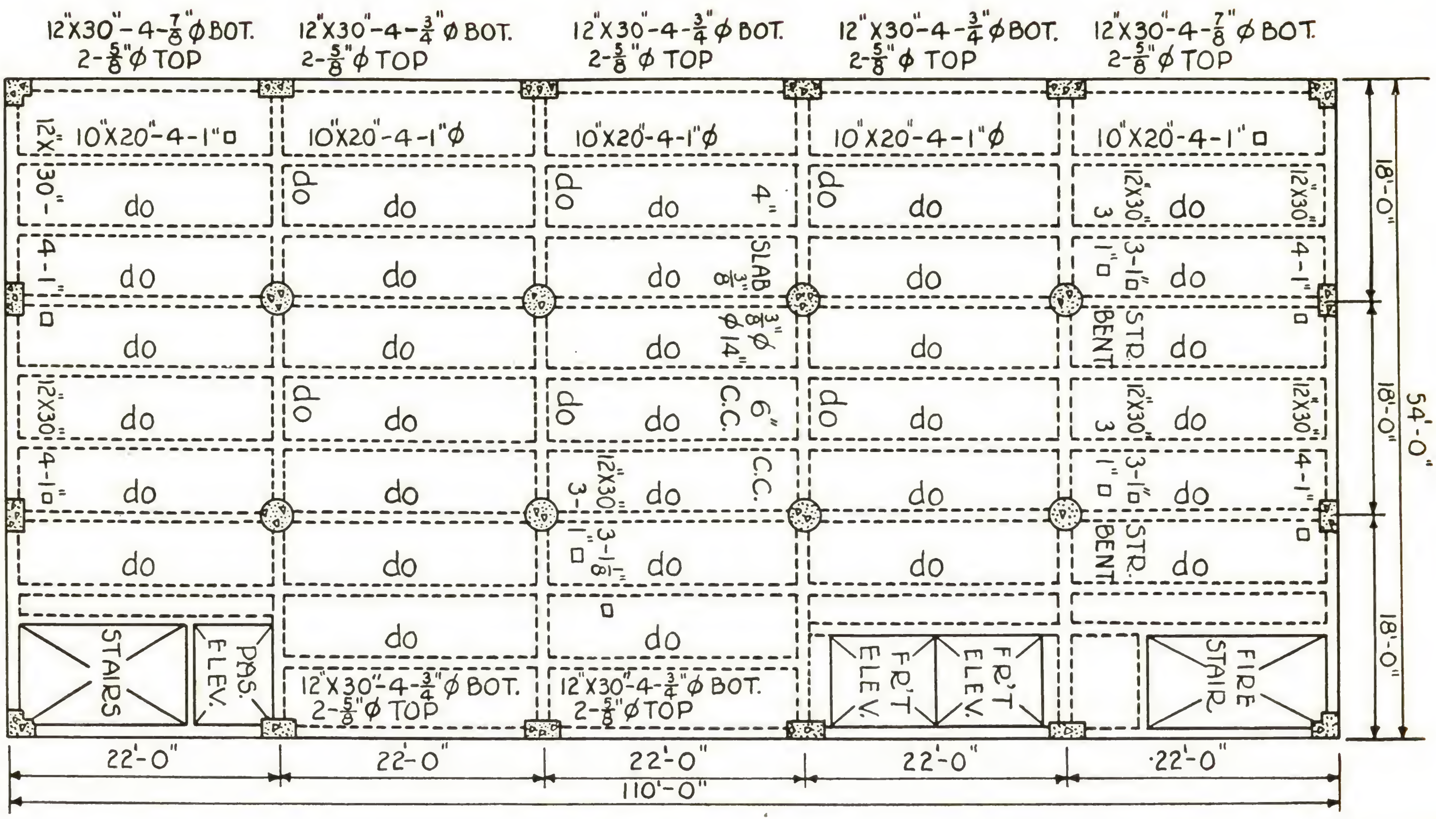



Fig. 133. Structural Floor Plan

Transposing, $M = Kbd^2$

$b = 12''$ (one foot in width)

$K = 131$

Substituting in the equation and solving for $d$ we have

$$M = Kbd^2$$
$$9900 = 131 \times 12 \times d^2$$
$$d^2 = 6.30$$
$$d = 2.51''$$

To this depth of 2.5″ must be added one-half the thickness of the steel bars plus one inch for fireproofing. The sum of these figures is less than 4″; but, as previously stated, the slab must be 4″ in thickness. The area of steel is found as follows:

$$A_s = \frac{M}{20{,}000 \times jd}$$
$$= \frac{9900}{20{,}000 \times 2.75 \times .87}$$
$$= .21^{\square''} \text{ or } \tfrac{3}{8}'' \text{ round bars, } 6'' \text{ center to center}$$

For temperature stresses ⅜″ round bars spaced 14 inches on centers will be satisfactory.

**Floor Beams.** The typical interior beams are spaced 6′ on centers and they have a span of 22′. Each beam would have a load of 200# (live load) plus 25# for floor finish, plus 48# (say 50) for floor slab ($12 \times 4 = 48$), plus the weight of the beam, which would be approximately 30#/□′ of floor area. Then the load on each beam would be:

$$6' \times 22' \times (200 + 25 + 50 + 30) = 40{,}260\#$$

$$M = \frac{Wl}{12} = \frac{40{,}260 \times 22 \times 12}{12} = 885{,}720''\#$$

Assume that the beam is 10″ wide and has a depth of 20″, which is less than one inch in depth for each foot of span. Then $d = 20'' - 3\tfrac{1}{2}'' = 16\tfrac{1}{2}''$ (center of steel to bottom of beam) and

$$jd = .87 \times (20 - 3\tfrac{1}{2}) = 14.4''$$

$$A_s = \frac{885{,}720}{14.4 \times 20{,}000} = 3.08^{\square''} \text{ of steel}$$

Four 1-inch round bars have the desired area. It will be noted that the above beam is fully continuous but that, in finding the

moment, the formula $\frac{Wl}{12}$ has been used while in this text is found the formula $\frac{Wl}{16}$ for the bending moment at the center of a fully continuous beam, point $C$, Fig. 134. Since the moment over the support, point $B$, Fig. 134, is $\frac{Wl}{12}$, it is common practice to use the same moment for the center of the beam. It is difficult to properly place and hold in place the additional steel that would be required over the beams to make up the difference between the two bending moments. The stress in the bottom of the beam at the support (point $A$, Fig. 134) is the same as the stress in the top of the beam over the support which is point $B$ in this figure. This stress, point $A$, is resisted by the concrete and the steel in the stem of the beam.

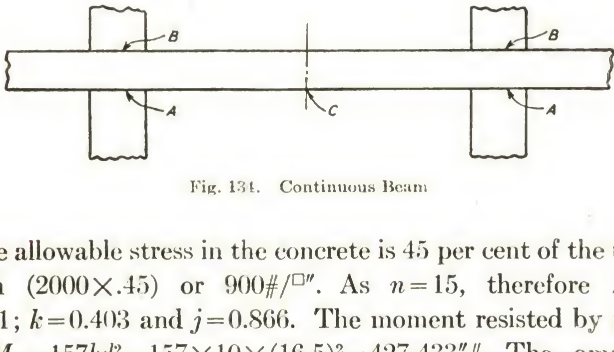



Fig. 134. Continuous Beam

The allowable stress in the concrete is 45 per cent of the ultimate strength $(2000\times.45)$ or 900#/□″. As $n=15$, therefore $K=157$; $p=.0091$; $k=0.403$ and $j=0.866$. The moment resisted by the concrete $M_c=157bd^2=157\times10\times(16.5)^2=427{,}433''\#$. The amount of steel required for this moment $=.0091\times10\times16.5=1.50$□″. As the total moment is 885,720″#, there is a moment of 458.287″# to be resisted by steel. Take $d'=2''$, then $(d-d')=14.5''$, and additional tension steel $=\frac{458{,}287}{20{,}000\times14.5}=1.58$□″. Using Equation (48) the amount of compressive steel, $A'=\frac{A^2(1-k)}{\left(k-\frac{d'}{d}\right)}=\frac{1.58(1-.403)}{\left(.403-\frac{2}{16.5}\right)}=\frac{1.58\times0.597}{0.282}=3.34$□″. As there are only four 1″$\phi$ bars in the bottom of the beam with an area of 3.14□″, it is necessary to supply an additional ½″ square bar, 6′ long, in the bottom of the beam over the support. A beam with dimensions of 10″ by 20″, reinforced with

four 1″ round bars and an additional ½″ square bar, 6′ long at the supports, will be used for the typical floor beams.

For end spans, the moment factor becomes $\frac{Wl}{10}$ and the moment will be increased in the ratio of $\frac{12}{10}$ and equals 1,062,864″#. This moment requires $\frac{1,062,864}{20,000\times 14.4}$ or 3.69□″ of steel. Four 1″□ bars will be used.

The shear in the end of the beam is found by dividing the end reaction, half the load, by the product of the width by the *jd* depth.

$$v=\frac{V}{b\times jd}=\frac{20,130}{10\times 14.4}$$
$$=140\#/\square''$$

This stress is satisfactory, provided that one-half of the bars in the bottom of the beams are turned up near the end of the beam, the end bars are hooked, and that a sufficient number of stirrups are used.

The total shear, $V$, at the end of the beam is equal to half of the uniformly distributed load, or $\frac{40,260}{2}=20,130\#$. This shear will be resisted by the concrete and stirrups. The amount of shear which the concrete will resist is equal to the area of the beam above the steel, multiplied by the allowable unit shear on plain concrete. Therefore, the shear resisted by the concrete $=10\times 14.4\times 60=8,640\#$. This leaves $20,130-8,640=11,490\#$ to be resisted by the stirrups. The required spacing for stirrups at any point is given by Equation (26) transposed for finding the value of $S$.

$$S=\frac{14,000A_v d}{V}$$

in which

$S$ = stirrup spacing in inches

$A_v$ = The cross-sectional area of the stirrup = 0.22□″ for a ⅜″ round stirrup

$d$ = the effective depth

$V$ = the vertical shear at the point under consideration to be carried by the stirrups

For practical purposes the stirrup spacing is computed at successive points along the beam 1 foot apart. The decrease in shear for each foot toward the center of the beam is $\frac{40,260}{22}=1830\#$. Using $\frac{3}{8}''\phi$ stirrups,

The stirrup spacing at any point $=\frac{0.22\times 14,000\times 14.4}{V}=\frac{44,352}{V}$

The spacing at the support $=\frac{44,352}{11,490}=3.9''$

The spacings for the rest of the beam as found by substituting the proper quantities in the formula $S=\frac{44,352}{V}$ are tabulated below:

| Distance "$a$" from Support, Feet | Shear ($V$) at Distance "$a$" from Support, Pounds | Stirrup Spacing ($s$) in Inches |
|---|---|---|
| 1 | 9660 | 4.6 |
| 2 | 7830 | 5.7 |
| 3 | 6000 | 7.4 |
| 4 | 4170 | 10.6 |
| 5 | 2340 | 19.4 |

The quantities in the second column are found by deducting 1830# (which is the decrease in shear for each foot toward the center of the beam) successively from the preceding value.

The stirrup spacing should not exceed $\frac{3}{4}d$, which in this case is $0.75\times 16.5=12.4''$. The spacing at a distance of 5′ from the support is computed as 19″, which is greater than 12.4″ and should be discarded. The spacing for stirrups is always given in even inches and very often in multiples of 3″. The spacings can be arranged so as to give approximately the computed spacings in the table. Stirrups should be used in the beam out to the point where the concrete alone is capable of resisting the shear at that point, which is 4′9″ from center line of beam, care being taken not to exceed the maximum spacing allowed. In Fig. 135 is shown a detail of this beam.

**Spandrel Beams.** The depth of the spandrel beams on the sides of this building must be the same as the depth of the spandrel girders at the ends of the building, so that the top of the sash will be on the same line. In this problem the depth of the spandrel girder is controlled by the amount of compression that can be de-

veloped in the 4″ floor slab. If the floor slab was increased to 6″ or 8″ for a width of about 30″, then the depth could be greatly reduced. The calculations will not be given here, but it was found that spandrel girders with a total depth of 30″ must be used, since the floor slab is only 4″ thick. If a greater amount of light is more important

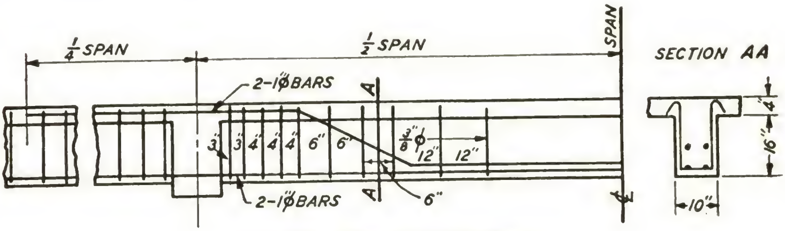



Fig. 135. Detail of Typical Beam

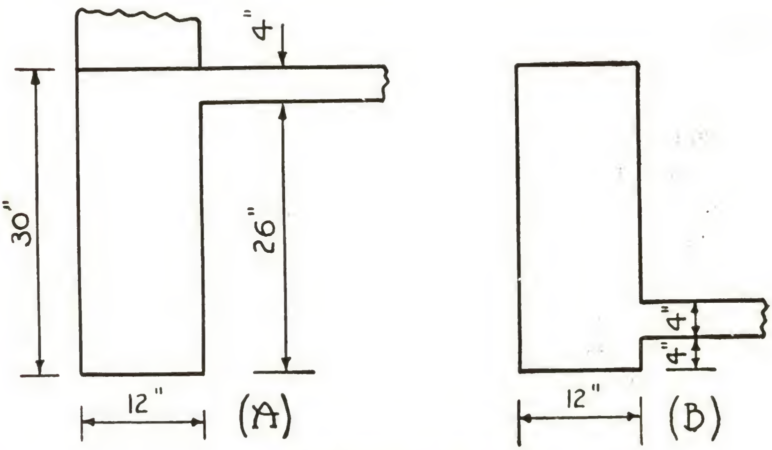



Fig. 136. Plain Rectangular Beams

than the appearance of the building, then a plain rectangular beam can be used for both sides of the building, as shown at (*B*), Fig. 136.

The height of all the stories of this building will be assumed as being 13′ from floor to floor, except the basement which will be 11′. If steel sash 8′ high is used, then the distance from the bottom of the sash to the top of the slab will be 2′6″, when type shown at (*A*), Fig. 136, is used. That is, the total depth of the beam 2′6″, plus 8′ for sash, plus 2′6″ for brickwork, equals 13′.

The floor load for the spandrel beams per square foot will be the same as the other beam, except the weight of the beam will be increased on account of the increased size. The load will be as follows:

| | | |
|---|---|---|
| Floor | $2.75' \times 22'0'' \times (200+25+50)$ | $=16,637\#$ |
| Beam | $1' \times 2.5' \times 18.8'$ (col. to col.) $\times 150 =$ | $7,050$ |
| Brick | $1' \times 2.5' \times 18.8'$ (col. to col.) $\times 120 =$ | $5,640$ |
| | | $29,327\#$ |

$$M=\frac{29,327\times 22\times 12}{12}$$
$$=645,194''\#$$

The total depth of the beam, as already stated, will be 30″. Then

$$jd=j(30-2\tfrac{1}{2}'')=27\tfrac{1}{2}\times .87=23.9''$$

Then
$$A_s=\frac{645,194}{23.9\times 20,000}$$
$$=1.35^{\square''} \text{ of steel}$$

Four ¾-inch round bars will be used for this beam, two bars being turned up and extended to the quarter point of the adjacent beams.

The compression in the slab and the shear stress are too small to be considered in this beam. But stirrups, ⅜″ round, should be placed about 12″ on centers near the ends and 18″ on centers near the center. Also, it would be well to place two ¾″$\phi$ bars in the top of the beam to assist in equalizing the temperature stresses.

**Girders.** The span of the girders is 18′ and, since the building is only three bays wide, the bending moment would be $\frac{Wl}{10}$ for a uniform load over the entire bay. These girders could be designed for the concentrated loads of the beams, but it would probably be more satisfactory to make the calculations as first suggested.

Assuming that the depth of the girder is 26″ below the slab and that the beam is 12″ wide and the girders are 22′ on centers, then the dead load of the girder per square foot of the floor is

$$\text{Weight of girder}=\frac{12''\times 26''}{144}\times 150\div 22'=15\#/^{\square'}$$

The load on the girder is

$$\text{Load} = 22' \times 18' \times (200\# + 50\# + 25\# + 15\# + 30\#)$$
$$= 126{,}720\# \text{ (say } 127{,}000\#)$$
$$M = \frac{127{,}000 \times 18 \times 12}{10}$$
$$= 2{,}743{,}200''\#$$

As a depth of 26″ below the slab has been assumed, the total depth is 30″, and allowing 3½″ as the distance from the bottom of the girder to the center of the steel, then

$$jd = (30 - 3\tfrac{1}{2}) \times .875 = 23.2''$$
$$A_s = \frac{2{,}743{,}200}{23.2 \times 20{,}000}$$
$$= 5.91^{\square''}$$

Six 1″ square bars may be used. Two bars should be bent up at the fifth point and one bar at the quarter point. The dimension of the girder below the slab will be 12″×26″.

The compression at the center of the span will next be examined. The allowable effective flange width is limited as follows: (*a*) it shall not exceed one-fourth of the span length of the beam; (*b*) its overhanging width on either side shall not exceed eight times the thickness of the slab, nor (*c*) one-half the clear distance to the next beam. Investigating all three conditions, the effective width of slab is limited to either

$$(a)\ \tfrac{1}{4} \times 18' \times 12 = 54''$$
$$\text{or } (b)\ (2 \times 8 \times 4) + 12 = 76''$$
$$\text{or } (c)\ (2 \times 126'') + 12'' = 264''$$

The minimum width of 54″ will be used. Substituting in the first part of Equation (**32**)

$$M_c = bt\frac{f_c}{kd}(kd - \tfrac{1}{2}t)jd$$

$$2{,}743{,}200 = 54 \times 4 \frac{f_c}{.375 \times 26\tfrac{1}{2}}(.375 \times 26\tfrac{1}{2} - 2)23.2$$

$$f_c = 685\#/^{\square''}$$

This value is satisfactory as 800#/□″ is permissible.

The shear load on each end of the girders is equivalent to the load of one beam plus half the weight of the girder, and is constant between the support and the first beam framing into the girder, except for the small uniform decrease due to the dead load of the

girder. For all practical purposes the shear can be considered constant.

$$\text{Beam load} \dots\dots\dots\dots\dots\dots = 40{,}260\#$$

$$\tfrac{1}{2}\text{ Weight of girder} = \frac{12'' \times 26''}{144} \times 150\# \times 9' = \frac{2{,}930}{43{,}190\#}$$

$$v = \frac{43{,}190}{12 \times 23.2} = 155\#/\square''$$

The amount of shear which will be resisted by the concrete equals $12'' \times 23.2'' \times 60\#/\square'' = 16{,}700\#$. This leaves $43{,}190 - 16{,}700 =$

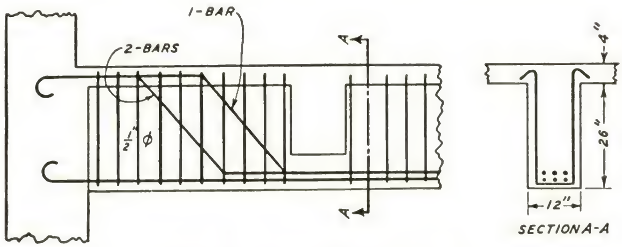



Fig. 137. Detail of End of Girder

26,490# to be resisted by the stirrups. The required spacing using one-half inch round stirrups is, therefore

$$\frac{0.39 \times 14{,}000 \times 26.5}{26{,}490} = 5.5''$$

One half of the bars in the bottom of the beams will be turned up near the ends of the girder and will be hooked on noncontinuous ends. They will therefore assist in the shear stresses. Fig. 137 gives the details of the girder where it connects with an exterior column.

**Columns.** The interior columns will first be considered. The area that each column supports will be $18' \times 22'$. Assume that the live and dead loads for the roof will be $125\#/\square'$. This will allow 30# for live load, 5# for roofing, 20# for cinder fill to slope the roof, and 70# for the slab, beams, and girders. The floor load for the interior columns will be the same as the girder load, $320\#/\square'$.

All interior columns will be round; steel forms will be used and, to reduce the number of sizes of forms needed for the building, two adjacent floors will have columns of the same diameter. That is, the basement and first-floor columns will be made the same size,

the second and third-floor columns the same, etc.; but the reinforcing steel will be reduced in the upper tier of columns each time.

| | |
|---|---|
| **Sixth Floor Columns** | |
| Roof load 18′×22′×125#............. | 49,500# |
| **Fifth Floor Columns** | |
| Load from column above.............. | 49,500# |
| 6th floor load, 18′×22′×320#......... | 126,720 |
| Weight of col. above—allow........... | 2,000 |
| | 178,220# |
| **Fourth Floor Columns** | |
| Load from column above.............. | 178,220# |
| 5th floor load........................ | 126,720 |
| Weight of col. above—allow........... | 3,000 |
| | 307,940# |
| **Third Floor Columns** | |
| Load from column above.............. | 307,940# |
| 4th floor load........................ | 126,720 |
| Weight of col. above—allow........... | 4,000 |
| | 438,660# |
| **Second Floor Columns** | |
| Load from column above.............. | 438,660# |
| 3rd floor load........................ | 126,720 |
| Weight of col. above—allow........... | 5,000 |
| | 570,380# |
| **First Floor Columns** | |
| Load from column above.............. | 570,380# |
| 2nd floor load........................ | 126,720 |
| Weight of col. above—allow........... | 7,000 |
| | 704,100# |
| **Basement Columns** | |
| Load from column above.............. | 704,100# |
| 1st floor load........................ | 126,720 |
| Weight of col. above—allow........... | 8,000 |
| | 838,820# |

The interior columns will be designed by Equation (**61**).

$$P=A_c[1+(n-1)p]f_c$$

and Equation (**62**) $\quad f_c=[300+(0.10+4p)f'_c]$

Investigating a column with a core of 28″ diameter, and allowing 2″ for fireproofing, the outside diameter will be 32″. Use 3000# concrete.

$$
\begin{aligned}
A_c &= \pi \times (14)^2 = 615\square'' \\
p &= .03 \\
f_c &= 300 + [0.10 + (4 \times .03)]3000 = 960 \\
n &= 10 \\
P &= 615[1 + (10 - 1).03]960 \\
&= 615 \times 1.27 \times 960 \\
&= 749{,}800\#
\end{aligned}
$$

This is found to be somewhat less than the load on the column.

Another computation increasing the ratio of the steel to .037 discloses that such a column will carry a load of 855,900#, which is greater than the load on the column. The amount of vertical reinforcing required is $(615 \times .037) = 22.75\square''$. This can be furnished by twenty-three 1″ square bars. The ratio of the spiral reinforcement should be not less than one-fourth the ratio of the longitudinal reinforcement. The volume of one-fourth of the vertical steel for one foot in height is $\frac{23 \times 1.00 \times 12}{4} = 69.0$ cubic inches. The circumference of a 28″ round column is $\pi 28'' = 88''$. The area of a ⅜″ round bar is $0.11\square''$. The volume of a ⅜″ round bar 88″ long is 9.68 cubic inches. Then $69 \div 9.68 = 7.1$. That is, 7.1 spirals will be required for each foot in height of the column or 12″ (height) divided by $7.1 = 1.69''$. A ⅜″ round spiral with a 1½″ pitch will be satisfactory. Therefore, the basement column will be 32″ outside diameter, reinforced with twenty-three 1″ square bars and a 28″ diameter ⅜″ spiral with a spacing of 1½″.

The column in the first floor must support a load of 704,100#. It is to be the same diameter as the basement column, and the same grade of concrete will be used; therefore the reinforcing steel can be reduced. The values to be substituted in Equation (**61**) are as follows:

$$
\begin{aligned}
A_c &= \pi \times (14)^2 = 615 \\
\text{Try} \qquad p &= .027 \\
f_c &= [300 + [0.10 + (4 \times .027)]3000] = 924\#/\square''
\end{aligned}
$$

Substituting $P = 615 \times [1 + (10-1).027]924$
$= 615 \times 1.243 \times 924$
$= 706{,}300\#$

The vertical steel required is $615 \times .027 = 16.61^{\square\prime\prime}$. This can be furnished by twenty-two 1″ round bars.

The volume required for the spiral will be $\frac{22 \times .785 \times 12}{4} = 51.8$ cubic inches per foot. Following the preceding solution, it was found that the volume of a ⅜″ round bar per foot is 9.68 cubic inches. Then $51.8 \div 9.68 = 5.4$. That is, 5.4 spirals will be required for each foot. Then 12″ divided by $5.4 = 2.22''$ or 2¼″ for the pitch of the spiral.

**Wall Columns.** The exterior column will form part of the wall and therefore will be exposed on the exterior surface of the building. All windows are to be the same size and extend from column to column. This condition makes it necessary to have a constant dimension for the width of these columns. The column loads are as follows:

| | |
|---|---|
| Sixth Floor Columns | |
| Roof load 9′×22′×125# | 24,750# |
| Parapet 3′×22′×90# (8″ wall) | 5,940 |
| | 30,690# |
| Fifth Floor Columns | |
| Load from column above | 30,690# |
| 6th floor load, 9′×22′×320# | 63,360 |
| Wall 2′6″×18′10″×(12″ wall) 120# | 5,650 |
| Weight of column (assumed) | 5,000 |
| | 104,700# |
| Fourth Floor Columns | |
| Load from column above | 104,700# |
| 5th floor load | 63,360 |
| Wall | 5,650 |
| Weight of column assumed | 6,000 |
| | 179,710# |

| | |
|---|---|
| **Third Floor Columns** | |
| Load from column above | 179,710# |
| 4th floor load | 63,360 |
| Wall | 5,650 |
| Weight of column assumed | 8,000 |
| | 256,720# |
| **Second Floor Columns** | |
| Load from column above | 256,720# |
| 3rd floor load | 63,360 |
| Wall | 5,650 |
| Weight of column assumed | 10,000 |
| | 335,730# |
| **First Floor Columns** | |
| Load from column above | 335,730# |
| 2nd floor load | 63,360 |
| Wall | 5,650 |
| Weight of column assumed | 12,000 |
| | 416,740# |
| **Basement Columns** | |
| Load from column above | 416,740# |
| 1st floor load | 63,360 |
| Wall | 5,650 |
| Weight of column assumed | 14,000 |
| | 499,750# |

Since the walls of the building are to be composed of the columns, steel sash, and brick spandrel walls below the sash, take a standard width of steel sash of 18′10″ that will leave a width of 3′2″ (22′−18′10″=3′2″) for the column. Therefore all columns will have a constant width of 3′2″, or 38″.

In designing these columns, Equation (**60**) will be used and a concrete with a 2500# strength at the age of 28 days will be used for the lower floors.

$$P=0.225f'_c\ Ag[1+(n-1)p]$$

Assume that the basement column will be 20″ by 38″, reinforced with twelve 1″ square bars. For the basement column the smaller dimension should not be less than half the larger dimension.

Then $0.225f'_c = 0.225 \times 2500 = 562\#/\square''$

$$Ag = 20 \times 38 = 760\square''$$
$$A_s = 12 \times 1 = 12.00\square''$$
$$p = \frac{12.00}{760} = .0158$$

and as $n = 12$, $(n-1)p = .1896$. Substituting in the equation

$$P = 562 \times 760 \times 1.1896$$
$$= 508{,}100\#$$

The dimensions of the concrete and reinforcing steel assumed for this column give satisfactory results. The bands are to be ⅜″ round, spaced 8″ on centers. One band around the column does not add much to the strength of the column, but if the bands are made in two units or more for each spacing, a much better column is secured at a very small additional cost.

The columns in the first story must support a load of 416,740#. Assume the dimensions of the column to be 16″×38″ and reinforced with eight 1⅛″□ and two 1″□ bars.

Then $Ag = 16 \times 38 = 608$

$$A_s = 12.16$$
$$p = \frac{12.16}{608} = .020, \text{ and } (n-1)p = .220$$

Then $P = 562 \times 608 \times 1.22$

$$= 416{,}900\#$$

This section will therefore be used.

The assumed dimensions for the column in the second floor are as follows: 14″×38″ reinforced with eight 1″ round bars. Substituting in Equation (**60**), $P$ is found to be 337,800# which is larger than the load 335,730 and this assumed section is therefore satisfactory.

A column 12½″ by 38″ reinforced with six ¾″ round bars is satisfactory for the third floor. Since the columns cannot be reduced in size to less than 12½″×38″, the grade of concrete will be reduced from 2500# to 2000#/□″. For the reinforcing steel, use six ¾″ round bars.

Then $Ag = 12\frac{1}{2} \times 38 = 475\#''$

$$f_c = 0.225 \times 2000 = 450\#$$
$$p = \frac{2.64}{475} = .00556; \ (n-1)p = .061$$
$$P = 450 \times 475 \times 1.061$$
$$= 226{,}800\#$$

The value of $P$, as found here, is 27,000# greater than necessary to support the load. But it is inadvisable to further reduce the quality of the concrete. Accordingly, the columns in the fifth and sixth floors would be the same as those in the fourth floor. The columns in the upper floors are made 12½″ wide, so that the column will be flush with the wall. A brick wall called 12″ thick usually measures 12½″. There would be no objection to making the wall columns in the upper stories of such a thickness that they would have the same dimension as the walls, provided that the wall was not less than 12″ thick.

**Footings.** The load for the interior columns is 838,820#. See interior column load. The weight of the footing will be about 60,000#. Then the total load on the ground will be 838,820+60,000=898,820# or, say, 899,000#.

Assuming that this building is to be constructed on a soil that will safely support a load of 6,000#/□′, then the area of footing will be

$$899{,}000 \div 6000 = 149.8^{\square\prime}.$$

Extracting the square root of 149.8, we have 12.24′ for each side, or, say, 12′3″.

The column is round in shape, has a high unit stress in the concrete, and the load is to be transferred to the footing in which the concrete is used at a much lower stress. To reduce this unit stress on the footing and also to give the column a better shape to act with the footing, we will construct a pedestal at the bottom of the column. The column load 838,820÷500□″=1,678□″. The square root of 1,678 is 41. Allowing 3″ for protection, the pedestal will be 47″ by 47″. In height the pedestal should be at least 18″ and will be reinforced with vertical bars and a spiral. The vertical bars in area will equal the area of the bars in the column. The spiral steel can be slightly reduced from the area used in the column. Because the pedestal is larger than the column, the unit stress is reduced in the pedestal. The concrete mix used in the pedestal will be the same as used in the columns.

The equation for finding the bending moment in the footings is Equation (**66**).

$$M = W\left(\frac{bc^2}{2}\right)$$

In this case $W = 6000\#/^{\square\prime}$; $b = 12'3''$, $c = 4'2''$ or $4.17'$

Substituting $$M=6000\left(\frac{12.25'\times(4.17)^2}{2}\right)$$

$$=639,040'\#$$

or $$M=639,040\times 12=7,668,480''\#$$

It has been stated before that the depth of the footing is controlled by the shear and bond. Assume a depth to the steel, $d=24''$. Then the critical section for shear is 24″ from the face of the column. This means that the critical section is along the perimeter of a square $[47+(2\times 24)]=95''$ on a side. The area of a square 95″ or 7.92′ on a side$=62.73^{\square\prime}$. The total area of the footing is $(12.25)^2=150.06^{\square\prime}$. The difference is $150.06-62.73=87.33^{\square\prime}$, and the total shear on the critical section is $87.33\times 6,000=524,000\#$. Substituting in Equation (22), $v=\frac{8V}{7bd}$; where $b=4\times 95=380''$, $v=\frac{8\times 524,000}{7\times 380\times 24}=66\#/^{\square''}$. As this is more than $60\#/^{\square''}$, the depth will be increased to 25″; then the unit shear $v=60\#/^{\square''}$, which is permissible. The area of the steel required with a depth of 25″ is

$$A_s=\frac{7,668,480}{20,000\times 25\times .875}=17.5^{\square''}$$

This area can be supplied by twenty-three $1''\phi$ bars. The $\Sigma o$ for these bars is 72.25″.

The critical section for bond is at the face of the column. The total shear at the face of the column is $6000\times(4.17\times 12.25)=306,500\#$. Substituting in Equation (23), $u=\frac{8V}{7d\Sigma o}=\frac{8\times 306,500}{7\times 25\times 72.25}=194\#/^{\square''}$. The allowable stress is only $150\#/^{\square''}$; so the $\Sigma o$ value of the steel must be increased or the depth must be increased. Increase the depth to 28″ and use thirty $\frac{7}{8}''\phi$ bars with $\Sigma o=82.50''$. Then $u=\frac{8\times 306,500}{7\times 28\times 82.50}=152\#/^{\square''}$. As this is so nearly $150\#/^{\square''}$ this section will be used. Since the depth is increased to 28″, the $A_s=\frac{7,668,480}{20,000\times 28\times .875}=15.45^{\square''}$, and as thirty $\frac{7}{8}''\phi$ bars have an area of $18^{\square''}$, the moment consideration is satisfied. Allowing 3″ for protection and 1″ for bottom row of bars, the total depth of the footing will be 32″. See Fig. 138. The footing could be designed with 25″ depth by using a large number of $\frac{3}{4}''\phi$ bars, but the spacing of the

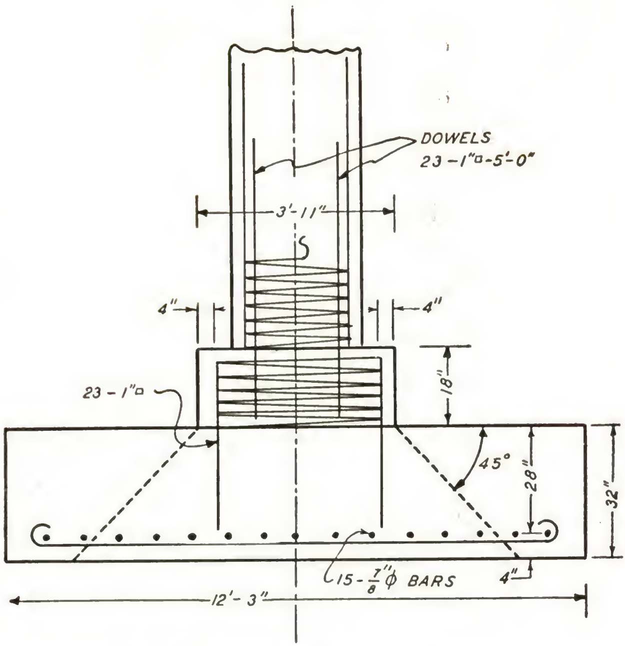



Fig. 138. Detail of Interior Column Footing

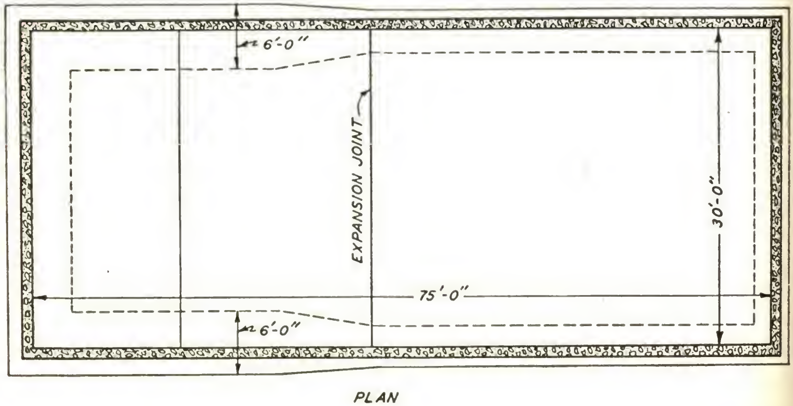



Fig. 139. Swimming Pool

bars would be too close, so it is better to increase the depth and furnish a little more steel than is theoretically required for the moment.

The footings for the exterior column can be designed in a similar manner; or if the building is located on a property line, then it will be necessary to have a combination footing as explained in another part of the text.

## SWIMMING POOL

**Plan.** In Fig. 139 is shown the plan and longitudinal section of a swimming pool with inside length of 75′, width of 30′ and a maximum depth of 9′. Pools may be constructed in any desired size, shape and depth if the walls have sufficient strength to support the water and earth pressure and if provision is made for temperature stresses either by reinforcing steel, expansion joints, or both. Many successful pools have been constructed without expansion joints. Often small cracks occur, which do not leak and which in the opinion of many people, are preferable to mastic joints which show.

Occasionally the entire pool, including walls and floor, is poured at one operation. This is expensive as the interior wall forms must be suspended from supports above the pool. Generally, pools are poured in two operations. The wall footings and the floor are poured first, and the walls are poured when the forms are completed and the reinforcing steel is placed. In the design given in this text, it will be seen that a pool may be poured at several operations. A duplicate mixer should always be near so that, if the one in operation breaks down, the pouring may continue by using the duplicate machine.

**Mix.** The concrete used for pool construction should be a rich, dense, watertight mix with a minimum amount of mixing water. The mix should not be leaner than a 1:2½:3½ mix, as water-tight concrete must be produced. A richer mix, 1:2:3 is often used. The concrete must be well cured. It must be protected by canvas or burlap and kept wet for several days.

**Walls.** The walls of the pool must be designed to resist earth pressure when the pool is empty, and water pressure when the pool is full, as the water pressure is much greater than the earth pressure. Therefore, the wall must be reinforced vertically in both faces. The horizontal reinforcement must be sufficient to resist the tempera-

ture stresses. The walls of the pool are usually made twelve inches in thickness, but sometimes they are limited to ten inches. In either case the form work costs about the same; but when the thicker walls are used, the amount of vertical steel is reduced, the concrete is easier to spade, tight joints can be more readily secured, and the additional concrete assists in making a water-tight job.

**Design of Walls.** In designing for earth pressure, a high stress may be used in the steel; but when designing for water pressure, the stress should be reduced so that the deformation in the concrete will be kept low, resulting in a more water-tight concrete. In the demon-

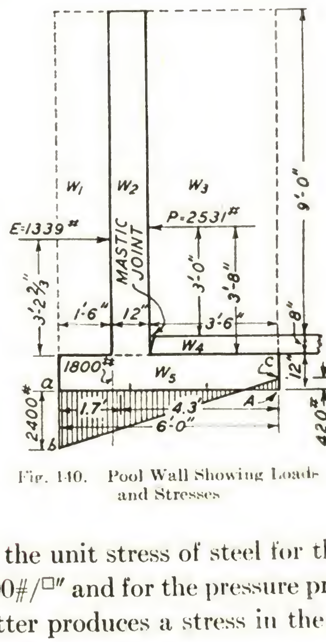



Fig. 140. Pool Wall Showing Loads and Stresses

stration given here, the unit stress of steel for the pressure produced by the earth is 18,000#/□″ and for the pressure produced by water it is 14,000#/□″. The latter produces a stress in the concrete of 535#/□″, which should be satisfactory.

In Fig. 140 is shown the maximum depth of the pool (outlined in Fig. 139) with the stresses indicated. The end walls and side walls (less in height) should be designed, as the vertical steel can be reduced materially.

Earth pressure produces a stress $= 0.286 \times \frac{Wh^2}{2}$, Equation (6a).

$$E = .286 \times \frac{100 \times 9\frac{2}{3}^2}{2} = 1,339\#$$

$$M = 1339\# \times 38\frac{2}{3}'' = 51{,}775''\#$$

$$A_s = \frac{51{,}775}{18000 \times 9\frac{1}{2} \times .87} = 0.35\square''$$

$\frac{5}{8}'' \phi$ bars $10\frac{1}{2}''$ center to center.

The water pressure is found from the equation $P = \frac{1}{2} W h^2$ in which $P$ is the pressure, $W$ is the weight of a cubic foot of water (62.5) and $h$ is the height of the water.

Substituting $\quad P = \frac{1}{2} \times 62.5 \times (9)^2 = 2{,}531\#$

The moment produced by this stress

$$M = 2531\# \times 36'' = 91{,}116''\#$$

$$A_s = \frac{91{,}116}{14{,}000 \times 9.5 \times .87} = 0.79\square''$$

Bars $\frac{7}{8}'' \phi$, spaced $9''$ center to center, are satisfactory for this condition. It is necessary to take only one-third of them to the top of the wall.

The stresses in the base will be investigated. A base $6'$ wide with a projection of $1'6''$ beyond the pool will be tested. The loads acting on the base (see Fig. 140) are $W_1$, weight of earth above the toe, 100# per cubic foot; $W_2$, weight of wall, 150# per cubic foot; $W_3$, weight of water above the base, 62.5# per cubic foot; $W_4$, weight of floor above the base; and $W_5$, weight of the base. Moments taken about the point $A$ are tabulated as shown.

**Moments Taken about Point *A***

| Section | Loads, Pounds | Arm, Ft. | Moment, Ft.-Lbs. |
|---|---|---|---|
| $W_1$ | $9\frac{2}{3} \times 1\frac{1}{2} \times 100 = 1450$ | $5\frac{1}{4}$ | 7610 |
| $W_2$ | $9\frac{2}{3} \times 1 \times 150 = 1450$ | 4 | 5800 |
| $W_3$ | $9 \times 3\frac{1}{2} \times 62.5 = 1970$ | $1\frac{3}{4}$ | 3450 |
| $W_4$ | $\frac{2}{3} \times 3\frac{1}{2} \times 150 = 350$ | $1\frac{3}{4}$ | 610 |
| $W_5$ | $1 \times 6 \times 150 = 900$ | 3 | 2700 |
| $\Sigma$ | Vertical loads = 6120 | | |
| $P$ | Water pressure = 2531 | $4\frac{2}{3}$ | 11810 |
| $E$ | Earth pressure = 1339 | $4\frac{2}{9}$ | —5650 |
| $\Sigma$ of Moment, Ft.-Lb. | | | 26330 |

The resultant cuts the base at a distance from $A$ equal to $\frac{26330\#}{6120\#} =$ 4.3′ which is 0.3′ out of the middle third, and an uplift is found at the point $A$. Substituting in Equation (6f) in which $Q = 6.0' - 4.3' = 1.7'$, pressure at toe =

$$\frac{2P}{3Q}=\frac{2\times6120}{3\times1.7}=2{,}400\#/\square'$$

which is a conservative load for a good soil.

The uplift at the heel is found by laying off *ab* at any convenient scale equal to 2,400#, and on the base laying off a distance equal to $3Q=3\times1.7'=5.1'$. From *b*, through this point, draw a line, and scale the force from *c* to the base line, which is 420#. The floor slab extending beyond the footing, and the water above it, will supply more than enough weight to overcome this force.

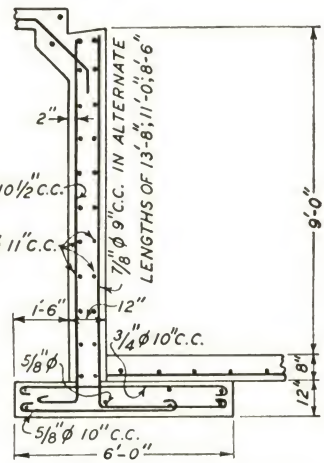



Fig. 141. Pool Wall Showing Reinforcing Steel

The base for the wall is designed the same as explained for retaining walls in Chapter XII. The heel of the base is designed for a load equal to the weight of water above it, the floor slab, and the base. From the tabulation shown, $W_3=1{,}970\#$, $W_4=350\#$ and the weight of the slab is $1'\times3.5'\times150=525\#$. $1970+350+525=2{,}845\#$.

$$M=2845\times21''=59{,}745''\#$$

$$A_s=\frac{59{,}745}{14{,}000\times9.5\times.87}=.52\square''$$

or ¾"$\phi$ bars 10" c.c.

The toe must have sufficient strength to transfer the load from the vertical shaft of the wall to the earth under the toe.

$$\frac{2400+1700}{2}=2050$$

This moment is $2050 \times 1\frac{1}{2} \times 12 = 36{,}900''\#$

$$A_s = \frac{36{,}900}{14{,}000 \times 9.5 \times .87} = 0.32''$$

or $\frac{5}{8}''\phi$ bars 12″ c.c.

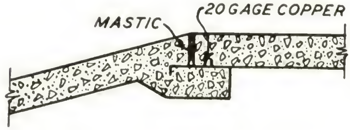



Fig. 142. Expansion Joint in Floor

Special attention should be given to the area of steel used for temperature stresses. For long walls or walls without frequent expansion joints, the ratio of steel should not be less than .003. In a 12″ wall, the steel required would be $12 \times 12 \times .003 = 0.43^{\square''}$, or two $\frac{1}{2}''\phi$ bars should be spaced not over 11″ on centers.

Joints in the wall can be made similar to other wall joints de-

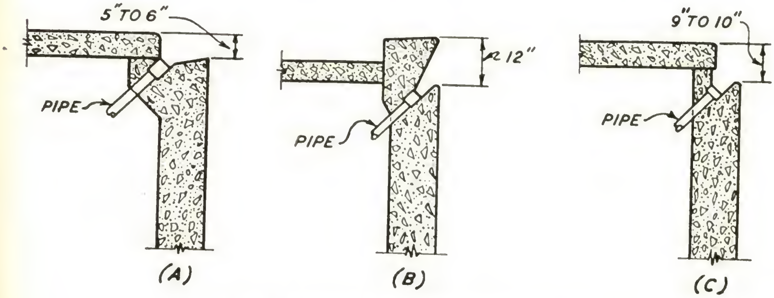



Fig. 143. Overflow Details for Swimming Pools

scribed in this text. In Fig. 141 is shown a section of the wall with the reinforcing steel indicated.

**Floor Slabs.** The minimum thickness for floor slabs is 6″. It is better to make them 7″ or 8″ in thickness. They must be well reinforced for temperature stresses in two directions. Expansion joints in the floor can be made as shown in Fig. 142. The surfaces of the first section poured should have a floated finish and should be coated with a good mastic cement before the second pouring. Crimped

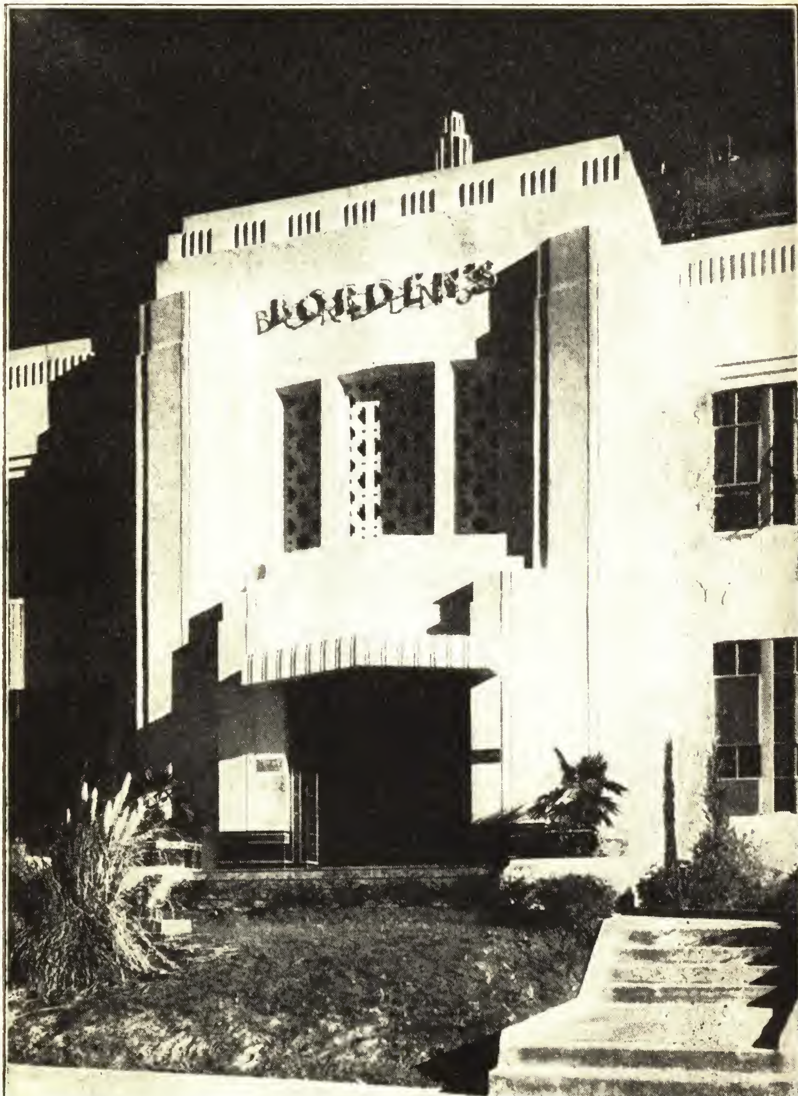



Fig. 144. An Example of Finished Concrete
*Courtesy of Architectural Concrete*

copper should be placed in the joints. The joint between the footing for the walls and the floor slab can be made in a similar manner, or the wall footing and the floor slab can be tied together by the use of **U**-stirrups.

**Overflow Drain.** In Fig. 143 are shown three types of overflow drains that are in common use. The first type (*A*) should be used for outdoor pools, as the water is only about 5″ or 6″ below the pavement and the water can easily be splashed on the pavement.

The drains shown in Fig. 143 at (*B*) and (*C*) are suitable for either indoor or outdoor pools. All overflow drains should be graded carefully to the outlet pipes, which usually are located 10′ to 12′ apart.

**Finished Surfaces.** The exposed walls in a pool may be finished in several ways. For outdoor pools, the finish usually consists of rubbing the structural concrete with carborundum blocks. If it is desirable to get away from the color of the concrete, then the pool may be painted; or if a light color is wanted, white cement may be used instead of the dark Portland cement. With the white cement, a light-colored sand should be used. For indoor pools, the finish can be made the same as described for outdoor pools, or they may be lined with tile, brick, precast concrete, etc. If any finishing material is to be added, the surface of the concrete should be rough and the finishing material should be well bonded to the structural concrete.

The total thickness of the floor should be poured at one operation, thoroughly compacted, floated with a wood float, lightly troweled with a steel trowel, and lightly brushed with a hair brush to give the surface a nonslip finish.

SWIMMING POOL OF CONCRETE CONSTRUCTION, SACRAMENTO, CALIFORNIA

*Courtesy of Portland Cement Association*

# CHAPTER XVII

## FINISHING SURFACES OF CONCRETE

**Imperfections.** To secure a satisfactory finish for exposed concrete surfaces, the materials must be carefully graded, must have uniform mixing and spading of the concrete, forms must be well constructed of dry lumber, and the concrete must be properly cured.

In many instances, when the forms are taken down, the surface of the concrete shows the joints, knots, and grain of the wood; it has more the appearance of a piece of rough carpentry work than that of finished masonry. Also, failure to tamp or flat-spade the surfaces next to the forms will result in rough places or stone pockets. Lack of homogeneity in the concrete will cause a variation in the surface texture of the concrete. Variation of color, or discoloration, is one of the most common imperfections. Old concrete adhering to the forms will leave pits in the surface; or the pulling-off of the concrete in spots, as a result of its adhering to the forms when they are removed, will cause a roughness.

The variation in color is due to the leaching-out of lime, which is deposited in the form of an efflorescence on the surface; or to the use of different cements in adjacent parts of the same work. The latter cause can almost always be avoided by using the same brand of cement on the entire work, and the former will be treated under the heading of "Efflorescence."

In Fig. 145 is shown the surface of a piece of concrete work in which the grain of the lumber, including the knots, is shown. The forms for this concrete were not of tight construction, therefore the joints are accentuated. The concrete appears to be dense, but the rough form work shows distinctly in the surface of the concrete.

To guard against these imperfections, the forms must be well constructed and have smooth, even surfaces and joints. As explained under the heading of "Forms", plywood, presdwood, etc., are much used to aid in securing a good surface. Form fillers will also assist in securing a better finish when applied to plywood or surfaced lumber.

Fig. 145. Concrete Surface Showing Joint Lines Left by Rough Forms
*Courtesy of Portland Cement Association*

Fig. 146. Surface Secured with Plywood or Presdwood
*Courtesy of Portland Cement Association*

The concrete must be a mix suitable to the exposure, thoroughly mixed, of the proper consistency and carefully but not over spaded. The different brands of cement vary in color; therefore the same cement should be used for the complete job where the concrete is exposed. Sands from different sources likewise vary in color, and the same should be used throughout the job.

In Fig. 146 is illustrated the surface usually found when plywood or presdwood is used for forms. The joining of the boards always shows. Since the boards are 3 to 4 feet wide and 6 to 8 feet long

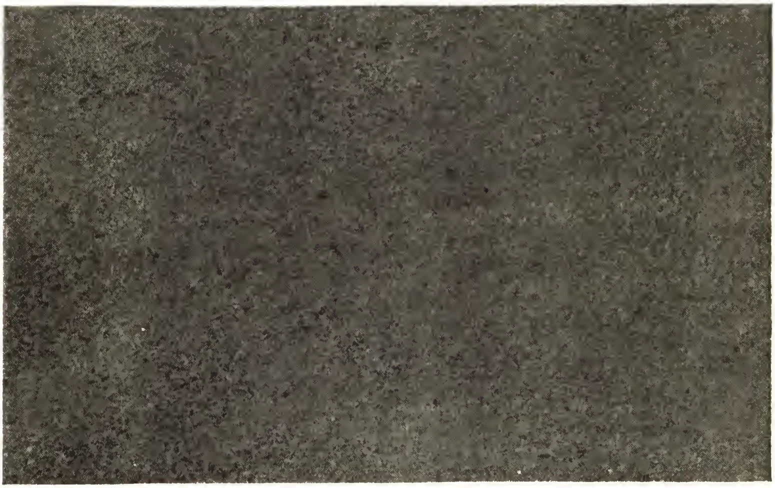


Fig. 147. Tooled Surface
*Courtesy of Portland Cement Association*

there are not so many joints to be treated. When the joints are rubbed down, a slightly different texture is shown. If such a surface is tooled, then a surface as shown in Fig. 147 will be secured.

When a smooth concrete ceiling is desired, the joints are rubbed and a good cement paint will usually fill the small air holes shown in Fig. 146 to such an extent that they will not be noticeable. Coarse-grained plywood will leave a slight impression of the grain of the wood unless well treated with a preparation to prevent it.

**Plastering.** The plastering of concrete surfaces is usually unsatisfactory unless the surfaces are specially prepared for such a finish. Plaster applied to the ordinary concrete will often scale off

in a few months, making the appearance worse than if nothing had been done to it.

By proper preparation, concrete walls can be plastered successfully. If a very dry mixture is placed against the face forms and not spaded, the face of the wall will be rough (honeycombed) making a mechanical bond for the plaster. All smooth places can be roughened and the wall should be well soaked with water several hours before the plaster is applied. However, no water should be used on the wall for at least two hours before plastering. The concrete surface must have sufficient moisture so that it will not absorb water from the plaster, but this moisture must not be freshly applied. If the wall has been recently poured, the surfaces can easily be picked or hammered; but if the concrete is hard, a weak solution of muriatic acid will eat out the cement and leave the sand and stone. The acid must be thoroughly washed off before the plaster is applied.

The plaster should not be of a richer mixture than 1 part cement and 3 parts sand, and should be applied with force or pressure so that a bond will be secured between the two materials. The plaster must be cured carefully or it will crack. It should be protected from the sun for at least 3 days and kept moist for several days. The curing will have much to do with the success of the finish.

**Rubbed Surfaces.** The forms for the exposed surfaces should be constructed of plywood or presdwood. They should be substantial and well finished. The concrete should be plastic and carefully placed in the forms and spaded. As soon as the concrete has hardened sufficiently to support itself, the forms should be removed, any pointing required should be done neatly, and the entire surface rubbed with carborundum brick and water.

Mortar made with ordinary Portland cement and sand will produce a much darker finish when applied to concrete in which the same cement and sand have been used for the concrete. However, the same color can be reproduced if part of the ordinary cement is replaced with white Portland cement. The amount of white cement required can be determined by experimenting with the materials to be used.

**Painting Concrete.** Portland cement paint is composed of Portland cement and other materials especially prepared and ground for applying to concrete and other masonry surfaces. For color, lime-

proof and sun-proof mineral pigments and other materials are finely ground and added to the cement, etc. The paint can be purchased in powdered form and mixed with water at the site of the work.


Fig. 148. Post Office—West Palm Beach, Florida
*Courtesy of Architectural Concrete; A. Farnell Blair, Contractor; Treanor and Fatio, Architects*

This paint bonds with concrete surfaces, stucco, brick and other masonry materials. The surfaces must be clean and uniformly damp when the paint is applied. When the first coat has hardened, it should be sprayed with water and a second coat applied. As soon as the second coat has hardened sufficiently so that the surface is not damaged by applying water, it should be sprayed with water and kept

damp for several days. During hot or windy weather, it should be protected so that it will not dry out too quickly.

In preparing a concrete surface for painting, the color of the patching must be made the same as the mass. To secure the same color see heading "Rubbed Surfaces." The paint will fill most of the pinholes and many of the holes caused by air bubbles.

In Fig. 148 is shown an elevation of a portion of the front wall of the post-office building at West Palm Beach, Florida, which illustrates a combination of rubbed surface and painting. The walls were constructed of poured concrete. Plywood was used for forms that were exposed to the concrete, and the finish was made by wet-rubbing the concrete finish with carborundum stone, after which cement paint was applied.

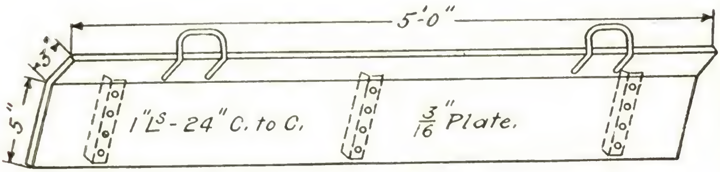



Fig. 149. Steel Plate, an Aid in Placing Facing Material

**Picked Surface Finish.** In Fig. 147 is shown a surface treatment that can be secured by bush-hammering or tooling with pneumatic tools on a well-seasoned concrete surface. Well-constructed forms with tight joints are necessary if only a small amount of material is to be removed from the surface to expose the coarse aggregates. If pebbles of different colors are exposed, it will add interest to the texture of the wall.

**Granolithic Finish.** Granolithic or pebble finish can be secured by placing about one inch or more of mortar (composed of cement, sand and granolithic chips or any other desired material) against the forms on the side of the structure exposed in the finished work. The coarse material should be graded from $\frac{1}{8}$ to $\frac{1}{4}$ inch. In Fig. 149 is shown a steel form that will aid greatly in placing the facing material. The angles, shown in dotted lines, can be made the depth wanted for the facing concrete. The angles are placed against the face forms and the space is filled. The concrete is then placed in

the main part of the wall, the steel forms are removed and the two mixes tamped together. The face forms are removed the day after concreting, and the cement is washed out leaving the small stones or gravel exposed.

Several years ago some concrete bridges in Philadelphia, Pa., were finished with the granolithic surface. The specifications for this work are quoted as follows:

Granolithic surfacing, where required, shall be composed of 1 part cement, 2 parts coarse sand or gravel, and 2 parts granolithic grit, made into a stiff mortar. Granolithic grit shall be granite or trap rock, crushed to pass a ¼-inch sieve, and screened of dust. For vertical surfaces, the mixture shall be deposited against the face forms to a minimum thickness of 1 inch, by skilled workmen,

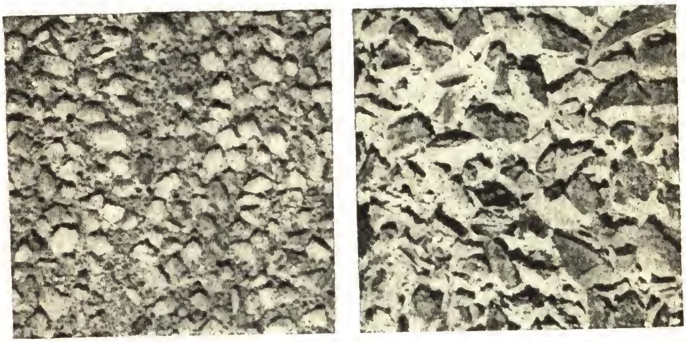


Fig. 150. Quimby's Finish on Concrete Surfaces. Left—Aggregate 3/16-Inch White Pebbles; Right—Aggregate 3/8-Inch Screened Stone

as the placing of the concrete proceeds; and it thus forms a part of the body of the work. Care must be taken to prevent the occurrence of air space or voids in the surface. The face shall be removed as soon as the concrete has sufficiently hardened; and any voids that may appear shall be filled with the mixture. The surface shall then be immediately washed with water until the grit is exposed and rinsed clean, and shall be protected from the sun and kept moist for three days. For bridge-seat courses and other horizontal surfaces, the granolithic mixture shall be deposited on the concrete to a thickness of at least 1½ inches, immediately after the concrete has been tamped and before it has set, and shall be troweled to an even surface, and, after it has set sufficiently hard, shall be washed until the grit is exposed.

The success of this method depends greatly on the removal of the forms at the proper time. In general, the washing is done the day following that on which the concrete is deposited. The fresh concrete is scrubbed with an ordinary scrubbing brush, removing the film and the impressions of the forms, and exposing the sand and stone of the

concrete. If this is done when the material is at the proper degree of hardness, merely a few rubs of an ordinary house scrubbing brush, with a free flow of water to cut and to rinse clean, constitutes all the work and apparatus required. The cost of scrubbing is small if done at the right time. A laborer will wash 100 square feet in an hour; but if that same area is permitted to get hard, it may require two men a day, with wire brushes, to secure the desired results. The practicability of removing the forms at the proper time for such treatment depends upon the character of the structure and the conditions under which the work must be done. This method is applicable to vertical walls, but it would not be applicable to the soffit of an arch. See Fig. 150.

**The Acid Treatment.** This treatment consists in washing the surface of the concrete with diluted acid, then with an alkaline solution. The diluted acid is applied first, to remove the cement and expose the sand and stone; the alkaline solution is then applied to remove all of the free acid; and, finally, the surface is washed with clear water. The treatment may be applied at any time after the forms are removed; it is simple and effective. Limestone cannot be used in the concrete for any surfaces that are to have this treatment, as the limestone would be affected by the acid. This process has been used very successfully.

**Masonry Facing.** Concrete surfaces may be finished to represent ashlar masonry. The process is similar to stone dressing; and any of the forms of finish employed for cut stone can be used for concrete. Very often, when the surface is finished to represent ashlar masonry, vertical and horizontal three-sided pieces of wood are fastened to the forms to make **V**-shaped depressions in the concrete, as shown in Fig. 151.

**Cast-Slab Veneer.** Cast-concrete-slab veneer can be made any desired thickness, size or shape. It is set and anchored to the concrete or brick used for the back part of the wall, and is usually cast in wood, sand or plaster molds. It can have a special face, or the entire slab may be of the same grade of concrete. The cast stone can be reinforced for handling. The reinforcing steel must be placed so that it will make the stone safe for handling with a derrick and so that it will reduce temperature stresses in the concrete. If desired, this material can be hammer dressed after it has hardened. It is

cheaper than cut stone and is much used. A typical facing hammer is shown in Fig. 152.

**Moldings and Ornamental Shapes.** Ornamental concrete units for buildings and bridges may be constructed in place or may be cast in molds, then erected and concreted in place. In Fig. 153 is shown a precast baluster. It is cast with a bar in the center which pro-

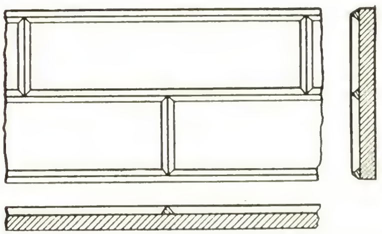


Fig. 151. Diagram Showing Method of Giving Masonry Facing to Concrete

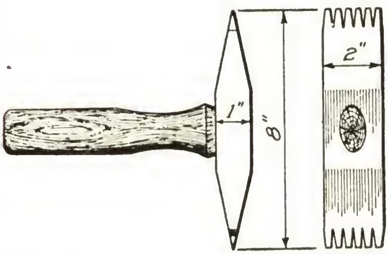



Fig. 152. Typical Facing Hammer

Fig. 153. Molded Concrete Baluster

jects a few inches from both ends for anchorage in the top and bottom rails.

**Coloring Materials.** Coloring materials mixed in concrete should not be used in excess of 10% of the amount of cement used, or it reduces the strength of the concrete. Commercially pure mineral colors should be used. Oxides are apt to fade or reduce the strength of the concrete. After the color has been decided on for a job, the cement and coloring materials should be carefully measured or weighed, so that every batch used will be the same. The materials

should be mixed dry until the batch has a uniform color and then the water may be added.

For a white finish and for all light shades, white cement and white sand should be used.

In the July 1937 number of *Concrete* the following guide was given for the selection of colors and coloring.

For Blue shades, use a good grade of ultramarine blue, free from sulfur in its free state, and free from soluble salts and organic matter.

For Browns use chemically-pure brown oxide of iron with a minimum iron oxide content of 96 per cent. Burnt umber should never be used.

For Buffs use a synthetic oxide of iron with a minimum iron oxide content of 80 per cent, or a natural yellow oxide with a minimum iron oxide content of 35 per cent.

For Grays use small quantities of chemically-pure black iron oxide, or germantown lamp black, preferably black iron oxide. Common lamp black should never be used.

For Greens use chemically-pure green chromium oxide, free from sulfur in the free state, and free from soluble salts and organic matter.

For Pinks use small quantities of the colors here designated for the Red shades.

For Red shades such as light brick or terra cotta, use chemically-pure red oxide of iron with a minimum iron oxide content of 96 per cent, free from sulfur in its free state, and free from soluble salts and organic matter; or a natural Spanish red oxide of iron with a minimum iron oxide content of 82 per cent. Venetian red should be avoided.

For Slate effects, use the colors designated for the Grays, but in greater quantities.

Buy good coloring materials—avoid the cheap ones—and follow the manufacturer's instructions.

**Efflorescence.** The white deposit found on the surface of concrete and other masonry structures is called efflorescence. This deposit seems to be composed of soluble salts carried to the surface by water through the pores of the masonry and left there when the water evaporates. The amount of this deposit varies and seems to depend on the nature of the soluble materials and the condition of the atmosphere.

To guard against efflorescence in concrete construction, watertight concrete and watertight joints must be obtained. This is done with a good mix, proper placing and curing. It may be necessary to use metal in the joints to make them watertight. It is difficult to keep a wall clear of efflorescence with an earth fill behind it. This will be discussed in the following paragraph.

**Laitance.** Laitance is the soft material found on the top surface after the concrete has been deposited and set. It is not confined to the top of the concrete, but may be found against the side forms. When additional concrete is to be poured against an earlier pouring, this soft material should be removed. For example, if a floor is being poured and a stop form is erected at the end of the day's work, the following morning this form should be removed and the surface cleaned of all soft material right back to the good material. If a concrete wearing surface is to be applied to the top surface at a later date, the entire top surface must be removed so that the finish coat will have a bond with good concrete.

In constructing walls, it is important to clean the stop ends thoroughly before pouring the next section, as laitance is one of the causes for efflorescence and should be avoided. In constructing the Grand Coulee Dam, each block of concrete was thoroughly cleaned before the adjoining block was poured.

When concrete is deposited under water, laitance gives the water a milky appearance. After some time it settles on the concrete to make a weak joint if not removed.

## EXPANSION AND CONSTRUCTION JOINTS

**Expansion Joints.** Expansion joints are often required in concrete structures. This is especially true of long buildings. To be effective, the joint in the structural steel must extend through the floors, roof, exterior walls including the basement walls, and generally the footings.

In Fig. 154 at (*A*) and (*B*) are shown expansion joint details for reinforced concrete building walls. Copper (16-ounce) is used in both of these joints to secure a weather stop. An elastic cement also can be used in addition to the copper. The copper never should be straight, but shaped as shown in Fig. 154 at (*C*).

The size and number of joints required will depend on the number of degrees change in temperature anticipated. The thermal coefficient of expansion of concrete is .000006. Multiplying this coefficient by the number of degrees change in temperature and by the length of the building gives the change in length. For example, if a building is 100 feet long and there is a change of temperature of 100 degrees, then the change in length would be equal to

$$.000006\times100°\times100'\times12''=\tfrac{3}{4}''$$

Actually the change is less than these figures indicate, probably not more than half the amount shown by the example just given. Build-

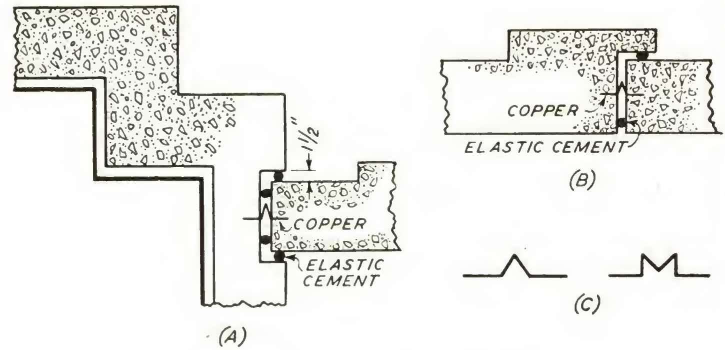



Fig. 154. Expansion Joints for Walls

ings 200 feet or less in length seldom are built with expansion joints, but longer buildings should have them.

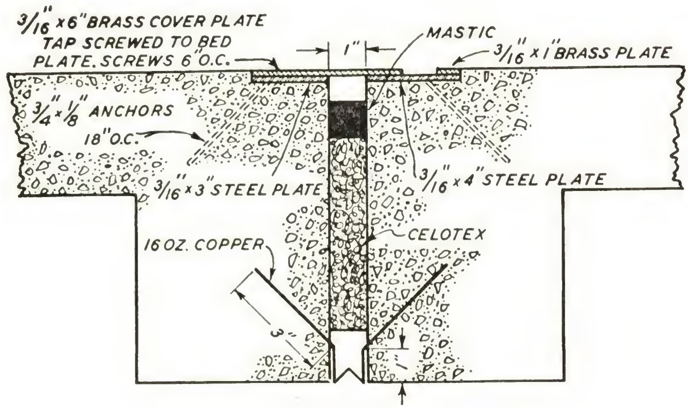



Fig. 155. Expansion Joint, Concrete Floor

Satisfactory floor joints are more difficult to make than wall joints. It is practically impossible to make a floor joint without a bump in the floor. A joint is especially bad when a watertight floor is required. In Fig. 155 is a floor joint suggested by the Portland Cement Association. Steel plates, $3''\times\frac{3}{16}''$ are anchored to the beams,

the anchors being welded to the plates. These plates are set so that a $\frac{3}{16}''$ brass plate, when placed on the steel plate, will be flush with the floor as shown in the figure. The brass cover plate is tap-screwed to the steel plate. In place of the copper at the bottom of the beam, a wood mold can be used having only one side secured to the concrete.

**Construction Joints.** Construction joints are generally necessary. It seldom occurs that an entire operation can be concreted at one operation. In floor construction, the joints are generally at the center of the span. At this point the shear is zero. While the maximum bending moments occur at this point, the steel provides for tensile stress and, since the joint is between the pourings of the concrete,

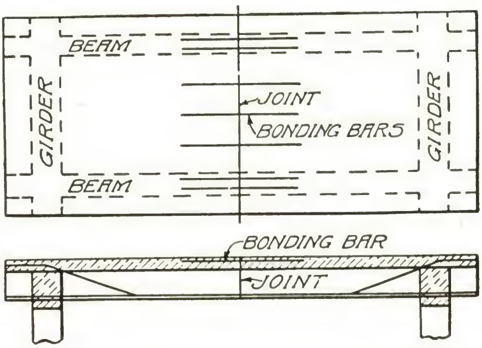



Fig. 156. Construction Joint in a Floor

it is tight and the compression is not affected, except for the fact that the joint may not be watertight.

In Fig. 156 is shown a joint in a typical floor panel. To assist in bonding the two pourings together, $\frac{3}{8}''$ round bars, spaced 12″ on centers are placed one inch from the top of the slab. Two bars are placed near the top of each beam. As soon as the concrete has set up, the stop at the end of the pourings should be removed and all soft material removed until the stone is exposed. Before concreting, the forms should be cleaned thoroughly and the exposed face of the previously poured concrete should be washed with cement grout and the new concrete well worked against it.

When structural steel is used for the framework, and reinforced concrete for the slabs and fireproofing, the joints in the slabs can be made over the beams as well as the center of the slabs. The fire-

proofing on one side of the beam or girder is completed and the other side is poured to a height of 2″ to 6″ above the top of the bottom flange at the first pouring. At the next pouring, the fireproofing is completed. This method is advantageous when the exposed surfaces are to be finished for painting. It is easier to finish a joint on the side of a beam or girder than at the center of the span of the slab, and it shows less in the finished work.

In pouring concrete walls, the construction joint should never be made straight. In both the vertical and the horizontal joints a groove should be made as shown in Fig. 157. The size of the groove

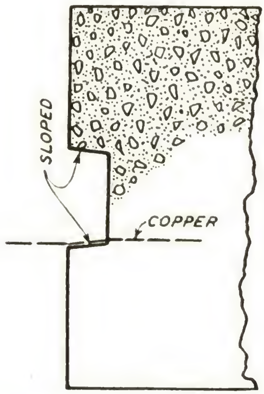



Fig. 157. Wall Construction Joint

will depend on the thickness of the wall, but it must not be less than 2″×4″ and much larger whenever possible. Before resuming concreting, the joint should be cleaned as described for floor construction. If a watertight joint is required, a piece of copper can be placed as shown by the dotted line in the figure.

## REPRESENTATIVE EXAMPLES OF REINFORCED CONCRETE WORK

**Dormitory at Bryn Mawr College.** Fig. 158 is an exterior view of a dormitory building constructed for Bryn Mawr College, Bryn Mawr, Pennsylvania. In constructing this building, reinforced concrete beams and slabs were used for the lower stories where plastered ceilings were not required; but for stories requiring plaster,

Fig. 158. Dormitory Building, Bryn Mawr College, Thomas & Martin, Architects

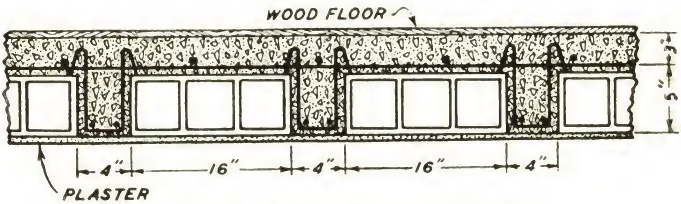



Fig. 159. Section—Floor Joists and Tile

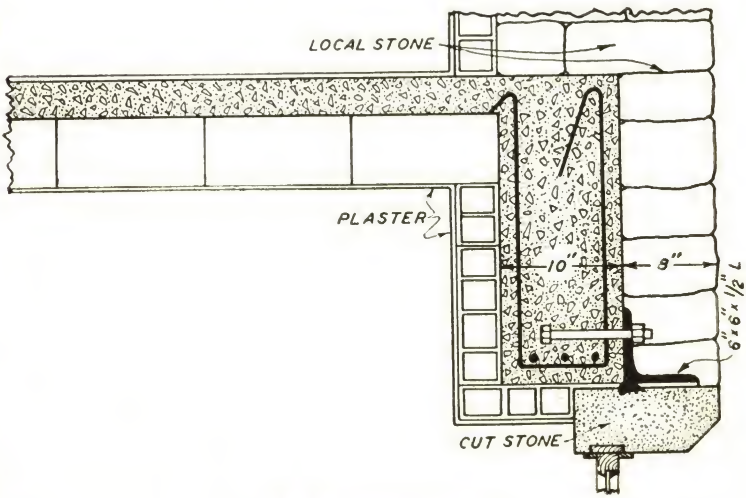



Fig. 160. Detail of Lintel

the floors were constructed of reinforced concrete joists with terra cotta tile placed between the joists, as shown in Fig. 159. The width and depth of the tile depended on the span and on the loading. In designing the building, a row of concrete columns was placed on each side of a center corridor in the building. These columns, where possible, were placed in closets or in corners of the rooms. The typical joists (spanning from the beams to the exterior walls and having a length of 13′6″) were 4″×8″, reinforced with two ½″ square bars. Tiles 3″ less in depth and 16″ wide were placed between the joists, making the center to center of joists 20″. The tops of the slabs were screeded and troweled so as to secure a smooth surface. The ¾″ wood floor was glued directly to the top of the slab.

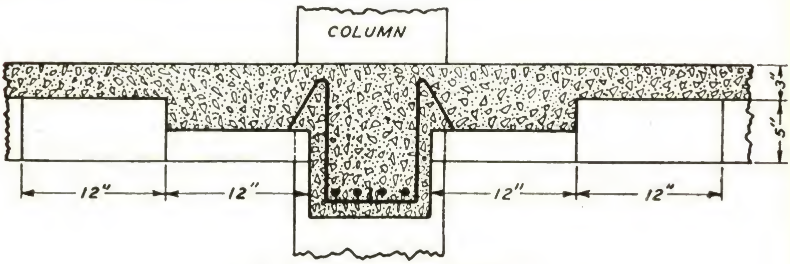



Fig. 161. Section—Typical Beam

The lintels, as shown in Fig. 160, were made to fill the space between the cut stone above the windows and the finished floor above.

In designing the beams, it was necessary to reduce the thickness of the tiles adjacent to the beam as shown in Fig. 161. This was done to secure the compression required for the beam. The slabs were made 3″ thick to permit the placing of the electric conduits in them. A slab of less thickness would provide sufficient compression for the joists, but extra depth was necessary for the conduits.

Floors of this construction often are used for buildings of this type. In using corrugated tile, a good surface is secured for bonding the plaster. This type of floor is economical for one-way spans up to about 14′ or 15′. For spans greater than 15′, the steel tile and joists become more economical than the terra cotta tile and joists. This is due to the lighter weight of the steel tile and joists compared with the terra cotta joists.

**Concrete Reservoir.** In 1930 a twenty-million-gallon reinforced concrete reservoir was completed by the city of Detroit, which contains many features of interest to the student and engineer. This work is described by Mr. Arthur B. Morrill in the October, 1931, *Journal of the American Concrete Institute.* It will be briefly described herein. The reservoir is 315′ by 455′, and was covered with a flat reinforced concrete slab designed for a live load of 500#/□′. Earth to the depth of 2′ was placed over the slab, and an allowance was made for trucks or other live load. Wood piles 40′ long were used to assist in supporting the load on the clay soil. It is interesting to note that the total load of the reservoir filled with water and covered with the earth fill was less than the weight of the material excavated. The piles assist in distributing the load and, in case of leakage, will be of assistance in preventing unequal settlement and cracks in the structure. The roof is supported by columns 20″ in diameter and spaced 20′ on centers in two directions.

This structure was designed for 2000-pound concrete, except for the roof, which was based on 2500-pound concrete. The sand was measured by the inundation method. A mix of 1:2:4 was specified and used at the start of the work, but this mix was found to be harsh and, after experimenting, the proportions for a workable concrete were found to be 1:2.37:3.64. The amount of water used in the concrete was varied somewhat, depending on where the concrete was placed, ranging from 5.6 gallons to nearly 8 gallons per bag of cement, with an average of about 7.12 gallons.

The specifications permitted a slump of 6″, but it was found necessary to increase the slump to 7″ for the columns. In the floor slab, a slump of 4″ was found satisfactory. All materials were carefully graded and inspected and tests were constantly made of the materials and the concrete. The crushing strength of the test cylinders made at the age of 28 days was about 2560#/□″.

The specifications stated that the leakage under full head of water should not exceed 100 gallons per minute. On tests after completion, the leakage amounted to 25 gallons per minute, and after a few days was 19 gallons. The wall forms were held together by placing a ½″ bolt through a 2″ pipe. When the bolts were removed, the pipe was plugged. Much of the leakage seemed to be through or around these pipes, as wet spots were found where they were located.

No special waterproofing was used for this reservoir, except that the top and the side walls for a depth of 6′ from the top were treated with asphalt to prevent the possibility of the surface water leaking through the joints of the slab. For the balance of the work, dependence was placed on securing a strong, impervious concrete to resist the leakage. Grooves were left in all construction joints and the reinforcing steel was carried continuously through all joints. In the bottom slab, grooves were left in all joints; into these grooves hot asphalt was poured after the concrete had dried out. The grooves were ½″ wide and extended down to the reinforcing steel.

**Engineering Building.** Recently the main portion of an Engineering Building was completed for the Pennsylvania State College at State College, Pa., Charles Z. Klauder, Philadelphia, being the architect. In the future, end wings are to be constructed. Fig. 162 shows a structural plan of one of the typical floors. The walls of the building are constructed of brick and limestone and the floors are of reinforced concrete. Because the major part of this building will be used for laboratories and classrooms, the structural work and finish were made as economically as possible. The concrete surfaces were rubbed and painted.

In making an examination of the structural plan, it will be noted that the front and rear elevations do not contain the same number of windows; also, the stairways (at each end) to some extent control the layout of the front of the building. In general, the beams are located in the center of the brick piers so that they are under the partitions. The beams having the greater amount of reinforcing steel show where the partitions are located. In laying out the columns it was desirable to have them opposite each other, as in the basement no partitions were used and in the sub-basement a hydraulic laboratory was located. Therefore, instead of the beams being framed directly into the columns, they are found to be partly on the columns and partly on the girder, or else entirely on the end of the girder. These girders are made deeper than usual in order to provide shear for the concentrated load near the columns.

The typical floors were designed for the live load of 75#/□′, and the weight of the floor finished was 25# in the classrooms and 30# in the corridor. The stress allowed in the reinforcing steel in tension was 18,000#/□″. The extreme fiber stress on concrete in compression

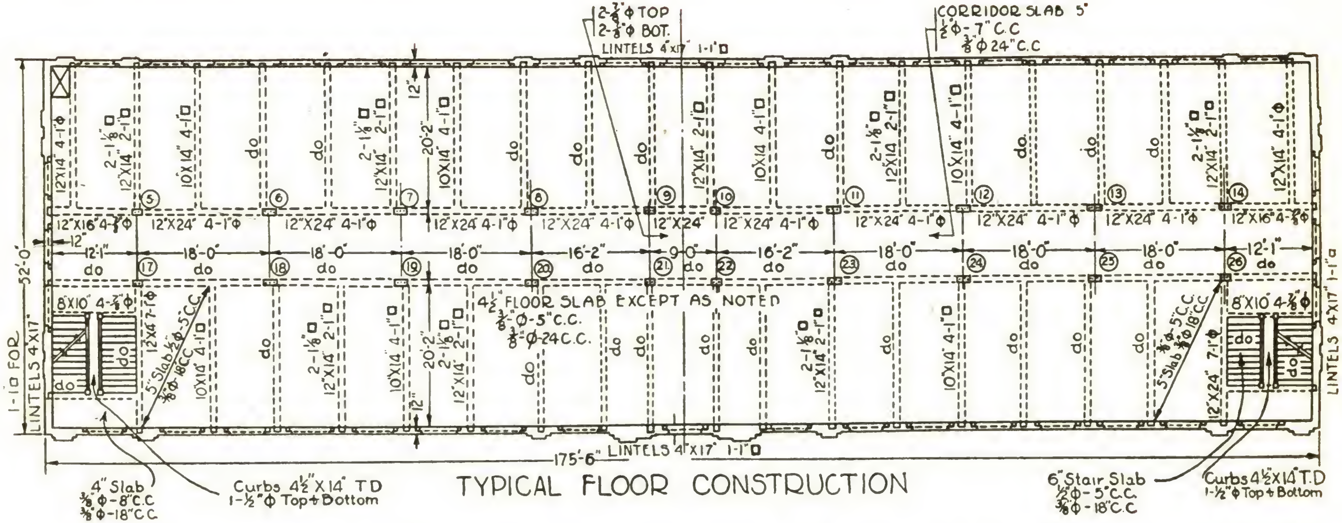



Fig. 162. Structural Plan of a Typical Floor in Engineering Building at Pennsylvania State College

was 650#/□″. The direct compression of the concrete in the column was 650# and on the reinforcing steel 8000#/□″. The mix of concrete for the floor was 1 part cement, 2½ parts sand, and 3½ parts stone. For the columns a mixture of 1 part cement, 1½ parts sand, and 2½ parts stone was used.

The only stone that could be secured at State College at a reasonable cost was limestone. This limestone has a good texture, is hard, and gave good results in the tests that were made. Specimens of the concrete secured from the floors were taken at each pouring and the ultimate strength at 28 days averaged approximately 2600#/□″. The stresses used in the design were based on a 2000-pound concrete for the floors and 2500-pound for the columns at the age of 28 days.

This building is three stories in height. As previously stated, there is a basement under the entire building and a sub-basement under half of it, which is used as a hydraulic laboratory. It often facilitates the construction of a building of this height to use columns in the exterior walls and support the brickwork on the spandrel beams constructed between the columns. In this case, the walls were constructed of solid masonry and had more than sufficient strength to support the floors and roof, therefore it seemed more economical to omit the wall columns and bear the beams on the walls. The delivery of the cut stone often controls the speed of the construction of a building.

A factory building of this size and height undoubtedly would have a complete framework of reinforced concrete. The floor loads would be greater, the amount of brickwork would be much reduced as the window area would be greater, therefore the columns may become a necessity. The exterior walls can be reduced to eight inches in thickness. Time can be saved in using a complete framework for a factory building.

To economize on the cost of the building, it was decided to rub the exposed surfaces in the ceiling rather than to plaster them. Planed lumber was used for all exposed surfaces of concrete and greater care was taken in constructing the form work to assist in getting an acceptable face. As soon as the forms were taken down, all projecting ridges were removed and the rough spots rubbed over with cement mortar composed of one part cement and five parts of very fine sand. After the concrete had hardened for 30 days or

Fig. 163. Interior of H. T. Heinz Company Warehouse, Showing Example of Flat-Slab Ceiling Construction
*Courtesy of The Condron Company, Chicago*

more, the entire surface was rubbed with an electric machine which produced fair results.

For another building being erected at State College, the contractor covered the slab forms with steel plates of about No. 16 gage. He claimed to have saved money in finishing the surfaces, as he had to touch them up in only a few spots. Before re-using the plates, all nail holes were sheared out and the plates were run through a tinner's roll, which put them in good condition for the next floor. The finish

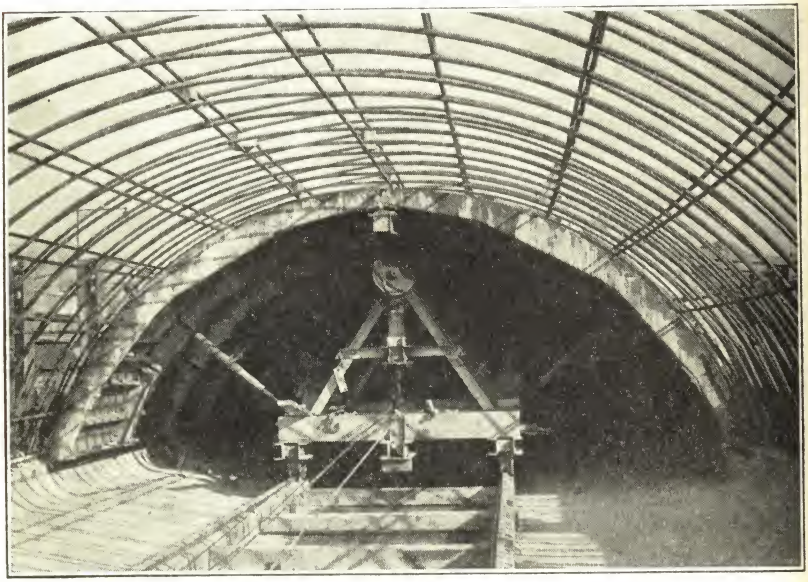


Fig. 164. Trunk Sewer, Borough of Queens, New York
*Courtesy of Blaw-Knox Company, Pittsburgh, Pa.*

secured by covering the wood forms with steel plates was very satisfactory, being much better than that obtained where the surfaced lumber was used and the concrete rubbed afterward.

**Heinz Warehouse.** In Fig. 163 is shown an interior view of a warehouse in which the floors are of the flat-slab construction. The bays are 18′6″ square and are designed for a live load of 300 pounds per square foot. The columns and column heads are octagonal and the drop panels are chamferred.

In Fig. 164 is illustrated a large trunk sewer in the Borough of Queens, New York City. The width was 22′ and the maximum

height was 10′. The material for the concrete was carefully graded so that a dense impervious concrete would be secured. The concrete was poured at two operations. After the bottom was concreted, a track was constructed for erecting the forms for the upper part of the sewer.

**Girder Bridge.** The reinforced-concrete bridge shown in Fig. 165A was constructed near Allentown, Pennsylvania. This type of bridge has been found to be economical for short spans. Worn-out wood and steel highway bridges are in general being replaced with reinforced-concrete bridges, and usually at a cost less than that of

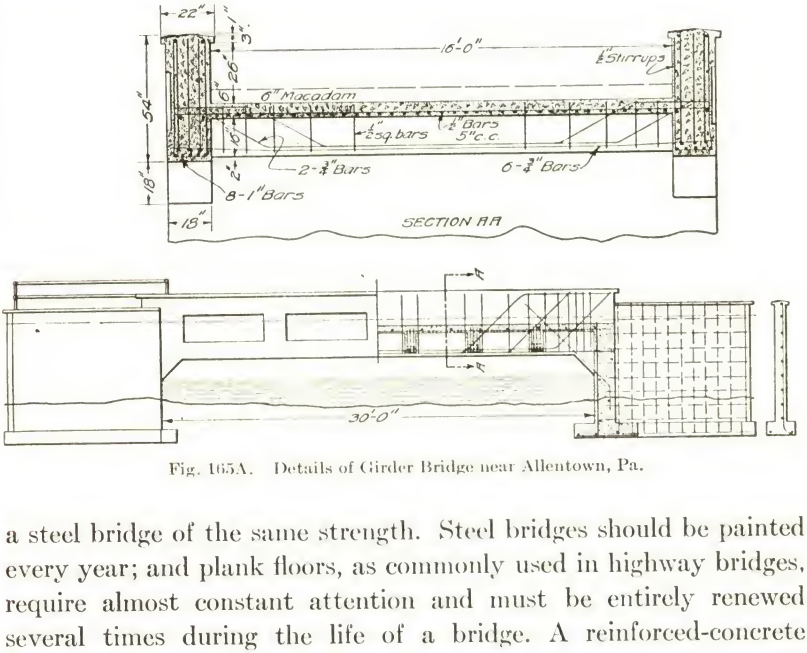



Fig. 165A. Details of Girder Bridge near Allentown, Pa.

a steel bridge of the same strength. Steel bridges should be painted every year; and plank floors, as commonly used in highway bridges, require almost constant attention and must be entirely renewed several times during the life of a bridge. A reinforced-concrete bridge, however, is entirely free of these expenses, and its life should at least be equal to that of a stone arch. From an architectural standpoint, a well-finished concrete bridge compares very favorably with a cut-stone arch.

The bridge shown in Fig. 165A is 16′ wide and has a clear span of 30′. It is designed to carry a uniformly distributed load of 150#/□′, or a steel road roller weighing 15 tons, the road roller having the following dimensions: the width of the front roller is 4′, and of each rear

roller, 20″; the two rear rollers are 5′ apart, center to center; and the distance between front and rear rollers is 11′, center to center; the weight on the front roller is 6 tons, with 4.5 tons weight on each of the rear rollers.

In designing this bridge, the slab was designed to carry a live load of 4.5 tons on a width of 20″ (when placed at the middle of the span), together with the dead load consisting of the weight of the macadam and the slab. The load considered in designing the cross-beams consisted of the dead load—weight of the macadam, slab, and

**Fig. 165B. Highway Bridge near Loudon, N. H.**
*Courtesy of Portland Cement Association*

beam—and a live load of 6 tons placed at the center of the span of the beam, which was designed as a **T**-beam. In designing each of the longitudinal girders, the live load was taken as a uniformly distributed load of 150#/□′ over one-half of the floor area of the bridge. The live load was increased 20 per cent over the live load given above, to allow for impact.

In a bridge of this type, longitudinal girders act as a parapet, as well as the main members of the bridge. The concrete for this work was composed of 1 part Portland cement, 2 parts sand, and 4 parts 1″ stone. Corrugated bars were used as the reinforcing steel.

When there is sufficient headroom, all the beams can be constructed in the longitudinal direction of the bridge, and are under the

slab. The parapet may be constructed of concrete; or a cheaper method is to construct a handrailing with 1½″ or 2″ pipe.

In Fig. 165B is shown a reinforced concrete highway bridge constructed near Loudon, N. H. It was designed by the New Hampshire State Highway Commission, John W. Childs bridge engineer, H. E. Langley designing engineer. The design is based on the continuous beam theory in which the girders are supported by individual piers.

The width of the bridge is 28′ supported by three girders spaced 9′6″ on centers. The center span is 56′ in length and the end spans are 42′ in the clear, the total length of the bridge being 143′6″. The design of the bridge was based on 3000-pound concrete with a minimum cement content of 1.60 barrels per cubic yard of concrete. The bridge cost $16,100.00 or $4.10 per square foot of deck.

TRUNK SEWER, BOROUGH OF QUEENS, NEW YORK

*Courtesy of Blaw-Knox Company, Pittsburgh, Pa.*

# CHAPTER XVIII

## BENDING DETAILS FOR REINFORCING STEEL

**Bending Details.** Drawings showing all the bending details of the bars, for all reinforced-concrete work, should be made before the steel is ordered. The designing engineer should detail a few of the

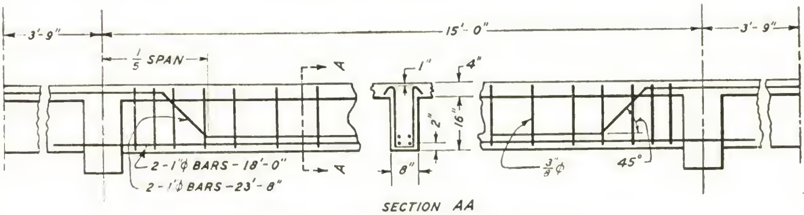



Fig. 166. Details of Beam Construction

| MARK | Nº OF BEAMS | Nº OF BARS IN EACH BEAM | SHAPE | STIRRUPS |
|---|---|---|---|---|
| B 2 | 64 | 2-1"φ 18'-0" | STRAIGHT | 3/4", 5 1/2", 17" |
| | | 2-1"φ 23'-8" | 5'-4", 2'-0", 45°, 9'-0", 2'-0", 5'-4", 1'-5" | 14-3/8" 4'-0" |

Fig. 167. Bending Details for Beams

typical beams and girders to show, in a general way, what length of bars will be required; the number of turned-up bars; the number, size, and spacing of stirrups required; and the dimensions of the concrete. These details will then be a guide for the construction engineer to make up the details required to properly construct the work. Fig. 166 shows the manner in which the designing engineer should detail a typical beam so that the constructing engineer can develop these details as shown in Fig. 167.

**Tables for Bending Bars.** A simple outfit for bending the bars cold consists of a strong table, the top of which is constructed as shown in Fig. 168. The outline to which the bar is to be bent is laid out on the table, and holes are bored at the points where the

bends are to be made. Steel plugs 5 to 6 inches long are then placed in these holes. Short pieces of boards are nailed to the table where necessary, to hold the bar in place while being bent. The bar is then placed in the position *AB*, Fig. 168, and bent around the plugs *C* and *D*, and then around the plugs *E* and *F*, until the ends *EH* and *FG* are parallel to *AB*. When bends with a short radius are required, the bars are placed in the vise, near the point where the bend is wanted, and the end of the bar is pulled around until the required angle is

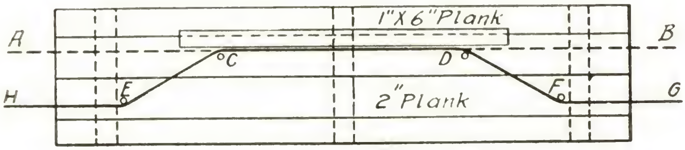



Fig. 168. Plan of Bending Table


Fig. 169. Type of Lever Bender

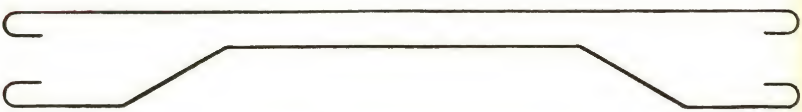


Fig. 170. Bars with Hooked Ends

secured. The vise is usually fastened to the table. The lever shown in Fig. 169 is also used in making bends of short radii. This is done by placing the bar between the prongs and pulling the end of the lever around until the required shape is secured.

**Bars with Hooked Ends.** Architects and engineers generally require that the ends of all the bars in the beams and girders shall be hooked as shown in Fig. 170. This is done to prevent the bars from slipping before their tensile strength is fully developed and to increase bond between steel and concrete.

**Slab Bars.** To secure the advantage of continuous slab construction, one-half of the bars are turned up over all supports such as beams, walls, etc., as shown in Fig. 81A. By cutting half of the bars to a length equal to the span of the slab plus one-fourth of the span of each adjoining slab and turning the ends up over the support-

ing beams from each adjoining slab, the same area of steel is secured over the beam as found in the center of the span.

The bending details for the slab bars are the same as shown for beam bars. For slabs 4 inches or less in thickness, straight bars are often shipped to the job, cut to the correct length and bent with a bending lever (Fig. 169) after being placed in position on the floor. Generally the bending is more satisfactorily done on a bending table

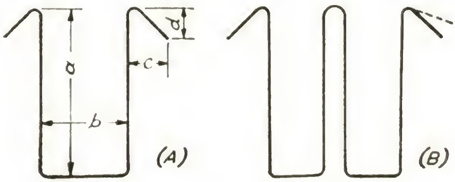



Fig. 171. Diagram Showing Bending Bars for Stirrups

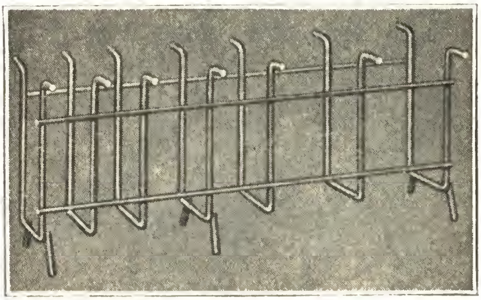


Fig. 172. Welded Stirrup Unit
*Courtesy of Universal Form Clamp Company*

(Fig. 168), or at a warehouse prepared for this work. The temperature bars are always straight.

**Stirrups.** Fig. 171 shows the bending of the bars for stirrups. The stirrups can be wired to the main bars in the beam or girder and the ends turned up into the slab. If the ends rest on the forms, they will not show on the finished slab unless there is moisture in the building. In that case, rust spots will show on the ceiling. The prongs may be cut one inch short and the stirrups held in place by wiring them to the main bars and to a quarter-inch bar at the top. The prongs or ends of the stirrups may be turned up so that they will be protected by two inches of concrete, as shown by the dotted line on the right prong in Fig. 171.

In Fig. 172 is shown a continuous welded stirrup unit for the end of a beam. Such a unit insures the correct spacing and position of all stirrups.

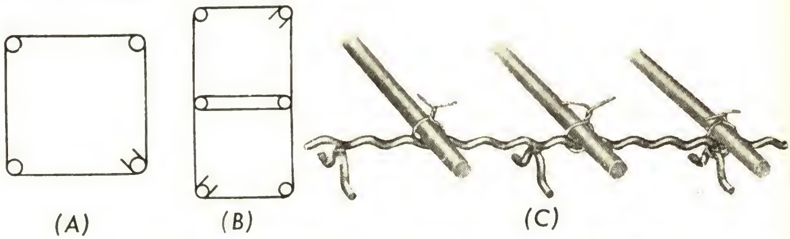



Fig. 173. (A) and (B) Column Bands and (C) Slab Bolster
(C) *Courtesy of Universal Form Clamp Company*

**Column Bands.** In Fig. 173 two types of column bands are shown. Fig. 173, at (A), shows bands for a square or a round column;

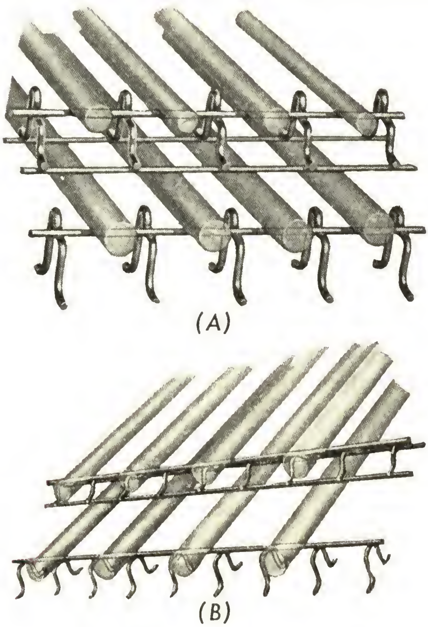



Fig. 174. (A) and (B) Beam Spacers
*Courtesy of Universal Form Clamp Company*

and Fig. 173 at (B), bands for a rectangular column. The bar which forms the band is bent close around each vertical bar in the column, and therefore assists in holding these bars in place. The bands for

the rectangular column ($B$) are made up of two separate bands of the same size and shape. Fig. 173, at ($C$), shows a slab bolster.

**Spacers.** All reinforcing steel must be accurately located in the

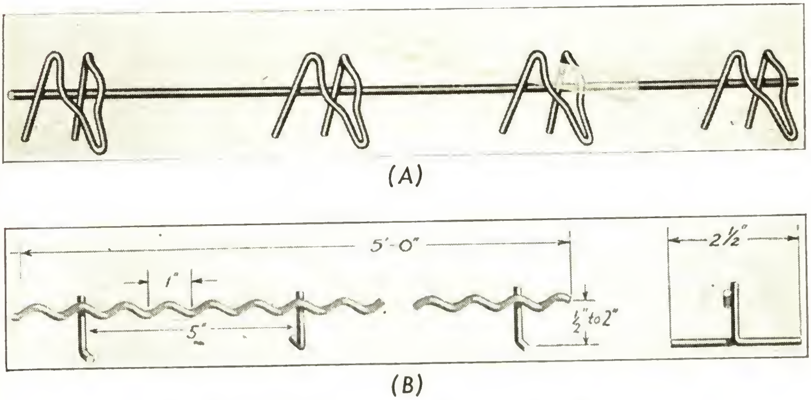



Fig. 175. At ($A$), Snap-in Slab Spacers; at ($B$) Corrugated Slab Spacers
*Courtesy of Union Steel Products Company*

forms and firmly held in place before and during the pouring of the concrete. This may be accomplished by making cement briquettes of the proper thickness for blocking up the beam and slab bars and

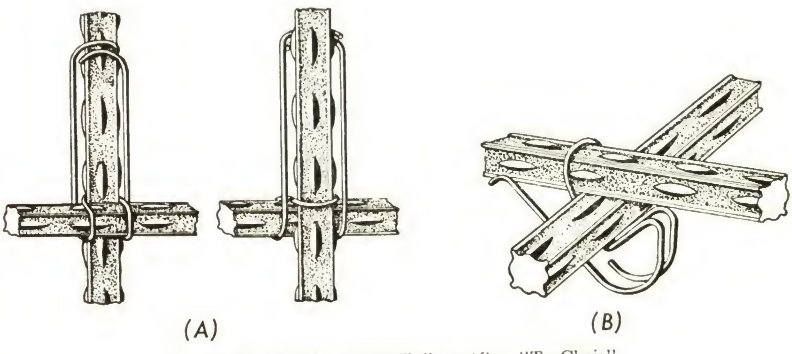



Fig. 176. At ($A$) a "Bar-Ty"; at ($B$) a "Ty-Chair"

doing such wiring as is necessary; or it may be accomplished by spacers. Many inexpensive devices are made for holding bars in place and are being used successfully.

In Fig. 174 are shown two types of beam spacers. Both types

can be ordered in the exact length required, or ordered in lengths of 5 feet and then cut to the desired length at the job.

Fig. 175 shows two types of slab spacers. In type (*A*), supporting clips are welded to the longitudinal bar to suit the spacing of the

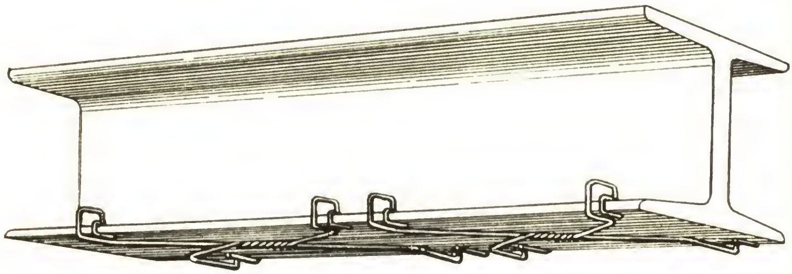


Fig. 177. Havemeyer X-Tension Clip

slab bars. The slab bars are snapped into the upper part of the vertical member and are well secured. In type (*B*), slab bars are wired to the corrugated horizontal bar. These spacers are made also with longer supporting legs and then used as a "hi-chair."

In Fig. 176 is shown a "Bar-Ty" (*A*) and "Ty-Chair" (*B*). The

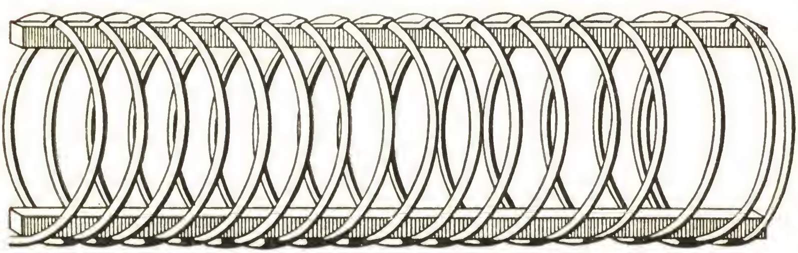


Fig. 178. Havemeyer Collapsible Spiral

"Bar-Ty" is used to fasten two bars together, while the "Ty-Chair" secures two bars and also supports them above the forms. They are made of spring steel and can be secured in several sizes.

**Beam and Column Ties for Structural Steel.** When structural steel columns and beams are fireproofed with concrete, the concrete must be bonded or tied to steel work with bars, mesh or clips. For columns of a moderate size, a small bar can be placed at each corner and held in position by quarter-inch ty-bars spaced 12 inches on

center. In fireproofing beams, two small bars (one-half inch or five-eighths inch in diameter) are placed below the lower flange of the beam and then the entire beam is wrapped with quarter-inch round bars spaced 12 inches apart. Special clips also are made to be attached to the column flanges and the bottom flange of the beams. They should be used only for the smaller size beams, 12 inches or

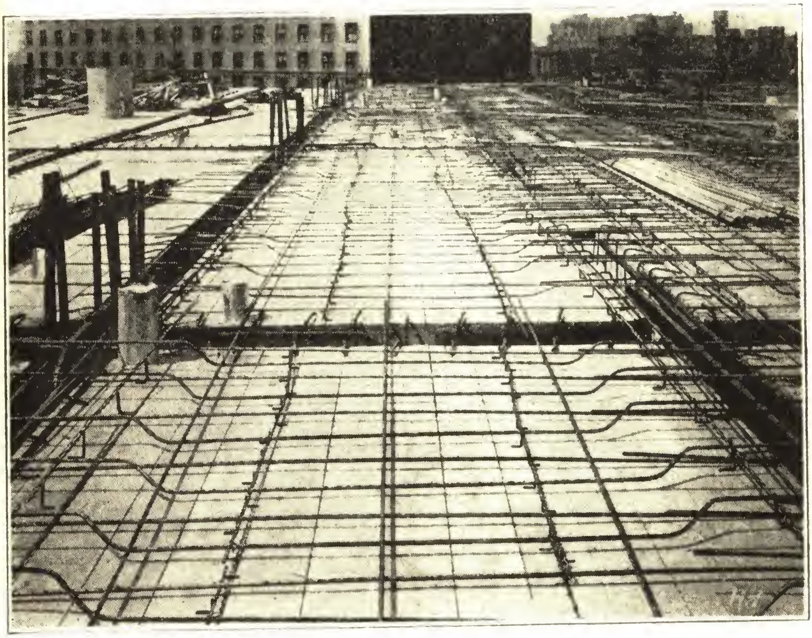


Fig. 179. Reinforcing Steel in Place, Ready for Concreting
*Courtesy of Union Steel Products Company*

under. Securing of the concrete to steel beams may be accomplished also by wrapping the bottom flange with wire mesh.

In Fig. 177 is an "X-Tension Clip" on the bottom flange of a steel beam that is to be fireproofed. They are used also for steel columns. The clips are 12 inches long, and should be practically continuous on the flange of a beam or column.

Fig. 178 shows a collapsible spiral column. The spiral is made up with bars of the desired size set to the necessary pitch and bent to suit the diameter of the column. The two longitudinal bars (shown in the illustration) are punched to suit the pitch of the spiral and are

clamped to the spiral to hold it in the proper position. It can be folded up for shipment to the job. If the column is large, additional spacing bars are added at the job. The vertical reinforcing bars are wired to the spiral either before or after it is placed in the column forms.

In Fig. 179 the reinforcing steel is shown in place, ready for concreting for a slab, beam and girder floor. Two rows of spacers are shown supporting the slab bars, and "hi-chairs" are seen near the beams holding the turned up ends of the bars. The chairs supporting the beam and girder bars cannot be seen in this figure.

## CHAPTER XIX

# FORM CONSTRUCTION

**General Requirements.** In actual construction work, the cost of forms is a large item of expense and offers the best field for the exercise of ingenuity. For economical work, the design of the structural members should consist of a repetition of identical units and the forms should be so designed that they will require a minimum of nailing to hold them, and a minimum of labor to make and handle them.

Forms usually are constructed of the cheaper grades of lumber. To secure a smooth concrete face, surfaced planks are used with the smooth side exposed to the concrete. Dry lumber is preferable to green, as it will not warp so badly in the sun.

If the surface of the planks placed next to the concrete is well oiled, the planks can be taken down much more easily, and, if kept from the sun, they can be used several times. Crude oil is an excellent and cheap material for greasing forms, and it can be applied with a whitewash brush. The oil should be applied every time the forms are used. The object is to fill the pores of the wood, rather than to cover it with a film of grease. Thin soft soap, or a paste made from soap and water, is also sometimes used.

In constructing a factory building of two or three stories, usually the same set of forms is used for the different floors; but when the building is more than three stories high, two or more sets of forms are required so that there will always be one set of forms ready to move.

The forms should be tight in order to prevent water and thin mortar from running through and carrying off the cement. This is accomplished by means of boards which are tongued-and-grooved, plywood, or presdwood.

Lumber for forms may be made of 1-inch, 1½-inch, or 2-inch planks. The spacing of sills depends in part upon the thickness of concrete to be supported, and upon the thickness of the boards on which the concrete is to be placed.

When concrete is to be vibrated, it is necessary for the forms to be much stronger than when spaded in the ordinary way.

## FORM WORK

**Forms for Floors.** In Fig. 180 is shown a typical layout for the

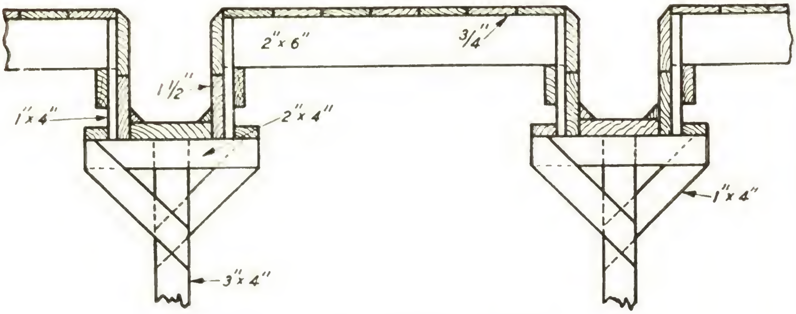



Fig. 180. Forms for Beams and Slabs

form work for slab and beam construction. The slab forms consist of 3/4-inch tongued-and-grooved boards 3 to 6 inches wide. The sides of the beams are made of 1½-inch material and the bottoms are usu-

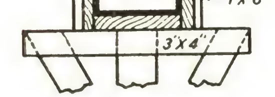



Fig. 181. Slab and Beam Forms Lined with Plywood or Masonite

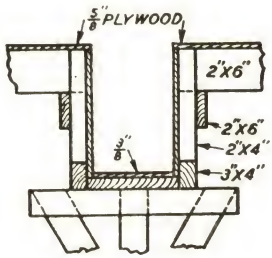



Fig. 182. Slab and Beam Forms Constructed of Heavy Plywood

ally 2 inches in thickness. These beams are supported by 3×4-inch struts and are braced as shown in Fig. 180. In addition to the bracing shown in the several figures for floor forms, the 3×4-inch struts must be braced horizontally in two directions. This bracing is usually placed about 6 feet above the floor and 1×6-inch planks are generally used.

In Fig. 181 is shown form work for floor construction in which a smooth ceiling is required and the slab and beam forms are lined with ⅜-inch plywood or 5/16-inch hard, tempered presdwood. The slab boards under the finished material need not be close together, but the

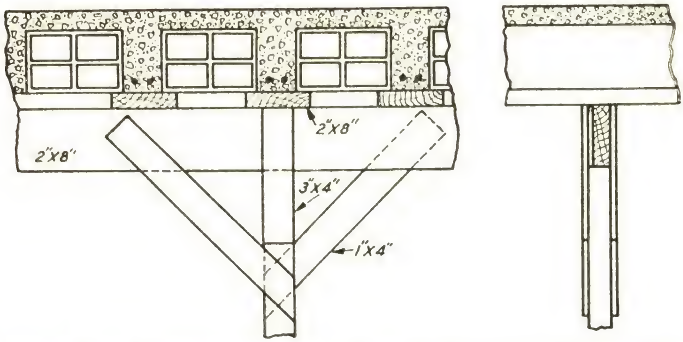



Fig. 183. Forms for Reinforced Concrete Joists and Terra Cotta Tile Construction

space between the boards should not be more than 3 to 4 inches. These boards are tacked to the materials supporting them.

In Fig. 182 are shown forms constructed of ⅝-inch plywood. The supporting material for the plywood must be placed sufficiently

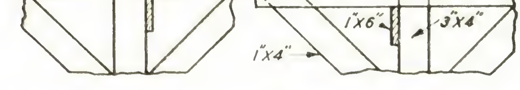



Fig. 184. Forms for Reinforced Concrete Joist and Steel Tile Construction

close to properly support the plywood. A wide plank or two pieces, 2×4", placed under the bottom form will assist materially in reducing the number of vertical struts.

Floors constructed of reinforced concrete joists with terra cotta tile between the joists are usually formed by placing under each joist a 2-inch plank of sufficient width to support the walls of the tile,

as shown in Fig. 183. The tiles are supported by 2×8-inch or 2×10-inch timbers and 3×4-inch supports. The spacing of these timbers depends on the weight of the floors and the span of the timber.

Forms for floors in which steel tile and concrete joists are employed are usually similar to the form work shown in Fig. 183

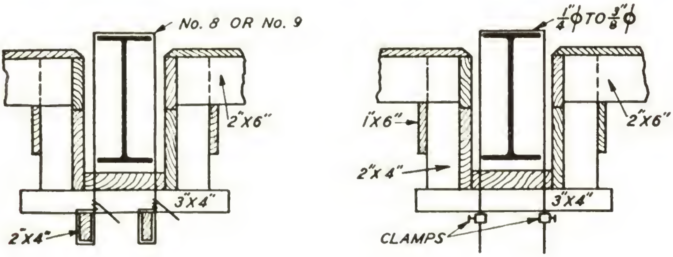



Fig. 185. Suspended Forms

for terra cotta tile. Different manufacturers of steel tile vary the details in their tile; therefore, variations must be made in the wood forms to suit these differences. The steel tile vary in width from 20 to 36 inches. For the 30-inch and 36-inch tiles, a line of supports should be placed under all joists as shown in Fig. 184.

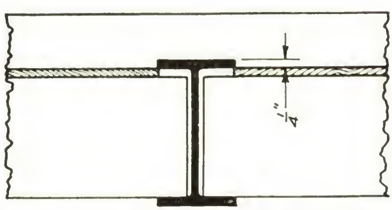


Fig. 186. Forms for Slab Supported by I-Beams

In Fig. 185 are shown the forms for a reinforced concrete slab and the fireproofing of steel beams and girders. These forms are supported entirely by the structural steel, being suspended by heavy wire or **U**-shaped bars spaced about 2 feet on centers. The size and spacing of the wires or the **U**-shaped bars will depend on the load to be supported.

Fig. 186 shows forms for a slab supported by steel beams. The top of the slab form should be set at least ¼ inch below the top of the beam so that the slab will hold the top of the beam in line when loaded.

This type of form is used often where slabs are placed over light-weight steel beams which are placed 2½ to 3 feet on centers.

**Forms for Columns.** For square or rectangular columns, forms are usually constructed of wood; but, for round columns, steel is generally used. In constructing wood columns, the vertical boards are one inch in thickness for an ordinary size column. They are

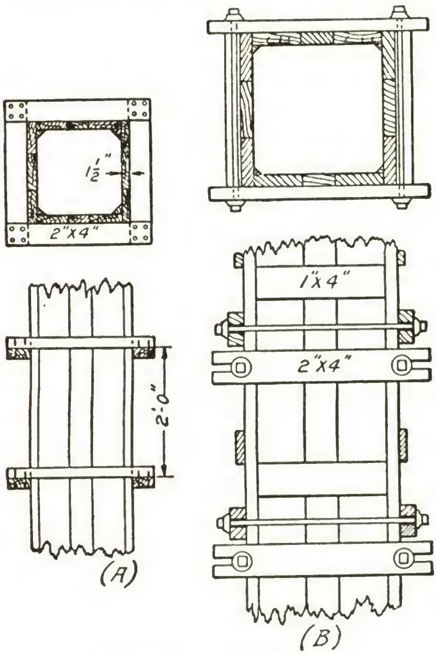



Fig. 187. Forms for Columns

clamped together with wood or steel on centers of about 2 feet. Column forms must be strong and tight to hold concrete. Poor forms will bulge, leak mortar, and even fail.

An opening is always left at the bottom of the column forms so that all shavings and sawdust can be removed. The opening is closed just before the column is concreted. The triangular strips shown in the corners of the forms may be omitted if square-cornered columns are preferred.

In Fig. 187 are shown column forms with two types of clamps. The type shown at (*B*) is preferable to that shown at (*A*). They can be tightened if necessary and can be removed easily when the con-

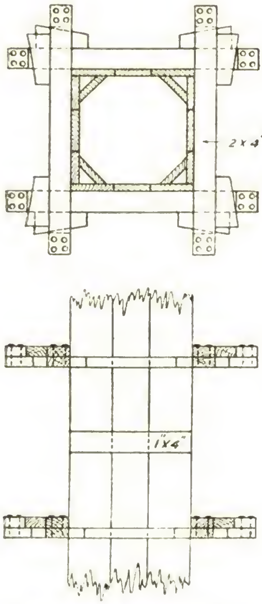



Fig. 188. Column Forms

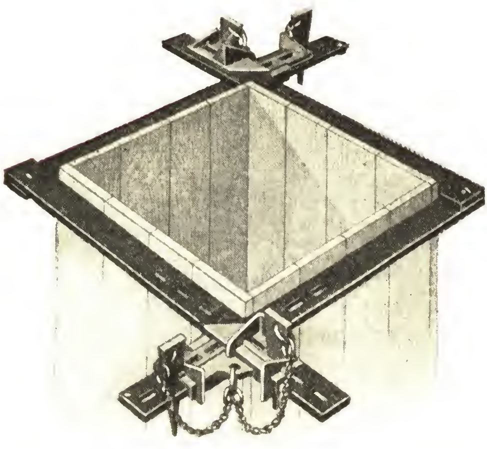


Fig. 189. New Ever-Square Column Clamp
*Courtesy of Symons Clamp & Manufacturing Company, Chicago*

crete has hardened. In the form shown at (A), the boards are destroyed after being used a few times.

Fig. 188 shows the forms for columns. The planks for each side of the column are held together by the $1\times4$-inch strip, and when erected in place, are clamped by the $2\times4$-inch strip.

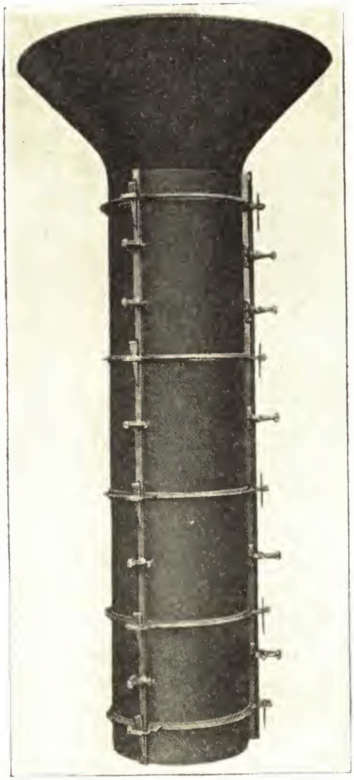


Fig. 190. Typical Column Form Assembly
*Courtesy of Bethlehem Steel Company, Chicago*

Fig. 189 shows the new ever-square column clamp, which is extensively used. These clamps are quickly placed around the column form and are more dependable than wood clamps which are nailed together. The use of steel clamps prevents the wear and tear of nailing into the column forms, therefore assisting in keeping the lumber used in the main shaft of the form in better shape.

Forms for round columns are always constructed of lightweight

steel plates. The column shafts are made in halves to simplify assembling and removing them. The metal plates are held by steel rings that are clamped together. They are easily erected and very smooth concrete faces are secured if the concrete is properly spaded.

Fig. 191. Deslauriers Column Form
*Courtesy of Deslauriers Metal Products Company, Detroit, Michigan*

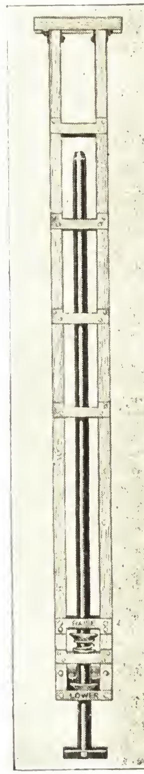


Fig. 192. Red Star Jack-Shore
*Courtesy of Red Star Products Corporation, Cleveland, Ohio*

Fig. 190 shows a steel form as manufactured by the Bethlehem Steel Company. This figure shows the flared head which is used on flat-slab construction. The head of the form is constructed in two sections, the same as the shaft. The flanges which bear on the wood forms of the floor are of a minimum thickness so as to eliminate, as much as possible, a break in the ceiling surface between the metal and wood forms.

Fig. 191 shows the Deslauriers column form. The outlines of this form are those followed in designing flat-slab construction. These forms can be secured with straight lines or molded effects.

Adjustable shores are now being much used instead of the 3×4-inch struts that were commonly used. These shores are adjustable in length, well constructed, and can be used for a long time if proper care is taken of them.

Fig. 192 shows the Red Star Jack-Shore as manufactured by the Red Star Products Corporation. This shore is automatic in its operation and can be raised and lowered by the use of an ordinary pinch bar. It is quickly erected or lowered and makes a substantial support. In building work these shores are used in place of the 3×4-inch struts shown in Fig. 180.

**Removal of Forms.** The length of time required for concrete to set depends upon the weather, the consistency of the concrete, and the strain which is to come on it. In good drying weather, and for very light work, it is often possible to remove the forms in 24 to 36 hours after placing the concrete, if there is no load placed on it. The setting of concrete is greatly retarded by cold or wet weather. Forms for concrete arches and beams must be left in place longer than in wall work, because of the tendency to fail by rupture across the arch or beam. In small circular arches, like sewers, the forms may be removed in 18 to 24 hours if the concrete is mixed dry; but in 24 to 48 hours if wet concrete is used. Forms for large arch culverts and arch bridges are seldom taken down in less than 28 days. The minimum time for the removal of forms should be:

For bottom of slabs (span 6 feet or less) and for sides of beams and girders, 5 days plus one additional day for each extra foot.

For bottom of beams and girders, and for spans 20 feet or less, 14 days.

For columns, 3 days.

For walls, not loaded, 1 to 2 days.

For bridge arches, 28 days.

When high early strength cement is used, the forms may be removed in less time than that given. With good curing conditions, the forms usually are removed 2 to 3 days after the concrete is placed. The time of removal of forms will not only depend on curing conditions, but on the length of the spans and general type of construction.

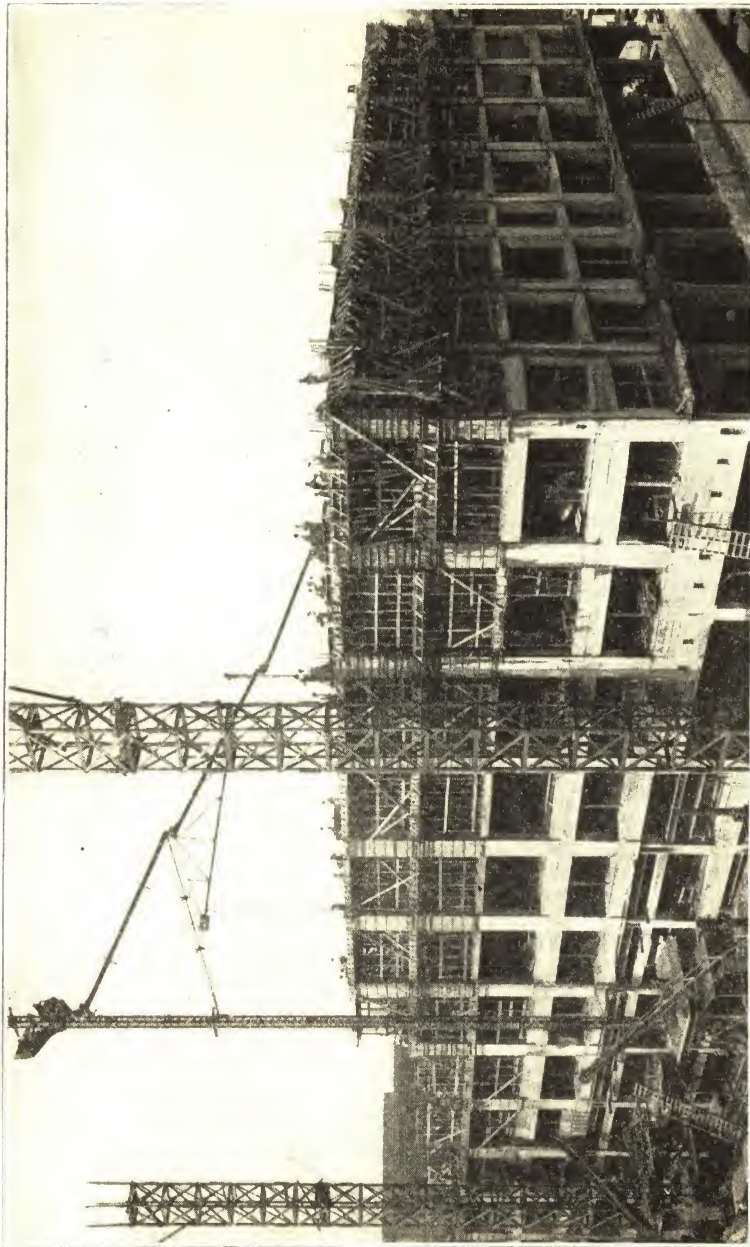


Fig. 193. Pennsylvania Railroad Freight Station, Philadelphia
*Courtesy of United Engineers & Constructors*

All concrete or reinforced concrete should be examined thoroughly before any forms are removed, and the engineer should be satisfied with the condition of the concrete before permission is given for the removal of the forms. All reshoring required should be promptly erected. At least two floors should be shored below the floor being concreted, unless a satisfactory support is secured with less shoring.

In Fig. 193 is shown the form work for columns, spandrel beams, etc., for a building. The planks used for the column forms are secured by battens and then clamped together with steel column clamps to resist the pressure when the concrete is poured. The struts supporting the beams are horizontally braced in two directions. Note also the bracing for the spandrel beam in the upper part of the picture.

## COST OF FORMS FOR BUILDINGS

(Revised by Mr. R. E. Lamb, February, 1930.)
(Reviewed by Mr. R. E. Lamb, September, 1938.)

An analysis of the cost of forms for an eight-story building is given by R. E. Lamb in *Concrete Engineering.* The basis of his estimate is made on using 7/8-inch by 6-inch tongued-and-grooved lumber for slab forms; 1¾-inch dressed plank for the sides and bottom of the beams and girders; and posts 4 by 4 inch, spaced 6 feet center to center. He makes the further assumption that it costs $35 per thousand feet of lumber to make and set one floor of forms; that it cost $28 per thousand feet of lumber to strip the forms and reset them on the next floor; and that it cost about $15 per thousand feet to strip the forms and lower them to the ground.

With the size of the beams and girders as shown in Fig. 194, Mr. Lamb states that it will take an average of 4 feet, board measure, to erect each square foot of floor area. The basis of his estimate is as follows: That 1.5 board feet of lumber per square foot of floor are required for the slab; that for every square foot of beam surface, including the bottom, 3.2 board feet per square foot are required; and that for each square foot of girder, including the bottom, 3.6 board feet of lumber are required. Taking these figures, for the panel shown, the slab will require 1.5 board feet per square foot; the beams, which are 8- by 18-inch, will have 3 feet 8 inches of surface per lineal foot; and multiplying this by 3.2 board feet per square

foot and dividing by 7.5 feet, the distance center to center of beams, we find that 1.56 board feet per square foot of floor surface are required.

Taking the girder in the same way, with 4 feet 8 inches of surface, multiplied by 3.6 board feet, and divided by 18 feet, the distance center to center of girders, we find that .94 board foot per square foot of floor is required. The total of the lumber required, then, is 1.5 board feet for the slab, 1.56 board feet for the beam, and .94 board foot for the girders—a total of 4 board feet per square foot of floor area.

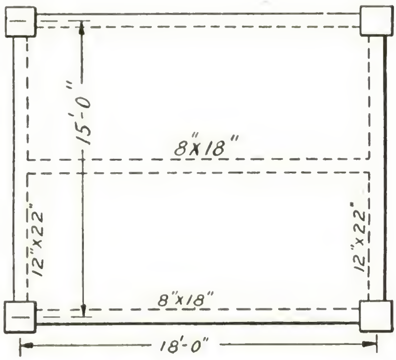



Fig. 194. Diagram of Forms

In this estimate for an eight-story building, three sets of forms were used:

| | | | |
|---|---|---|---|
| Roof | Stripping the sixth floor, resetting, altering to form valleys, and finally stripping roof and lowering forms to ground, 4 board feet at 4.8 cents | | $0.192 |
| Eighth Floor | Stripping the fifth floor, resetting, and finally stripping and lowering forms to ground, 4 board feet at 4.3 cents | | .172 |
| Seventh Floor | Stripping the fourth floor, resetting, and finally stripping and lowering forms to ground, 4 board feet at 4.3 cents | | .172 |
| Sixth Floor | Cost, same as for the fourth floor | | .132 |
| Fifth Floor | Cost, same as for the fourth floor | | .132 |
| Fourth Floor | Stripping the first floor, and resetting, 4 board feet at 3.3 cents | | .132 |
| Third Floor | Cost, same as for the first floor | | .292 |
| Second Floor | Cost, same as for the first floor | | .292 |
| First Floor | Making and setting forms, 4 board feet at 3.5 cents | $0.14 | |
| Material | 4 board feet at 3.8 cents | .152 | .292 |
| | | | 9)1.808 |

Average cost per square foot of surface ........ $0.201

To this average cost of 20.1 cents, 10 per cent should be added for waste, breakage, nails, etc.; and if two sets of forms are used, the third floor would cost 13.2 cents per square foot and the seventh floor 13.2 cents, giving an average of 17.9 cents per square foot.

In estimating the cost of the forms for the columns, it is assumed that making and placing the forms for the basement columns will cost about $42 per thousand; the cost of stripping and resetting, $30 per thousand and 3.1 square feet of lumber are required for each square foot of column surface.

| | | | |
|---|---|---|---|
| Eighth Story | Stripping sixth story, resetting and altering, finally stripping eighth story and lowering to ground, 3.1 board feet at 4 cents | | $0.124 |
| Seventh Story | Stripping fifth story, resetting, and finally stripping and lowering to ground, 3.1 board feet at 3.6 cents | | .112 |
| Sixth Story | Cost, same as second story | | .093 |
| Fifth Story | Cost, same as second story | | .093 |
| Fourth Story | Cost, same as second story | | .093 |
| Third Story | Cost, same as second story | | .093 |
| Second Story | Stripping basement columns and resetting, 3.1 board feet at .03 cents | | .093 |
| First Story | Cost, same as for the basement columns | | .248 |
| Basement | Material, 3.1 board feet at 3.8 cents | $0.118 | |
| | Making and setting, 3.1 board feet at 4.2 cents | .13 | .248 |
| | | | 9)1.197 |
| | Average cost per square foot of surface (cents) | | 13.3 |

To this average cost of 13.3 cents per square foot of column surface, should be added 10 per cent for bolts, nails, waste, etc. If three sets of forms are required, the second-story cost would be 24.8 cents and the sixth floor 9.3 cents, giving an average cost per square foot of 15.2 cents.

The student should remember that this lumber has a value after it has been removed from the building, and that this value should be deducted from the total to find the actual cost of the forms.

## WALL FORMS

Forms for all walls should be well constructed. They must be strong and kept in good alignment while the concrete is being poured. The pressure that is exerted on them is unknown, but some experiments that have been made indicate that the forms should be designed for a lateral pressure to withstand a hydrostatic pressure of a liquid of about 125 pounds a cubic foot. When concrete is placed at a

rapid rate and with a normal temperature, the maximum pressure will occur about thirty minutes after the concrete is placed. With a slow rate of pouring and a temperature of about fifty degrees, the maximum may be anticipated in 1½ hours to 2½ hours. If the concrete is vibrated, the pressure will be increased.

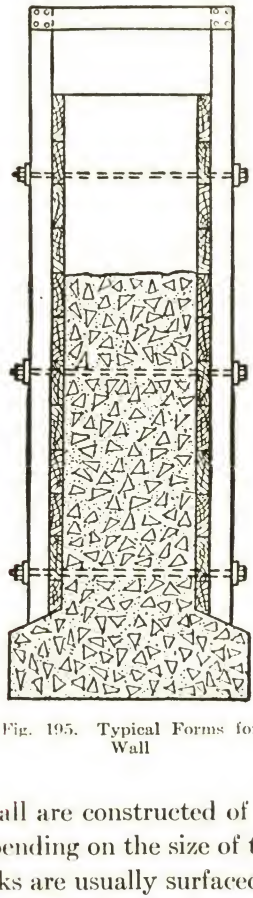


Fig. 195. Typical Forms for Wall

The forms for a wall are constructed of planks one inch to two inches in thickness, depending on the size of the wall and the spacing of the studs. The planks are usually surfaced on the side exposed to the concrete. The face forms may be lined with presdwood or plywood if a smooth finish is desired. The vertical timbers may be 4×4 inches; 4×6 inches or larger, depending on conditions and the size of the wall. These are tied together with bolts, the bolts being removed from the finished wall and the holes filled with concrete. If the bolts are well oiled before the concrete is placed, it will assist in

their removal. Special bolts are made for this purpose and are illustrated in this text. Fig. 195 shows in detail the form work for a wall as described here. Steel forms, backed with wood and plywood forms are manufactured and are successfully used. They are generally used on a rental basis.

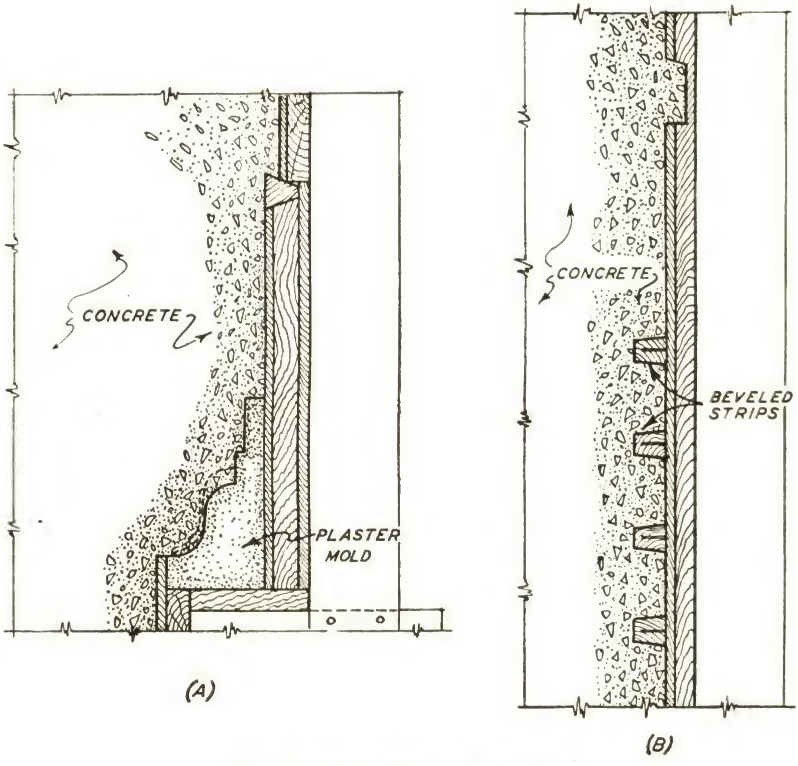



Fig. 196. Plaster Mold for Form Work

**Forms for Concrete Building Walls.** When buildings are constructed with monolithic concrete walls on which there are architectural details in the way of moldings, etc., the form work must be carefully constructed of kiln-dried lumber. The forms should be lined with presdwood, plywood, or similar material, to assist in securing a smooth and true surface. For complicated moldings, plaster molds are used. Fig. 196 (*A*) and (*B*) show some of these details.

The forms at the joints must be tight to prevent the leakage of

mortar and must be located where least noticeable in the finished work. Horizontal rustications make the location of horizontal joints a simple matter. The rustications are formed by nailing triangular strips on the forms and extending the joints through the wall at the center of any of these strips. Vertical joints can generally be concealed by locating them at re-entrant angles of the walls. All joints must be level.

In Fig. 197 are shown details for horizontal joints. Fig. 197 at (*A*) shows the completion of a pouring. The concrete is poured slightly above the bottom of a stop strip, say ¾ inch or more, and is

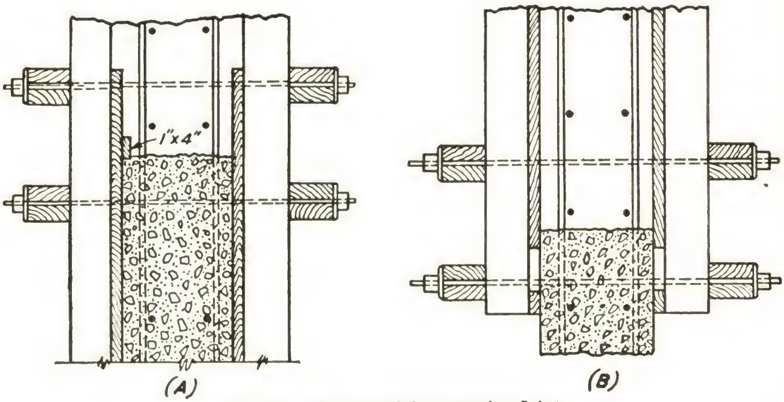


Fig. 197. Horizontal Construction Joint

allowed to settle for at least one hour. The top material is then removed to the bottom of the stop strip, and the surface is floated. If ⅜-inch bolts are used for the first operation, they can be replaced with 5/16-inch or ¼-inch bolts when the forms are moved up for the next operation. In Fig. 197 at (*B*) the forms are shown moved up ready for the next pouring.

**Steel Forms.** In Fig. 198 are shown steel forms being used for the construction of the Southwest Side Sewage Disposal Plant at Chicago. These forms were constructed entirely of steel made up in units and were handled by overhead cables supported by steel towers. For jobs of this character, steel forms made up in units would be more economical than wood forms.

Several manufacturers of specialties for concrete work have

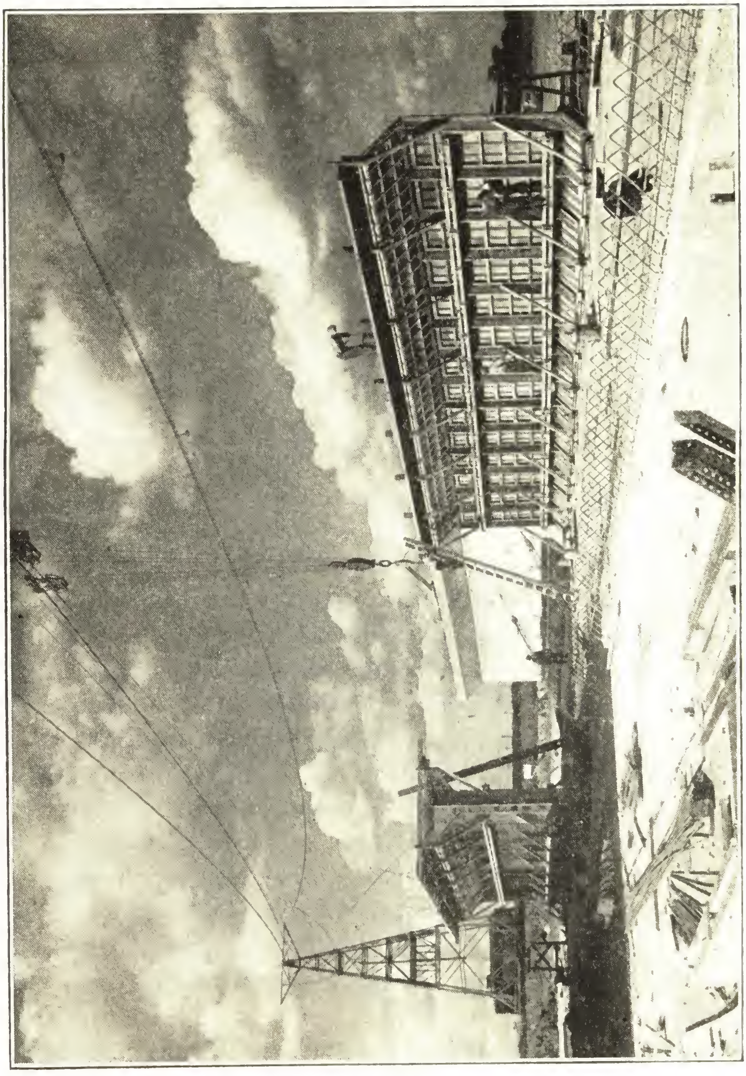


Fig. 198. Southwest Side Sewage Disposal Plant, Chicago
*Courtesy of Blaw-Knox Company*

devised special bolts or clamps for securing wall forms. In Fig. 199 is shown a "clamp assembly" made by the Universal Form Clamp Co. It consists of two clamps, two outside bolts drilled and tapped for an inside threaded bolt. The length of the bolts allow for necessary adjustment of the forms. The forms are bored at the desired points and the bolts are easily inserted. To remove the forms, the outer bolts are removed; the center bolts can be left in the wall or removed.

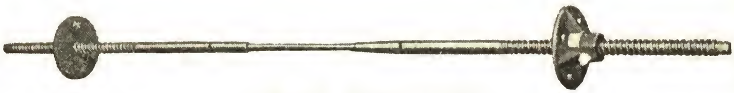


Fig. 199. Clamp Assembly
*Courtesy of Universal Form Clamp Company*

Where watertight walls are required, the center bolts are usually left in the walls and the holes are filled with mortar.

## FORMS FOR SEWERS

**Forms for Conduits and Sewers.** Forms for conduits and sewers must be strong enough not to give way, or to become deformed, while the concrete is being placed and rammed; and must be rigid enough not to warp from being alternately wet and dry. They must be constructed so that they can readily be put up and taken down, and can be used several times on the same job. The forms must give a smooth finish to the interior of the sewer. This has usually been done by covering the forms with light-weight sheet iron.

These forms are usually built in lengths of 16 feet, with one center at each end, and with three to five—depending on the size of the sewer or conduit—intermediate centers in the lengths of 15 feet. The segmental ribs are bolted together. The plank for these forms are made of 2- by 4-inch material, surfaced on the outer side, with the edge beveled to the radius of the conduit. The segmental ribs are bolted together, and are held in place by wood ties 2 by 4 inches or 2 by 6 inches.

**Forms of Torresdale Filters.** In constructing the Torresdale filters for supplying Philadelphia with water, several large sewers and conduits were built of concrete and reinforced with expanded metal. In section, the sewers were round and the conduits were horseshoe-shaped, with a comparatively flat bottom. The sewers were 6 feet

and 8 feet 6 inches, respectively, in diameter, and the forms were constructed similarly to the forms shown in Fig. 200, except that at

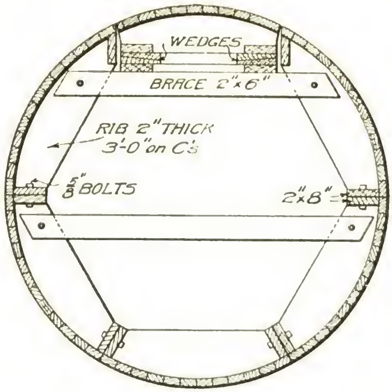



Fig. 200. Center for Round Sewer

the bottom the lower side ribs were connected to the bottom rib by a horizontal joint, and the spacing of the ribs was 2 feet 6 inches,

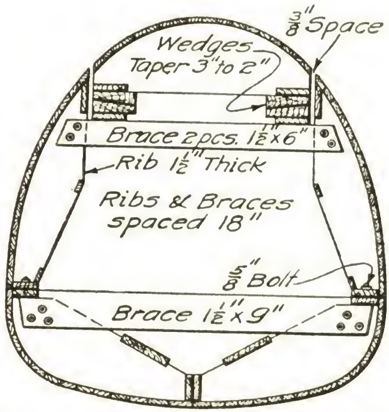



Fig. 201. Form for Construction of Horseshoe-Shaped Conduit

center to center. Fig. 201 shows the form for the 7-foot 6-inch conduit. The centering for the 9-foot and 10-foot conduits was constructed similarly to the 7-foot 6-inch conduit, except that the ribs

were divided into 7 parts instead of 5 parts as shown in Fig. 201. The spacing of the braces depended on the thickness of the lagging. For lagging 1 inch by 2½ inches, the braces were spaced 18 inches, center to center; and for 2- by 3-inch lagging, the spacing of the bracing was 2 feet 6 inches.

Fig. 202. Steel Forms for Sewer, McKeesport, Pa.
*Courtesy of Blaw-Knox Company*

These forms were constructed in lengths of 8 feet. The lagging for the smaller sizes of the conduits was 1 inch by 2½ inches, and for the larger sizes 2 by 3 inches; all of this was made of dressed lumber and covered with No. 27 galvanized sheet iron. The bracing of the forms was arranged to permit the centering to be taken apart and brought forward through the sections set in front of it. Three

sets of these forms were required for each conduit. The specifications required that the centering be left in place for at least 60 hours after the concrete had been placed. It was also required that this work should be constructed in monolithic sections—that is, the contractor could build as long a section as he could finish in a day—and that the sections should be securely keyed together.

**Steel Forms.** Steel forms are extensively used for sewer construction, large water conduits and similar work. The plates exposed to the concrete are usually about 1/8 inch in thickness and are carefully shaped to the design of the structure being erected. These plates are held in place by a framework of steel angles, turnbuckles, etc. They are always made so that they can be removed easily and can be used over and over again. Before being used, they should be cleaned and oiled carefully so that the new concrete will not stick to any old concrete on the forms.

In Fig. 202 is shown a steel form which was used in constructing a sewer at McKeesport, Pa. It was made up in units and moved on a track constructed for this purpose.

## FORMS FOR CENTER OF ARCHES

**General Specifications.** The centers for stone, plain concrete, and reinforced-concrete arches are constructed in a similar manner. A reinforced-concrete arch of the same span and designed for the same loading will not be so heavy as a plain concrete or stone arch, and the centers need not be so strong as for the other types of arches. One essential difference in the centering for stone arches and that for concrete or reinforced-concrete arches is that centering for the latter types of arches serves as a mold for shaping the soffit of the arch ring, the face of the arch ring, and the spandrel walls.

The successful construction of arches depends nearly as much on the centers and their supports as it does on the design of the arch. The centers should be as well constructed and the supports as unyielding as it is possible to make them. When it is necessary to use piles, they should be as well driven as permanent foundation piles, and the load, in most cases, should not be heavier than that on permanent piles.

**Classes of Centers.** There are two general classes of centers—those which act as a truss; and those in which the support, at the

intersection of braces, rests on a pile or footing. Trusses are used when it is necessary to span a stream or roadway. Sometimes the length of the span for the centering is very short, or there are a series of short spans, or the span may be equal to that of the arch. The trusses must be carefully designed, in order that the deflection and deformation due to the changes in the loading will be reduced to a minimum. By placing a temporary load on the centers at the crown, the deformation during construction may be very greatly reduced. This load is removed as the weight of the arches comes on the centers.

The lagging for concrete arches usually consists of 2×3-inch or 2×4-inch plank, either set on edge or laid flat, depending on the thickness of the arch and the spacing of the supports. The surface on which the concrete is laid is usually surfaced on the side on which

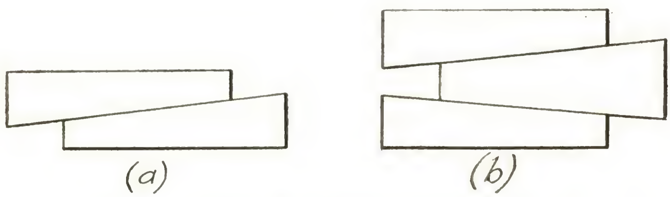



Fig. 203. Wedges Used in Placing and Removing Forms

the concrete is to be placed. The lagging is very often supported on ribs constructed of 2×12-inch plank, on the back of which is placed a 2-inch plank cut to a curve parallel with the intrados. These 2×12-inch planks are set on the timber used to cap the piles, and are usually spaced about 2 feet apart. All the supports should be well braced. The centers should be constructed to give a camber to the arch about equal to the deflection of the arch when under full load. It is, therefore, necessary to make an allowance for the settlement of centering, for the deflection of the arch after the removal of the centering, and for permanent camber.

The centers should be constructed so that they can easily be taken down. To facilitate the striking of centers, the practice is to support them on folding wedges or sand boxes. When the latter method is used, the sand should be fine, clean, and perfectly dry, and the boxes should be sealed around the plunger with cement mortar. Striking forms by means of wedges is the commoner method. The

type of wedges generally used is shown in Fig. 203 at (*a*), although sometimes three wedges are used, as shown by Fig. 203 at (*b*). They are from 1 to 2 feet long, 6 to 8 inches wide, and have a slope of from 1:6 to 1:10. The centering is lowered by driving back the wedges; and to do this slowly, it is necessary that the wedges have a very slight taper. All wedges should be driven equally when the centering is being lowered. The wedges should be made of hardwood, and are placed on top of the vertical supports or on timbers which rest on the

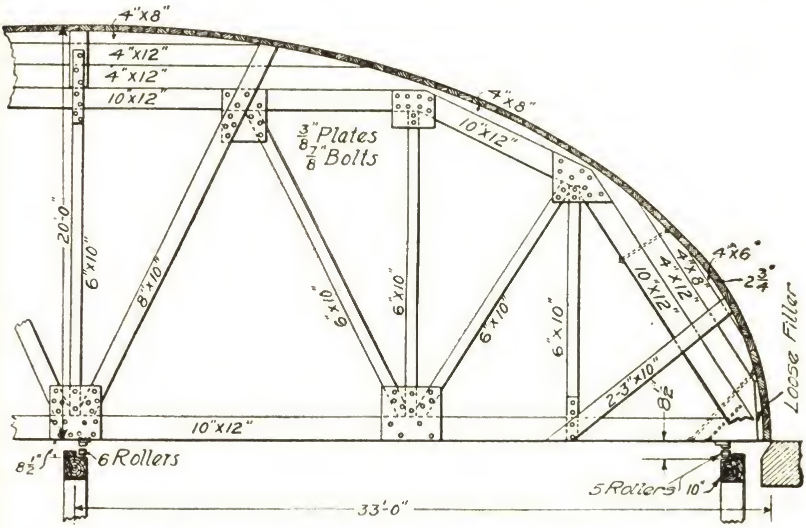



Fig. 204. Typical Arch Form Used at 175th Street, New York City

supports. The wedges are placed at about the same elevation as the springing line of the arch.

**Form for Arch at 175th Street, New York City.** In constructing the 175th Street Arch in New York City, the forms were so built that they could be moved easily. The arch is elliptical and is built of hard-burned brick and faced with granite. The span of the arch is 66 feet; the rise is 20 feet; the thickness of the arch ring is 40 inches and 48 inches, at the crown and the springing line, respectively; and the arch is built on a 9-degree skew. The total length of this arch is 800 feet.

The arch was constructed in sections, the centering being supported on 11 trusses placed perpendicular to the axis of the arch and

having the form and dimensions shown in Fig. 204. The trusses are placed 5 feet on centers, and are supported at the ends and middle by three lines of 12×12-inch yellow pine caps. The caps are supported by 12×12-inch posts, spaced 5 feet center to center, and rest on timber sills on concrete foundations. The upper and lower chord members of the trusses are of long-leaf yellow pine, but the diagonals

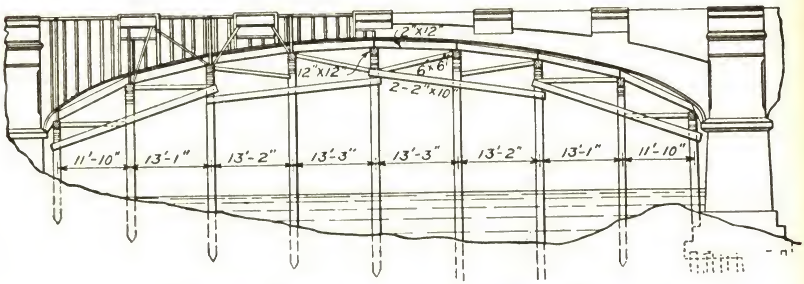



Fig. 205. Centers for Bridge at Canal Dover, Ohio

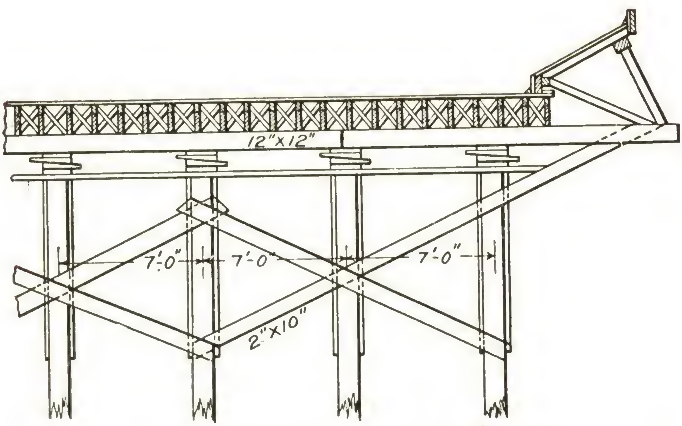



Fig. 206. Centers for Bridge at Canal Dover, Ohio

and verticals are of short-leaf yellow pine. The lagging is 2¾x6-inch, long-leaf yellow pine plank. The connections of the timbers are made by means of ⅜-inch steel plates and ⅞-inch bolts, arranged as shown in the illustration. As it was absolutely necessary to have the forms alike, to enable them to be moved along the arch and at all times fit the brickwork, they were built on the ground from the same pattern, and hoisted to their places by two guyed derricks with 70-foot booms.

On the $12\times12$-inch cap was a $3\times8$-inch timber, on which the double wedges were placed. When it was necessary to move the forms, the wedges were removed, the forms rested on the rollers, and there was then a clearance of about $2\frac{1}{4}$ inches between the brickwork and the lagging. The timber on which the rollers ran was faced with a steel plate $\frac{1}{4}\times4$ inches in dimensions. The forms were moved forward by means of the derricks. The settlement of the forms under the first section constructed was $\frac{1}{4}$ inch; and the settlement of the arch ring of that section, after the removal of forms, was $\frac{1}{4}$ inch.

**Forms for Bridge at Canal Dover, Ohio.*** The details of the centering used in erecting one of the 106-foot 8-inch spans of a reinforced-concrete bridge over the Tuscarawas River at Canal Dover, Ohio, are shown in Figs. 205 and 206. Besides this span,

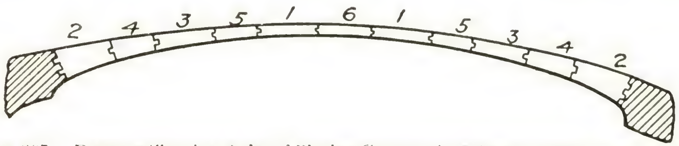



Fig. 207 Diagram Showing Order of Placing Concrete in Bridge at Canal Dover, Ohio

the bridge consisted of two other spans of 106 feet 8 inches each, and a canal span of 70 feet. The centering for the canal span was built in 6 bents, each bent having 7 piles. A clear waterway of 18 feet was required in the canal span by the State Canal Commissioner, and this passage was arranged under the center of the arch. The piles were driven by means of a scow. The cap for the piles was a $3\times12$-inch timber. Planks 2 inches thick were sawed to the correct curvature, and nailed to the $2\times12$-inch joists, which were spaced about 12 inches apart. The lagging was one inch thick, and was nailed to the curved plank. The wedges were made and used as shown. The centering was constantly checked; this was found important after a strong wind. The centering for the other two of the main arches was constructed similarly to that of the arch shown.

After some difficulty had been experienced in keeping the forms in place during the concreting of the first arch, the concrete for the other arches was placed in the order shown in Fig. 207, and no difficulty was encountered. Sections 1 and 1 were first placed, then 2 and 2, etc., finishing with section 6.

*Engineering Record.

The concreting on the canal span was begun in the late fall, and finished in 12 days; the forms were lowered by means of the wedges five weeks later. The deflection at the crown was 0.5 inch, and after the spandrel walls were built and the fill made, there was an additional deflection of 0.4 inch. In building the forms, an allowance of 1/800 part of the span was made, to allow for this deflection. The deflections at the crown of the other three arches were 0.6 inch, 1.45 inches, and 1.34 inches, respectively.

## CHAPTER XX

# MACHINERY FOR CONCRETE WORK

**Concrete Plant.** No general rule can be given for laying out a plant for concrete work. Every job is a problem in itself, and usually requires a careful analysis to secure the most economical results. Since it is much easier and cheaper to handle the cement, sand, and stone before they are mixed, the mixing should be done as near the point of installation as possible. All facilities for handling these materials, charging the mixer, and distributing the concrete after it is mixed, must be secured and maintained. The charging and distributing are often done by wheelbarrows or carts; and economy of operation depends largely upon system and regularity of operation. Simple cycles of operations, the maintenance of proper runways, together with clocklike regularity, are necessary for economy. To shorten the distance of wheeling the concrete, it is very often found, on large buildings, that it is more economical to have two medium-sized plants located some distance apart, than to have one large plant. In city work, where it is usually impossible to locate the hoist outside of the building, it is constructed in the elevator shaft or light well. In purchasing a new plant, care must be exercised in selecting machinery that will not only be satisfactory for the first job, but that will fulfill the general needs of the purchaser on other work. All parts of the plant, as well as all parts of any one machine, should be easy to duplicate from stock, so that there will not be any great delay from breakdowns or from the use of worn-out parts.

The design of a plant for handling the material and concrete, and the selection of a mixer, depend upon local conditions, the amount of concrete to be mixed per day, and the total amount required on the contract. It is very evident that on large jobs it pays to invest a large sum in machinery to expedite the work and to reduce the number of men; but if only a few cubic yards of concrete are to be mixed and deposited per day, then the investment in machinery must be in proportion to the amount of work to be done.

The interest on the plant must be charged against the number of cubic yards of concrete; that is, the interest on the plant for a year must be charged to the number of cubic yards of concrete laid in a year. The depreciation of the plant is found by taking the cost of the entire plant when new, and then appraising it after the contract is finished, and dividing the difference by the total cubic yards of concrete mixed. This will give the depreciation per cubic yard of concrete manufactured.

The introduction of central mixing and batching plants has been a great aid to contractors in cities where the space is limited for a mixing plant and storage of materials. A contractor purchasing ready-mixed concrete has only to order it when wanted and convey it from the trucks to the point of deposit. This is a convenience for the contractor, and should give a more uniform product. The cost of ready-made concrete is about the same as that of concrete mixed on the job, and it may be a trifle less. In purchasing ready-mixed concrete, the contractor will save the capital investment necessary for a mixer, its maintenance, moving and storage when not in use.

## CONCRETE MIXERS

**Characteristics.** The best concrete mixer is the one that turns out the maximum of thoroughly mixed concrete at the minimum of cost for power, interest, and maintenance. The type of mixer with a complicated motion gives better and quicker results than one with a simpler motion.

**Batch Mixers.** In the last few years many improvements have been made in the machinery for mixing and handling concrete. This is especially true of the concrete mixer. In size, mixers of today vary from small portable units that can mix only a few cubic feet of concrete, to machines which will mix at one time five cubic yards of concrete.

The drums in which the concrete is mixed are always constructed of steel. They vary in shape, but all are rigid in their construction. Inside, the drums are fitted with mixing blades, which greatly assist in mixing the cement, sand, stone, and water into a homogeneous concrete. The different manufacturers vary the shape, size, and position of these blades. The drums are usually mounted on a truck which can be handled by a traction engine or other power

unit, or, in the case of small mixers, can be moved about the job by man power.

The drums are often fitted with timing machines and the number of revolutions per minute is controlled. In their specifications, engineers often state the number of revolutions that the mixer shall

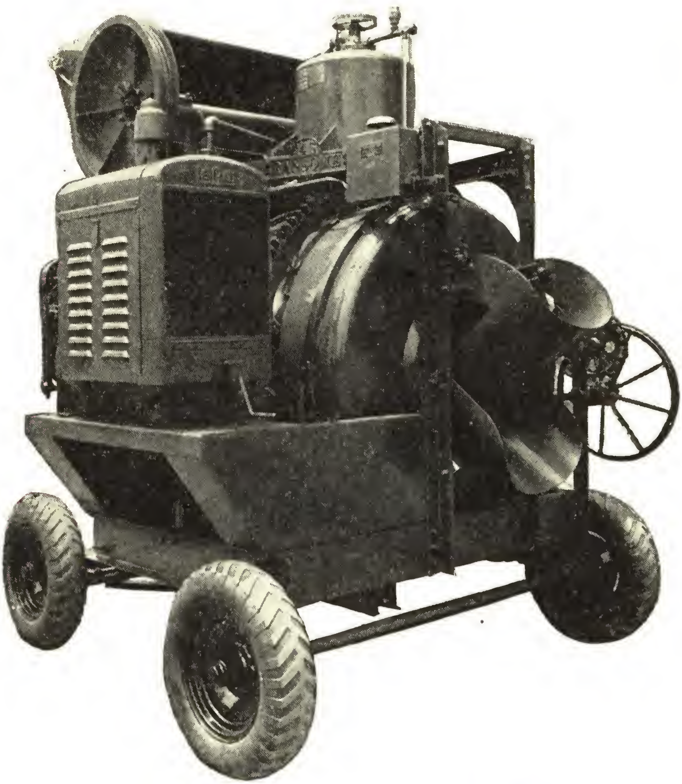


Fig. 208. No. 14-S Standard Building Mixer
*Courtesy of Ransome Concrete Machinery Company, Dunellen, New Jersey*

make per minute in mixing concrete. Usually about eighteen to twenty revolutions are made per minute and the concrete is kept in the drum for a period of not less than one minute, but sometimes specifications state that it shall be mixed for two minutes. On some jobs, three minutes have been specified.

Practically all the standard mixers are equipped with charging hoppers, which are operated by the power unit connected with

the mixer. These hoppers, while being loaded, are on the ground so that they can easily be charged with materials dumped from wheelbarrows. When the proper amount of cement, sand, and stone has been placed in the hoppers, it is raised and dumped into the mixer and the hoppers can be recharged while the concrete is being mixed.

As pointed out in another section of this book, a great deal of attention is being devoted to the amount of water used in mixing

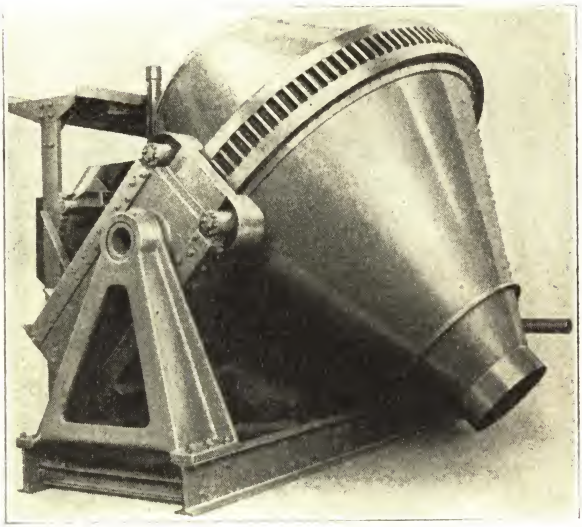


Fig. 209. Smith 28-S (28 Cu. Ft.) Tilter with Pneumatic Tilt Unit
*Courtesy of the T. L. Smith Company*

concrete. To assist in measuring the water, the concrete mixers are equipped with an automatic measuring tank. When the proper amount of water is determined, the gage of the tank is set, and thus the same amount of water will be used in each batch of concrete mixed. The regulation of the tank will depend on the amount of water in the sand and stone which is being used.

The drums are equipped with a discharging chute which discharges the concrete rapidly into a hopper or, if desired, it can be discharged in small amounts. On small jobs, where the concrete is wheeled to the place of deposit, the concrete may be discharged directly into the wheelbarrows.

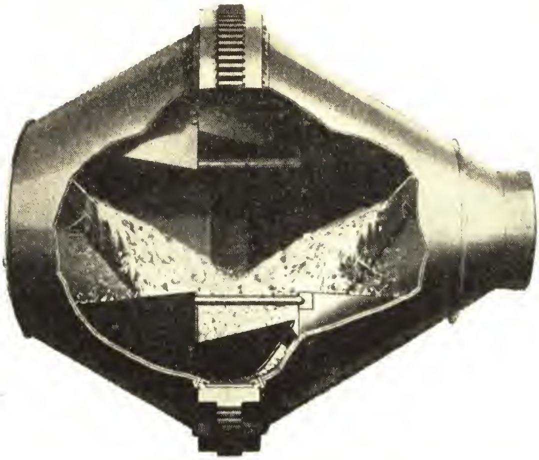


Fig. 210. Smith Tilting Drum
*Courtesy of National Equipment Company, Milwaukee, Wisconsin*

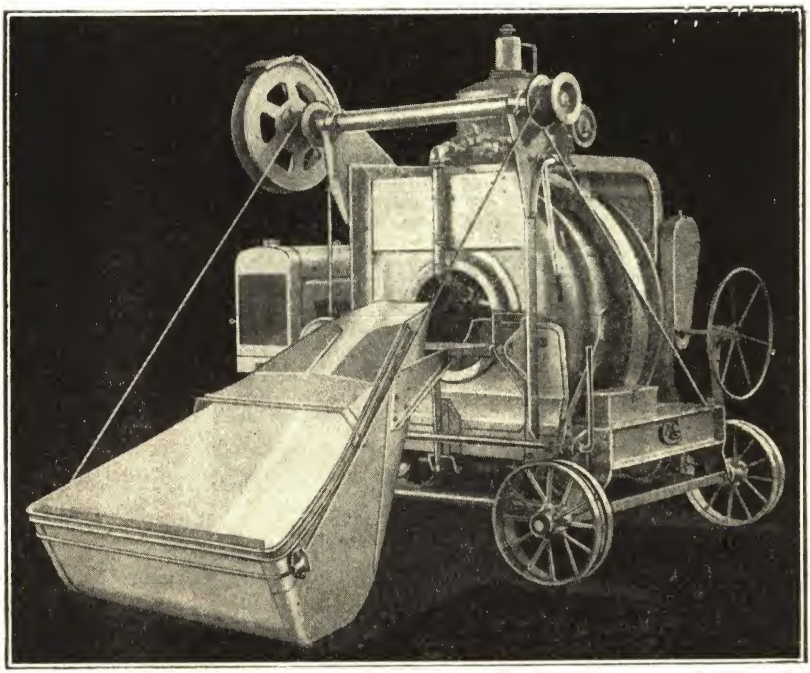


**Fig. 211. C. M. C. Mixer**
*Courtesy of Construction Machinery Company, Waterloo, Iowa*

Fig. 208 shows a Ransome concrete mixer, a good concrete mixer for general building work. The power unit for operating this mixer is a gas engine, shown at the left in Fig. 208.

The Smith mixers are made in both the tilting and non-tilting type. In Fig. 209 is shown the tilting type, and Fig. 210 shows an

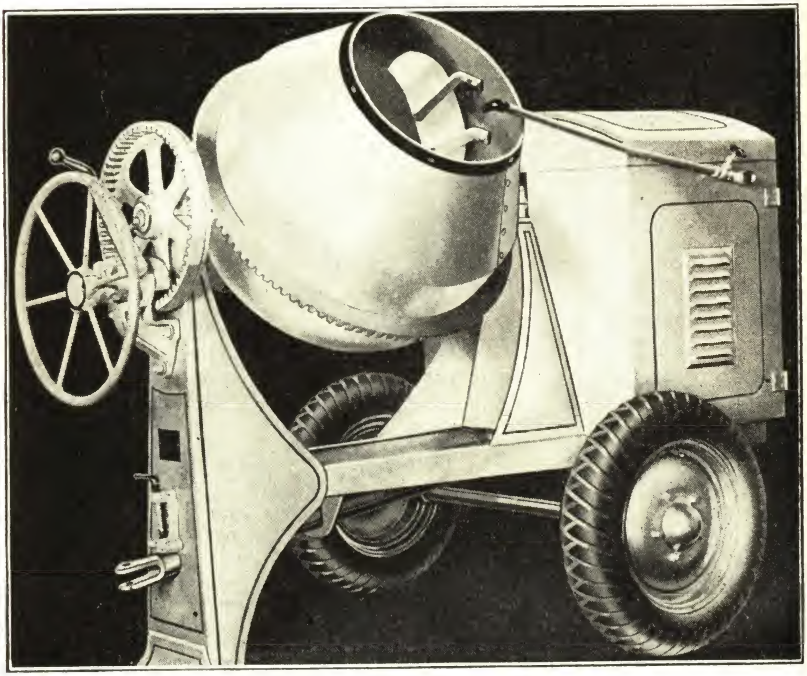


Fig. 212. "Half-Bag" Wonder Mixer
*Courtesy of Construction Machinery Company, Waterloo, Iowa*

interior view of the drum. The drum is a double cone in shape in the tilting type, and the concrete in being mixed has an "end to center action." The converging blades cut through the concrete and pull it from the ends to the center. This machine is quick in discharging the concrete. This mixer is made in large sizes and is used extensively on big jobs. Fig. 210 shows the mixing blades in a Smith tilting drum and the "end to center" movement.

Fig. 211 shows a C.M.C. concrete mixer. This mixer is made in many sizes, and can be operated by steam, gasoline, or motor. The drum of the mixer is made of steel, and for the one-yard mixer

the steel in the drum is $\frac{5}{16}$ inch in thickness. In the one-yard mixer, there are 10 mixing blades and pick-up buckets which are bolted to the drum. The speed of the drum of this size mixer is $14\frac{1}{2}$ revolutions per minute.

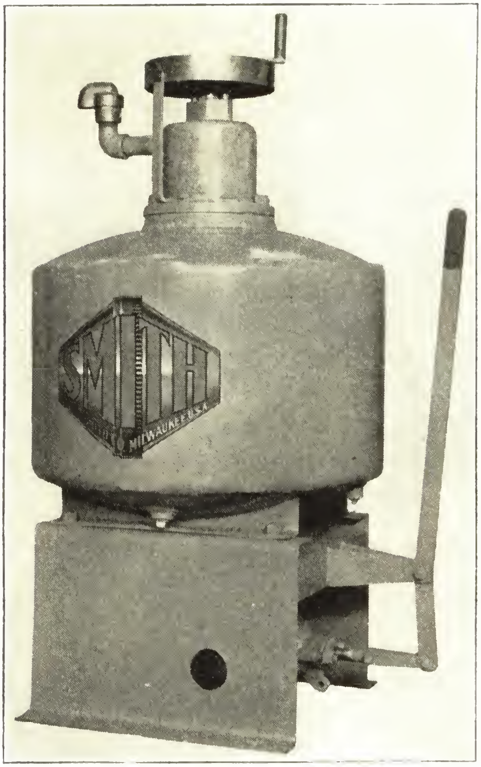



Fig. 213A. Syphon Type Tank

*Courtesy of T. L. Smith Company, Milwaukee, Wisconsin*

On small jobs and also on larger jobs where there is a small amount of concrete, such as foundations to be constructed at scattered points on the operation, a small mixer that can be moved easily is often found economical. It is cheaper to haul the sand and stone to scattered points than it is to wheel the concrete after being mixed. Fig. 212 shows a "half-bag" Wonder mixer. In charging this mixer with a 1:2:4 concrete, a half bag of cement, one cubic foot of sand, and two cubic feet of stone would be placed in the mixer at one time. The wheels have rubber tires and the spring cushion

frame makes it possible to tow one of these machines from job to job at a speed at which a truck can be operated. The drum of the mixer is fitted with blades and in all details it is very complete.

**Measurement of Water.** The manufacturers of concrete machinery have given much study to devices for the correct measurement of the amount of water used in mixing concrete. In Fig. 213A is shown a vertical syphon type of water tank having the outlet pipe located in the center. The control is set by a crank at the top of the tank.

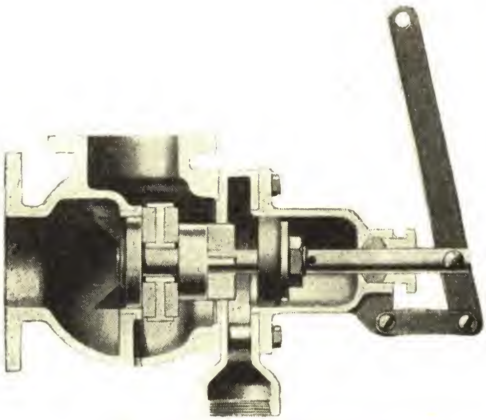


Fig. 213B. Non-by-Pass Valve
*Courtesy of T. L. Smith Company, Milwaukee, Wisconsin*

The gage is graduated in pounds, gallons, and quarts. The manufacturers claim that the tank will discharge the exact amount of water for which the gage is set. A three-way non-by-pass valve is used to control the flow of water. In Fig. 213B is shown a section with the valves seated vertically to prevent the sediment and dirt from lodging in them.

## POWER UNITS FOR OPERATING A CONCRETE PLANT

**General Considerations.** In every case the source of power for operating mixers, derricks, hoists, and cables must be considered. The three general sources of power for operating this machinery are the gasoline engine, electric motor, and steam engine. The plant may be operated by any one of these sources of power or a combination of all three of them. For example, concrete may be mixed by a gaso-

line engine and hoisted to a floor of a building by an electric motor. In mixing concrete, the engine is kept more or less continuously at work, as the drum is kept rotating; but in hoisting, the load is intermittent and therefore, if a gasoline engine is used, there is a small

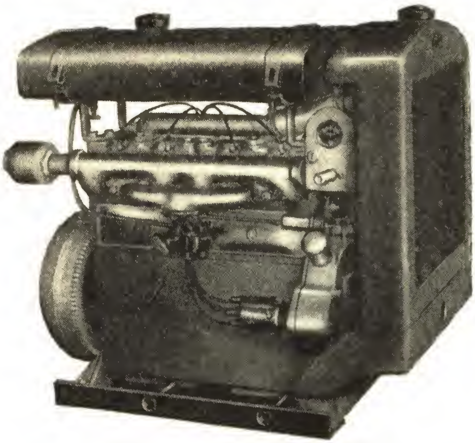


Fig. 214. Novo Gasoline Engine
*Courtesy of Novo Engine Company, Lansing, Michigan*

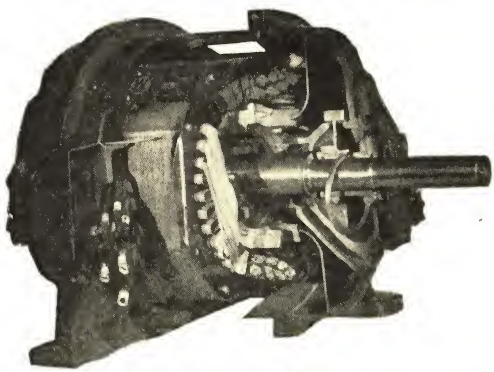


Fig. 215. Hoist Motor
*Courtesy of Westinghouse Electric and Manufacturing Company*

waste due to keeping the engine operating continuously. Generally each mixer, hoist, etc., forming a part of a concrete plant is operated by an individual power unit connected to it.

**Gasoline Engines.** In recent years there have been great improvements made in gasoline engines, and they are now used extensively for mixing concrete as well as for hoisting. Gasoline

engines are constructed with two or four cylinders, and for very small units sometimes a one-cylinder engine is used. Fig. 214 shows a four-cylinder engine with the housing removed. The flywheel is also exposed in this view. This is a variable speed engine with gasoline filter, spark plugs and radiator-type cooling. This engine is built in sizes ranging from 5 to 20 horsepower and is often used as a power unit in building operations.

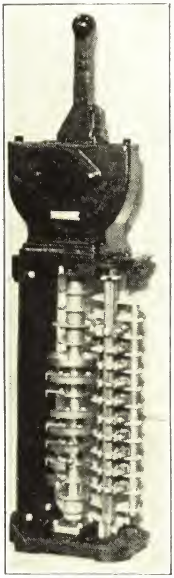


Fig. 216. Switch for Hoisting Machine
*Courtesy of Westinghouse Electric and Manufacturing Company*

**Electric Motors.** Electric motors are seldom used for mixing concrete, except in the cinder block industry, but are frequently used for hoisting materials.

A motor for hoisting purposes must have a substantial frame and be powerful enough for the load. The intermittent service is severe; frequent starts, stops, and reversals must be made. Heavy loads must be picked up and raised quickly to the desired height.

In Fig. 215 is shown a hoisting motor designed especially for this service. The main frame is cast in one piece with feet for bolting to the supporting members. The ends are removable so that the

interior parts are accessible. In these motors the high-resistance rotor provides the highest ratio of starting torque to starting current obtainable with a squirrel-cage motor. They cause minimum disturbance to the line and permit the use of a low-cost full voltage. Light loads can be raised speedily, but with heavy loads the speed is decreased with a corresponding increase in torque. Ball or sealed-sleeve bearings can be secured and either will exclude dirt particles and prevent the loss of lubricant.

In Fig. 216 is shown a switch, with cover removed, for operating a motor for hoisting.

**Steam Engines.** Steam power for mixing concrete is not used as much as formerly. While the steam power is very reliable, there are several drawbacks to its use for operating a concrete plant. The steam is usually generated by a small vertical boiler. In cities where smoke ordinances are in force, these boilers are not permitted as they produce a great deal of smoke at times. Also, a licensed engineer is required to operate the plant, which is an additional expense over a gasoline engine. Therefore, if steam power is used in the larger cities, it must be secured from some stationary plant so as to avoid the excessive smoke.

## HOISTING MACHINERY

**Hoisting Materials.** The hoisting of materials and concrete is a part of all building work. Included under the heading of materials to be lifted are forms and reinforcing bars, and it may be necessary at times to raise the cement, sand, and stone several feet above the ground. At central mixing and batching plants it is necessary to elevate the cement and aggregates. The hoisting of the concrete used in the construction of a building is an important item, and the equipment must be adequate for the work to be done.

**Typical Hoisting Engine.** Fig. 217 illustrates a typical double-drum hoisting engine. This engine is designed to fulfill the requirements of a general contractor for all classes of derrick work and hoisting. A double-drum unit is shown in the figure, but it is also manufactured with a single drum. A third drum may be connected to the double-drum unit, or one drum may be detached, making a single-drum hoist. The double-drums are independent of each other and therefore two hoists may be used at the same time. These units

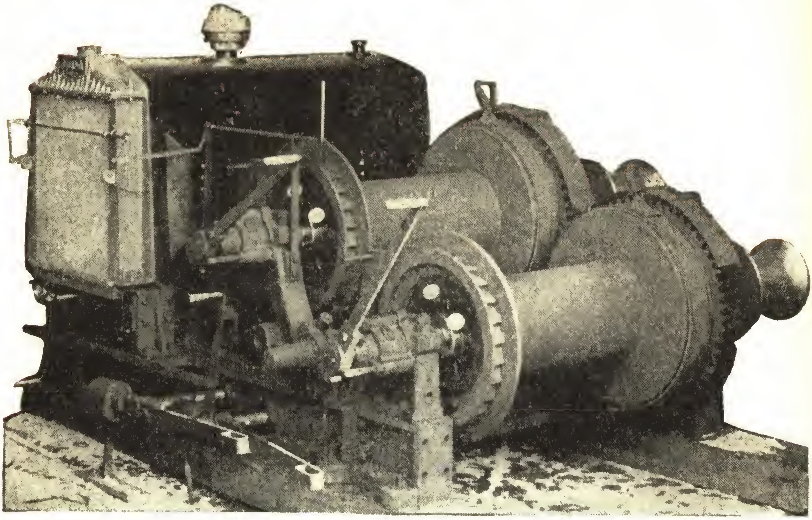


Fig. 217. Double-Drum Hoisting Engine
*Courtesy of Lambert National Equipment Company*

are generally operated by gasoline engines but electric power or steam may be used.

In Fig. 218 is shown a "speedy" hoist, of the screw-thrust type,

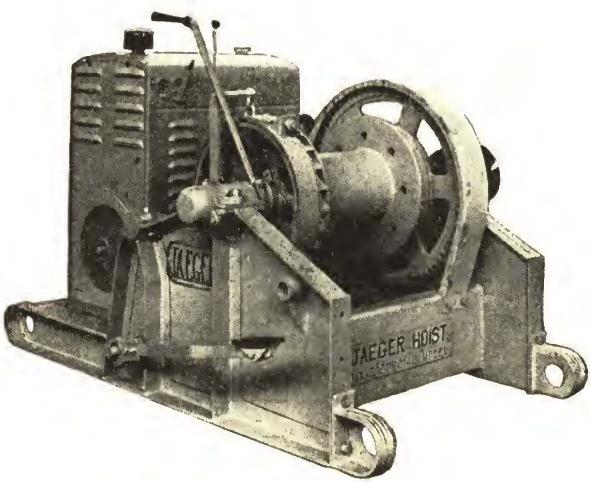



Fig. 218. Single-Drum Hoisting Engine
*Courtesy of Jaeger Machine Company*

with a single drum. These hoists can also be secured with double drums and can be driven by either gas or electric power. They are used for hoisting forms, reinforcing steel, etc.

**Charging Mixers.** For the ordinary building job, the charging skip attached to the mixer is usually loaded by means of wheelbarrows or concrete carts. In Fig. 211 the skip is shown in position for loading. It is raised by the power unit connected to the mixer.

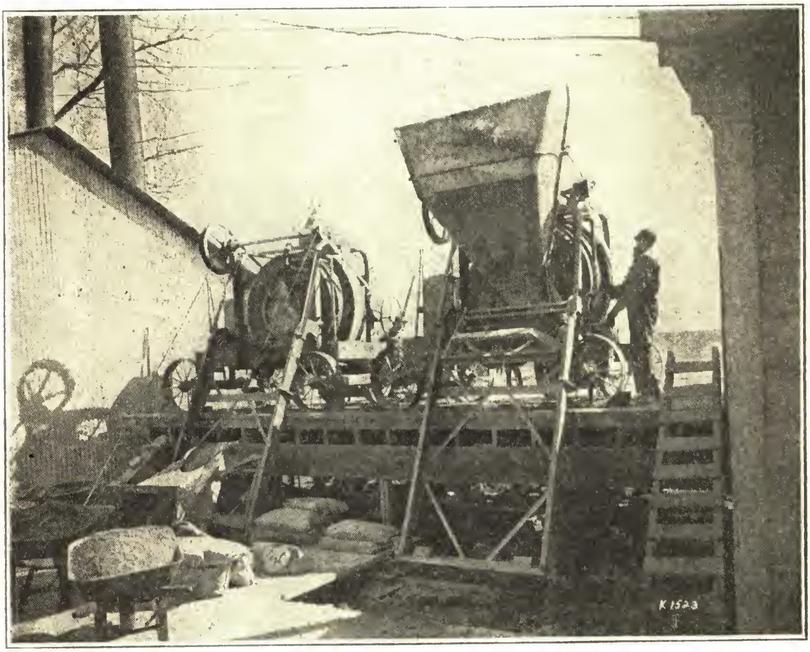



Fig. 219. Mixer with Extension Track for Loader
*Courtesy of Koehring Company*

On large jobs the sand and aggregates are raised to storage bins, weighed and dumped into the mixer.

**Extension Track for Raising Materials.** In constructing a retaining wall, it is often more convenient to have the materials delivered at the ground level on the low side of the wall and have the mixer located on top of the wall. In such a case, the charging skip can be raised on an extension track as shown in Fig. 219. From the lower level, the charging skip can be loaded easily, and in this case the materials will be hoisted instead of the mixed concrete.

In Fig. 219 is shown a mixer with an extension track for the elevation of the materials both being operated by the same gasoline engine.

## TRANSPORTING CONCRETE

Concrete is usually transported by wheelbarrows, carts, cars, or chutes, although other means are frequently used. It is essential, in handling or transporting concrete, that care be taken to prevent the separation of the stone from the mortar. In building operations, wheelbarrows or carts are generally used to transport the concrete. About 2½ cubic feet of wet concrete is the average load for a man to handle in a wheelbarrow.

Fig. 220. Wheelbarrow Having Wheel with Pneumatic Tire
*Courtesy of Sterling Wheelbarrow Company*

**Wheelbarrows, etc.** In Fig. 220 is shown a wheelbarrow with a pneumatic tire on its wheel. These tires are a new development in wheels for wheelbarrows and concrete carts and appear to be successful. A larger load can be transported at a faster speed over rougher surfaces and without the vibration of the ordinary wheelbarrow. The tires are made both two-ply and four-ply, with inner tubes, and can be secured in several sizes. It is important to keep the tires properly inflated to secure the best results.

Fig. 221 shows a cart for transporting concrete. These carts have a capacity of 6 cubic feet, and one man can push one over a plank runway. The carts are manufactured with pneumatic tires as well as plain steel wheels as shown in the figure.

The runways are made of two-inch plank, and in width are at least a foot wider than the distance between the wheels. The planks

are fastened together on the back with 2×6-inch cross pieces and are made in sections so that they can be handled by four men.

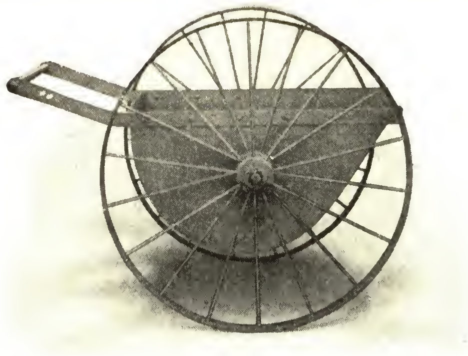


Fig. 221. Typical Cart for Concrete
*Courtesy of Ransome Concrete Machinery Company, Chicago*

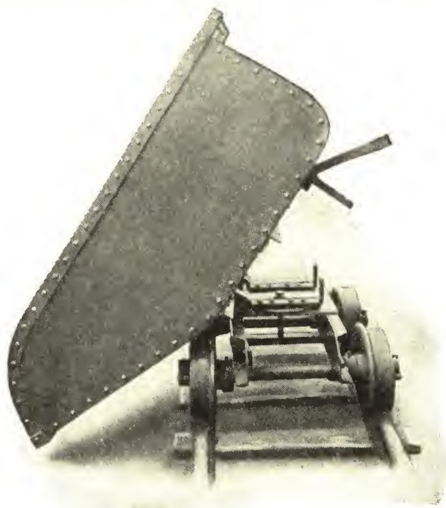


Fig. 222. Typical Rotary Dump Car
*Courtesy of Ransome Concrete Machinery Company, Chicago*

When it is necessary to convey concrete a longer distance than is economical by means of wheelbarrows or carts, a dumping car which is run on a track is often used. Fig. 222 shows a steel car made for this purpose. The capacity of these cars is from 10 to 40 cubic

feet, and the track gage is from 18 to 36 inches. Both end-dumping and side-dumping cars are made.

**Hoisting Tower and Buckets.** In building work, the concrete is usually hoisted in automatic dumping buckets. The bucket

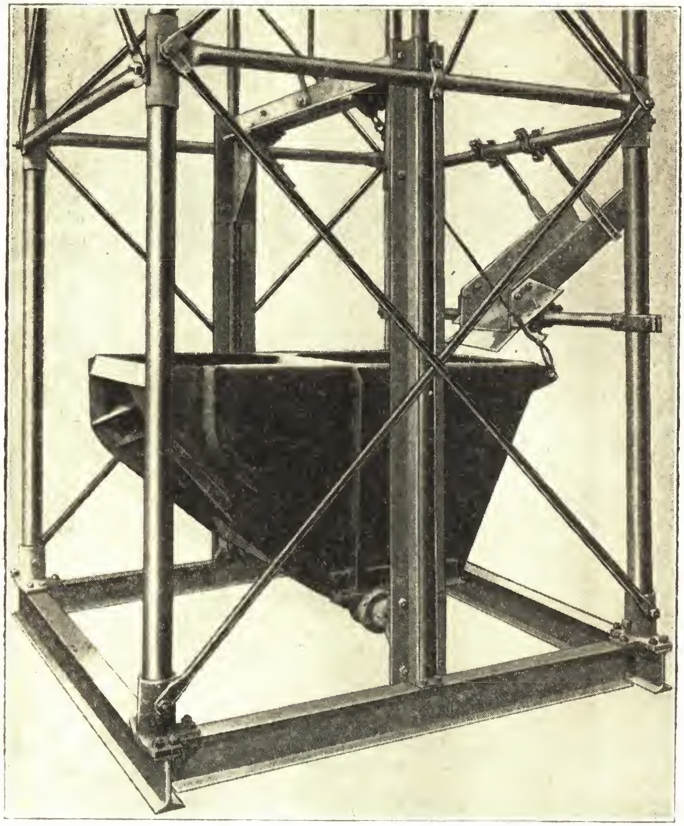


Fig. 223. Concrete Bucket and Loading Chute
*Courtesy of Dravo Equipment Company*

operates in a tower constructed of timber, structural steel, or pipes, and it dumps automatically by pitching forward when it reaches a dumping device set in the tower at the place where the concrete is wanted. The bucket rights itself automatically as soon as it begins to descend. In Fig. 223 is shown a bucket in position for loading at the bottom of the tower. In Fig. 224 it is shown in position for dumping into a hopper.

All vertical posts, bracing, and guides for the buckets shown in the figure are constructed of tubular (pipe) sections. These towers can be dismantled easily and used repeatedly, and have largely taken the

Fig. 224. Bucket in Dumping Position
*Courtesy of Draco Equipment Company*

place of towers constructed of wood. Towers of tubular sections several hundred feet in height have been erected and used. On ordinary jobs one tower will readily handle the concrete, but on large factory jobs several may be necessary to make the speed required in the construction of the building.

Fig. 225. Boom Chute Plant on Double Tower
*Courtesy of Dravo Equipment Company*

**Chuting Concrete.** Concrete is often placed by means of chutes. The concrete is elevated some distance above the point where it is to be placed, and then flows through a series of chutes to a point where it is to be deposited. The chutes are erected at an angle of about 25° to 30° so that the height the concrete is to be raised in the tower will depend upon the distance it is to be conveyed, and also upon the elevation where it is to be placed.

When concrete is conveyed by means of chutes, the system of chutes must be laid out well, the concrete must be made suitable for being placed in this manner, and the water content carefully watched, as there is tendency for the stone to separate from the mortar. The concrete should be discharged into a bin and partly remixed before it is placed in the forms unless it is being placed in a large mass where it can be spaded well.

In Fig. 225 is shown an outfit for chuting concrete. In this case a double tower is used. The upper section of the chutes is pivoted to the bottom of the tower bin so that it can be moved around. It is about 40 feet in length. The lower section is supported at the upper end by a pivoted joint to the first section so that it can be moved from place to place to meet the requirements of despositing. These chutes must be well guyed. It will be noted that the lower end of the upper chute is guyed to the top of the tower.

**Depositing Concrete with Dump Buckets.** On large dams the concrete is often placed by means of dump buckets operated on cables. The bucket is conveyed on a cable to the point above where it is wanted, then lowered to the surface of the concrete previously poured, and the new load is discharged.

## WOODWORKING PLANT

A portable woodworking plant can be used to advantage in shaping the lumber for the forms when a large building is to be erected. The plant can be set near the site of the building to be erected, and the woodworking done there. The machinery for such a plant should consist of a planer adapted for surfacing lumber on three sides, a ripsaw, and a crosscut circular saw; in some cases a band saw can be used to advantage. Usually the cost of surfaced lumber delivered at the job is less than rough lumber. This is due to the difference in the freight cost. That is, the saving made in freight

between rough lumber and lumber surfaced on one side is more than the cost of surfacing at the large mills. Under these conditions, a planer would be required only for a special job.

In such a plant the ripsaw and the crosscut saw would be found to be the most useful; and, if reasonable care is taken, this machinery will soon pay for itself. It is often difficult to get work done at a planing mill when it is wanted; and if a contractor has his own woodworking machinery, he will be independent of any planing mill. A plant of this kind can be operated by a steam or gasoline engine or by an electric motor.

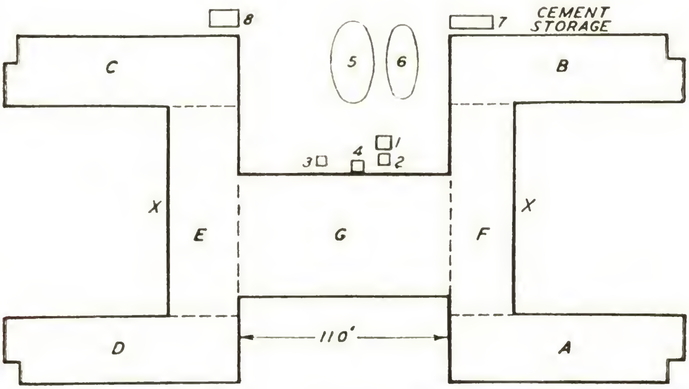



Fig. 226. Dormitory for Women, Pennsylvania State College; 1, Concrete Mixer; 2, Concrete Hoist; 3, Hoisting Engine; 4, Lumber and Steel Hoist; 5, Stone Storage; 6, Sand Storage; 7, Cement Storage (4 Cars); 8, Woodworking Plant

## EXAMPLES OF CONSTRUCTION PLANTS

**Plant for Dormitory Building.** The layout of the plant used for constructing the reinforced concrete work for a dormitory building to house 500 women at the Pennsylvania State College, State College, Pennsylvania, required a close decision by Henry E. Baton, Inc., the contractor. The foundations were constructed on rock and therefore were small. Small mixers that could be moved easily about the excavation were used for the footings and part of the basement walls.

Fig. 226 shows a general plan of the building and the final location of the plant for the reinforced concrete work. The contractor considered using two plants located at points "$X$". This would save much distance in carting the concrete but would require an

additional mixer, hoist, etc. The portions of the building marked *E-F-G* are higher than the wings *B* and *C*, and these wings are higher than wings *A* and *D*. Only one floor, the second, is at the same elevation for the entire building. Portions *E* and *F* are not at the same elevation as *G*, except at the second floor. The contractor purchased concrete buggies and wheelbarrows with pneumatic tires and decided to use one mixing plant. This arrangement worked out satisfactorily.

A ¾-inch yard capacity mixer operated by a four-cylinder gasoline engine was used to mix the concrete. The cement and aggregates were placed in the charger and hoisted and dumped into the mixer by the engine attached to the mixer. The water was measured in a tank on the mixer.

A woodworking plant consisting of a ripsaw and a crosscut saw was located adjacent to the stone storage.

Two hoists were used, and both were operated by the same engine. A steel tubular framed hoist was used for the concrete and it was placed between the mixer and the building. A bucket holding ¾ cubic yard was used to raise the concrete to the floor where wanted. The other hoist had a flat cage bottom and was used for lifting reinforcing steel, lumber, etc. A two-drum, six-cylinder gasoline engine, developing about 40 horse power, supplied power for both hoists.

The cement building was of sufficient size to store 4 carloads of cement. Pipes, 20 inches in diameter, were laid on the ground under the sand and stone storage piles. In cold weather, wood fires were built in the pipes to heat the sand and stone. The floors were poured in the late fall, winter, and early spring. The newly poured concrete was kept warm by enclosing the building with canvas and firing salamanders, coke being used for the fuel.

**Concrete Plant for Library Building, Drew University.** This building is approximately 85′9″ by 145′3″ and is three stories in height. The concrete plant was located at one end of the building, as that was the only convenient point that could be reached with trucks.

The concrete for this building was transit mixed. Therefore, the plant at the building consisted of a concrete hoist, a hoisting engine, table-saw, and winch. The concrete was elevated in an automatic dumping bucket of a standard make with a capacity of about

½ cubic yard, and the tower was constructed of wood. A 60-horsepower, single-drum, 4-cylinder gasoline engine was used for hoisting the concrete. The reinforcing steel was raised by hand or carried up ramps. The form lumber was raised by means of a Dobbie winch, with a ⅜″ steel cable, mounted on "A" frames placed on the floor where the material was needed. A table-saw, driven by a gasoline engine, was used for all wood cutting. Crosscut and ripping blades were interchangeable.

## CHAPTER XXI

# SCALES FOR PROPORTIONING CONCRETE MATERIALS

Concrete is a material that generally is used at the time it is manufactured and its quality is dependent on good materials and on proper mixing and curing. It is difficult and expensive to repair poor concrete. Sometimes it is impossible to make satisfactory repairs. Therefore it is important that good concrete is obtained at the time it is poured.

Much study and many experiments have been made to develop equipment that will secure better control of the measurement of aggregates and mixing water, the accurate determination of the amount of surface moisture in the aggregates, the length of mixing time, etc. The equipment developed consists of automatic, semiautomatic, and hand-controlled scales; gages for determining the specific. gravity and surface moisture in aggregates. Most of this work has been done by engineers employed by the manufacturers of this special equipment and such equipment is used in some of the central mixing and batching plants and on large construction jobs where accurate measurement of materials is required. Much credit is due to the Scientific Concrete Service Corp., Washington, D. C., for their work in this field.

When aggregates are controlled properly with respect to size and measurement, when the correct amount of mixing water is used and concrete is well mixed, the amount of cement can be reduced and a strength maintained that is comparable to a concrete produced with a richer mix by ordinary means. A reduction in the amount of cement used will slightly reduce the cost of the concrete. For example, if five bags of cement, at a cost of fifty cents per bag, are required to produce a cubic yard of concrete with an ultimate strength of 2000 pounds per square inch, and a concrete of the same strength can be produced by using only four bags of cement, then practically fifty cents per cubic yard will be saved. With a reduction of the amount of cement used, a reduction in the shrinkage of the concrete can be

anticipated, which is a more important factor than the saving in cost.

**Toledo Scales.** In Fig. 227 is shown a Toledo compensating auto-gage with three beams. It is made in two models, automatic and semi-automatic. For the larger construction operations the

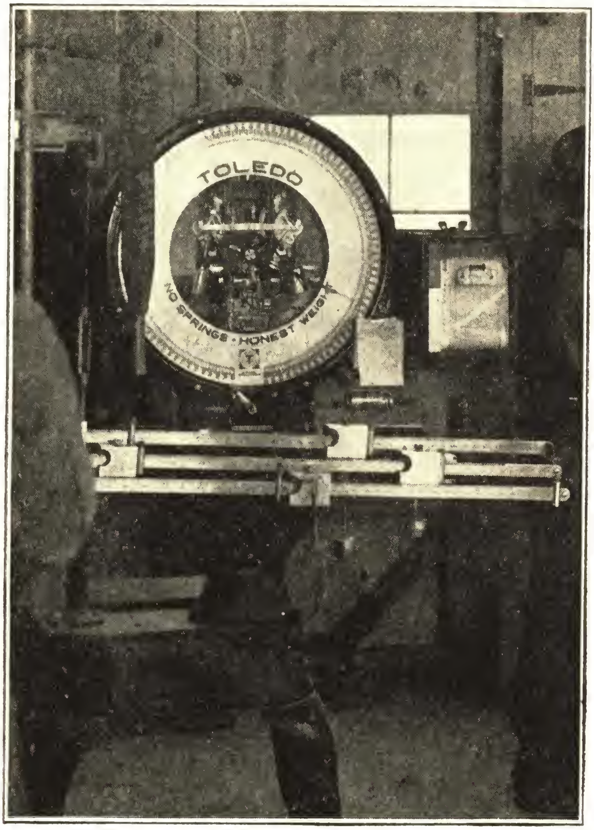



Fig. 227. Automatic Compensating Scale
*Courtesy of Scientific Concrete Service Corporation*

automatic generally is used. In Fig. 227 the three beams are exposed and the mechanism is enclosed in dust- and moisture-proof housing. The graphic recorder shown at the right of the indicator dial makes an ink record of the details in connection with every batch of concrete. The information on the record includes the weight of stone, sand, cement, water used, percentage of surface moisture, compensation for weight of surface moisture, time of day at which it

was weighed, etc. This auto-gage is equipped with an electric cut-off device so that the valves, gates, and conveyors can be operated speedily and accurately.

In operating these scales, each material to be weighed is assigned to a separate beam, including the cement if it is loose. These beams

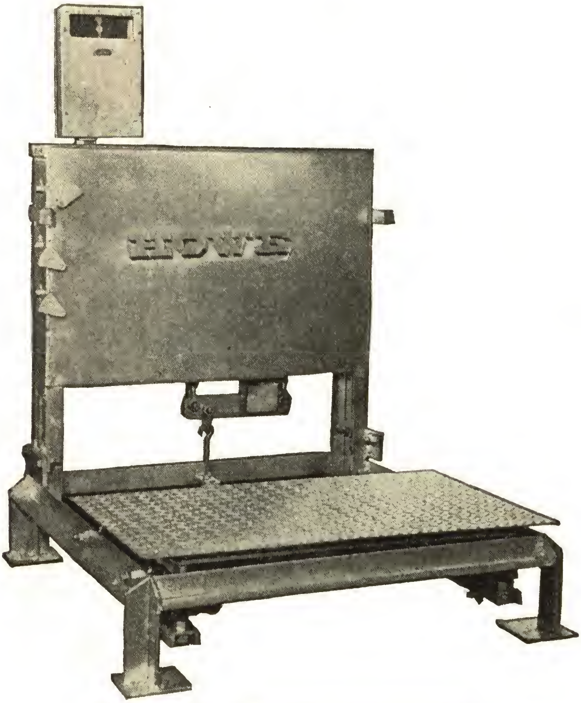


Fig. 228. Howe Wheelbarrow Batching Scale
*Courtesy of Howe Scale Company, Philadelphia, Pa.*

are set in the order in which the material is to be weighed. Before the beams are set, the moisture content in the aggregates is determined and the correction for the amount of material to be used is made. For example, if 2000 pounds of surface-dry sand is required for a batch of concrete and it is found that it contains 10% of moisture, which would be 200 pounds, then the beam is set at 2000 pounds plus 200 pounds, or 2200 pounds, to compensate for the moisture. Likewise, the moisture is determined in the coarse aggregate and compensation is made for the weight of the moisture by adding stone

equal to the weight of the moisture. In weighing the water, the amount of water found in the material is deducted from the amount of water required if surface-dry material is being used.

**Scales for Small Jobs.** Fig. 228 shows a scale with the beam box closed, developed for weighing the aggregate for concrete on the

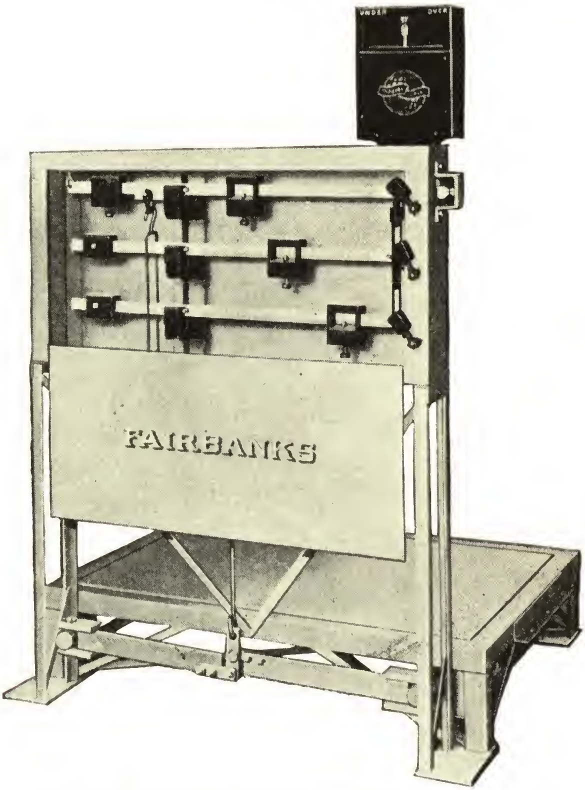



Fig. 229. Three-Beam Scale for Weighing Aggregates
*Courtesy of Fairbanks, Morse & Company, Chicago*

smaller jobs. Many specifications for public work, and some private work, demand that the aggregates shall be proportioned correctly, measurement by wheelbarrows not being acceptable. This requirement is met by weighing the sand and stone.

The platform of the scale is several inches above the ground and usually it is necessary to wheel the aggregates up a slight incline to reach the elevation of the platform. These scales are usually mounted

on wheels for moving them from job to job. Before being put into operation the wheels are removed to reduce the height of the incline.

Scales for weighing aggregates are manufactured with one to four beams. Probably the one with three beams (see Fig. 229) is the most popular. One beam is used to balance the empty wheelbarrow, one for sand and one for the stone. If two sizes of stone are required, then the four-beam scale is used. In operating these scales the tare beam poise is set to balance the empty wheelbarrows or carts and


Fig. 230. Small Concrete Plant in Operation
*Courtesy of Fairbanks, Morse & Company, Chicago*

locked into position. The aggregate beams are then set for the correct amount of material for each wheelbarrow load and locked into position independently of the others. The indicator registers zero when the correct amount of aggregates has been loaded in the wheelbarrows.

In Fig. 230 a small plant is shown in operation. The aggregates are wheeled up the incline to the platform of the scale, weighed and dumped into the hopper of the mixer. In laying out these plants the cement is often placed on the same side of the runway as the scales, and a supply of sand and one of stone are placed at the location of the cement in the figure. Any adjustment required in the aggregates can be made conveniently.

## CONCRETE VIBRATORS

**Use of Vibrators.** Vibrators should be used in compacting concrete of a stiff consistency (1-inch to 1½-inch slump) that cannot be spaded successfully by hand methods. There is usually no advantage in using them for medium or wet mixture. Concrete to be vibrated requires stronger forms than concrete that is hand-spaded. When concreting walls, if an internal vibrator is applied to the center

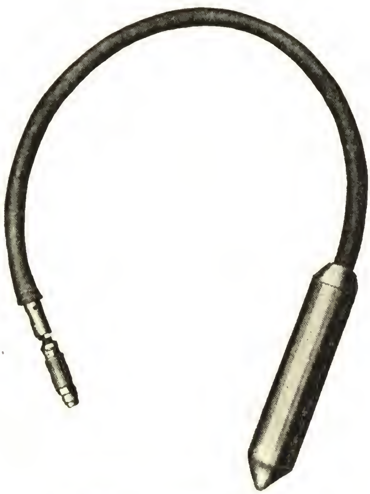


Fig. 231. Internal Vibrator
*Courtesy of Ingersoll-Rand*

of the mass, rough surfaces will be found unless some precaution is taken to avoid this condition, such as external vibrating or hammering of the forms. In operating vibrators, care must be taken to avoid damaging the forms. Vibrators have been much used on large dams and similar works. There is a variety of these machines on the market. The type and size of the vibrator depends on the size, shape, and use of the concrete to be placed.

**Types of Vibrators.** There are three general types or classes of vibrators for compacting concrete; (1) internal (2) external (3) surface vibrator. For general work, internal vibrators are preferred as they can be applied directly to the concrete. External vibrators are usually rigidly attached to the forms, and the concrete is vibrated through the forms. In placing slabs and fireproofing structural steel

for steel frame buildings, the vibrator may be attached to the steel work or to the forms, depending on the type of vibrator. The external machines are often used in the manufacture of cast stone, precast piles, sewer pipes, and like products. Surface vibrators are used for heavy work, usually in connection with internal vibrators. After the concrete has been vibrated internally, the surface vibrator is applied to the top of the concrete. This type is often used for pavements. It is attached to the screeds used for leveling the concrete and is applied to the concrete in this manner.

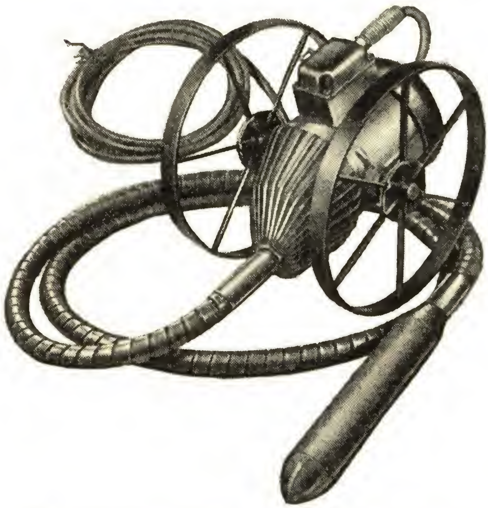


Fig. 232. Internal Vibrator, Electric-Motor Driven
*Courtesy of Syntron Company*

**Internal Vibrators.** Internal vibrators are manufactured differently in a number of points. In some of the electric machines the motor is located in the vibrating head and is equipped with a rigid or flexible handle. In the rigid type, an eccentric shaft is enclosed in a tube rigidly attached to the motor housing. In the flexible shaft type, an eccentric shaft is located in a tube which is operated through a flexible shaft. These machines may be operated by electric motor, gasoline engine, or air turbine.

In Fig. 231 is illustrated an internal vibrator operated by air. The vibrating unit (lower end in illustration) is 2¾ inches in diameter, 14¾ inches long, and weighs 15¾ pounds.

In Fig. 232 is shown an internal vibrator driven by an electric motor. This vibrator consists of rotating counterweights encased in a small steel cylinder and driven at high speed, through the medium of flexible shafts, by a single-phase 110-220-volt induction-repulsion electric motor geared up to high speed. The vibrator operates at 7,200 vibrations per minute. The rated horse power is 1½. This vibrator is also fitted for gasoline engines.

In Fig. 233 is illustrated an internal vibrator for heavy massed concrete. For the smaller sized vibrator, this manufacturer uses an eccentric weight mounted on ball bearings and driven by a pneumatic vane type of motor through a suitable coupling device or enclosed in the vibrator tube. Air to the motor travels through a small hose of a special resisting construction which is enclosed in a larger hose through

Fig. 233. Vibrator for Heavy Mass Concrete
*Courtesy of Chicago Pneumatic Tool Company*

which the exhaust air from the motor travels and into which it escapes. In the heavier machine, which is shown in the illustration, the motor is rigidly mounted on the opposite end of a spandrel tube connected to the upper end of the vibrator hose. A rigid shaft connects the rotor to the motor with the eccentric shaft through two couplings. The motor is located in this position to improve the balance for two-men operation. Thus a more powerful motor can be constructed.

**External Vibrators.** External or form vibrators vary in design. They may consist of an electric motor with an unbalanced member, pulsating electromagnets, or a reciprocating cylinder operated by compressed air. Air hammers are sometimes used as external vibrators. They may be applied to the exterior side of the forms but only short strokes should be permitted in order not to damage the forms. External vibrators may be operated by electric motor, gasoline engine, or air turbine. Fig. 234 illustrates an electrically operated external vibrator attached to wall forms. The vise clamp strikes the forms

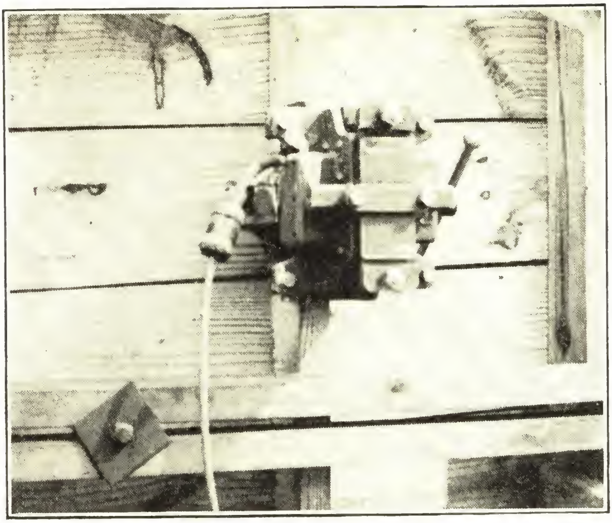


Fig. 234. Vibrating Wall Forms with Vise Clamp
*Courtesy of Syntron Company*

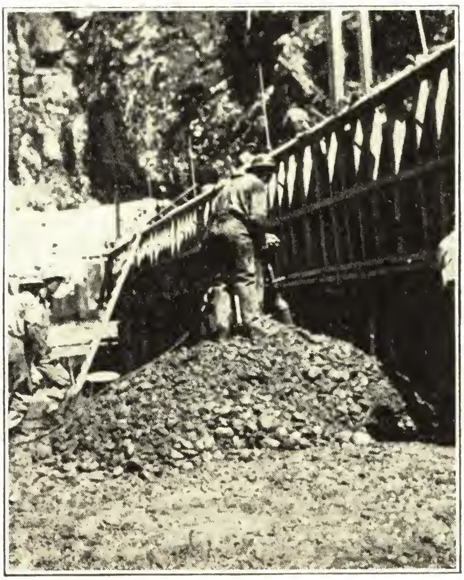


Fig. 235A. Concrete Dumped in Form, Ready for Placement
*Courtesy of Chicago Pneumatic Tool Company*

near the point where it is attached and in this way compacts the concrete.

Fig. 235B. Vibrator or Shimmy Spade, "Puddling" or Settling the Mix
*Courtesy of Chicago Pneumatic Tool Company*

**Surface Vibrators.** A surface vibrator is a type of external vibrator mounted on a heavy platform. It is used for leveling the

Fig. 235C. Vibrator or Shimmy Spade, Vibration Complete
*Courtesy of Chicago Pneumatic Tool Company*

surface of the concrete after the internal vibrator has been used. Surface vibrators are used for pavements.

**Capacity.** The capacity of vibrators varies from 5 cubic yards to 40 cubic yards per hour. The smaller machines may be operated from an outlet of an ordinary lighting system. The horse power of

these machines varies from $1\frac{1}{2}$ to 3 or 4 horse power. The speeds vary from 2,500 to 7,000 vibrations per minute.

Figs. 235A, 235B, 235C show concrete being placed with a vibrator or shimmy spade, as it is named by the manufacturer. In Fig. 235A it is noted that a stiff mix of concrete has been placed and the men are on top of it starting to operate the vibrators. In Fig. 235B the vibrators are operating at full speed and it can be noted that the men sink into the concrete to the depth of a few inches. In Fig. 235C the puddling is about completed. The top is smooth, the concrete is well against the forms, and the man operating the vibrator is standing on top of the concrete without sinking into it. This indicates that the concrete is compact.

## PLANT-MIXED, TRUCKED CONCRETE, PUMPED CONCRETE, AND CEMENT GUNS

**Ready-Mixed Concrete.** Ready-mixed concrete is now being extensively used. The term "ready-mixed concrete" is commonly known to mean concrete mixed at a plant or in transit and delivered to the job ready to be placed in the forms. In the last few years great advances have been made in plants for mixing concrete and in trucks for delivering the concrete to the site of the building operation. No additional material is added to this product at the job.

The quality of this concrete should be of the best when good materials, correct proportioning and proper mixing are employed. In practically all of these plants the aggregates are weighed. The former method of proportioning by volume is nearly obsolete in the newer and better plants. This is probably due to the development, by scale manufacturers, of equipment especially made for concrete plants. Scales for determining the weight may be beam scales or dial scales. Also the determination of the amount of moisture in the aggregates and the recording of the time of the mixing on a permanent record have helped advance plant batching and mixing materially. These plants may be automatic or manual, or sometimes a combination of the two methods is used.

In the larger plants an inspector, from some independent laboratory, is kept at the job to watch the materials, the proportioning, and the mixing. They certify that the concrete is according to the specifications under which it is being mixed. Complete laboratories

are often found at these plants. The inspectors make up the cylinders for the seven- and twenty-eight day tests. They really have a better chance of securing the proper curing of the cylinders than they do on the ordinary job, especially in cold weather. It is often quite difficult in the field to keep the cylinders at the proper temperature until they are delivered to the laboratory.

Fig. 236. Batching Plant
*Courtesy of C. S. Johnson Company*

**Central-Mixed Concrete.** Central-mixed concrete is mixed at a stationary plant and delivered to the job in an agitating truck. These plants are to be found in cities and larger towns. They are more or less of a permanent plant constructed to supply concrete in densely populated districts. Such a plant must be provided with elevated storage bins for aggregates, room for cement, water-tanks, scales for weighing materials, machines for mixing the concrete and trucks for delivering it to the site of the work. The aggregates and cement are usually elevated by means of derricks, although other means of elevating the materials are sometimes used. The concrete is generally delivered in closed trucks of the agitating type; although

where a stiff mixture is used and must be hauled only a short distance, open dump trucks are sometimes used. A good example of this type of delivery is for paving use.

The central-mixed plants are now apparently giving way to the batching plants, which appear to be more economical.

**Batching Plants.** Batching plants have increased rapidly in the last few years and, as already stated, have infringed on the central-mixed plant, although some of the central-mixed plants are now

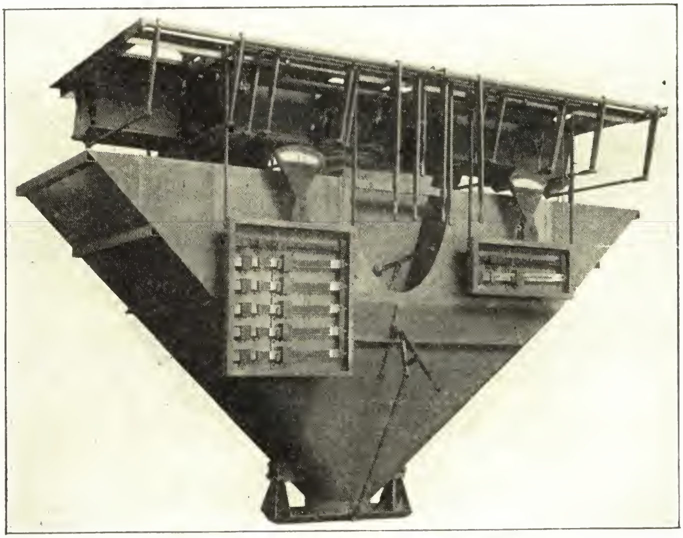


Fig. 237. Concentric Agg-Cement Batcher
*Courtesy of C. S. Johnson Company*

equipped for batching, too. The batching plant is generally supported by a steel frame, but wood is used in some cases. These plants are more or less portable and can be dismantled and moved to another location. They vary in size, depending on the amount of concrete to be poured. In the batching plant, the aggregates, cement, and water are carefully measured and placed in a truck with the water in a separate compartment. This concrete is mixed while in transit or after it reaches the job, depending on the length of haul and when it can be unloaded. Like the central-mixed concrete, the materials must be elevated and stored in large bins. If loose cement is used, it must be stored in weather-tight tanks, as it is a difficult material to

handle. When aerated, cement flows like water; but when allowed to settle in the bins, it is apt to arch and it is difficult to dislodge cement arched in this manner.

The batching plants may be single or multiple units and may be operated automatically, semi-automatically, or by hand. The single units are easier to handle and there is less danger of an error being made, as fewer levers are to be pulled by the operator.

In Fig. 236 is shown a batching plant, as already described. The aggregates are elevated and stored in the bins at the top of the plant.


Fig. 238. Rex Moto-Mixer
*Courtesy of Chain Belt Company*

Below the bins is found the platform on which are located the hoppers and scales for weighing the aggregates and cement, if loose. If bags of cement are used, they are usually opened and handled by the men.

Fig. 237 shows a concentric aggregate-cement batcher with manual control for all weighing. The beam scales are for weighing the aggregate; the pendulum dial scale is for weighing the cement.

**Delivery of Central-Mixed Concrete.** Concrete mixed at a central mixing plant is usually delivered to the job in closed trucks similar in appearance to the truck mixers shown in Fig. 238. The concrete is placed in the drum and kept in motion. The agitator body generally used is a revolving drum which may be driven by an

auxiliary engine, or the power may be supplied by the truck engine. These trucks are not equipped with a water tank; therefore, additional water cannot be added to the concrete while in transit. The capacity of these trucks varies from one to four cubic yards, depending on the capacity of the central mixing plant and also on the load permitted to be carted over the streets or highways. When possible, the capacity of the mixers in the central plant and the delivery trucks should be coordinated so that it will not be necessary to mix fractional batches to fill the drum of the truck. Concrete with a very small slump, to be transported only a short distance and used in large masses, may be transported in dump trucks. Concrete used for paving is an example of this delivery.

The length of haul, or time permitted from the time that the agitator truck is loaded until it is deposited in the forms, is usually 1½ hours. Under favorable conditions, this time may be extended; or in hot weather it may be necessary to reduce this time. The cost of long hauls will generally limit the time to a reasonable period.

**Truck Mixer.** Truck or transit methods of mixing concrete have made great progress in the last few years. Starting with the old pug mill, mounted on an open top truck, the progress has been rapid. When materials are proportioned at a central batching plant and then are truck-mixed, the concrete is often cheaper than concrete mixed at a central mixing plant with delivery of the concrete made by agitating trucks. This is largely so because the batching plant is more economical than the central mixing plant in construction and operation.

The modern truck concrete mixer is mounted on 4 wheels with pneumatic tires, larger trucks being equipped with 6 wheels. The drums are constructed of steel and contain blades which rotate on a horizontal axis to mix and discharge concrete. The blades and the shape of the drum cooperate to give an end-to-end mixing action which is assisted by the rotation of the drum. These mixers are loaded through an opening in the top and have a discharge door at the rear. The trucks must be strong enough to transport a heavy load over rough roads in order to reach many of the jobs.

Fig. 238 represents a Rex Moto-Mixer. Like many other mixers, in capacity they range from 1 to 5 cubic yards. The smaller sizes have greater speed and can be handled and placed where it would be inconvenient to operate the larger machines.

In charging the truck mixers, the stone is usually placed in the bottom, the sand next, and the cement is placed on top. On some trucks two compartments are provided for water, one for mixing and the other for washing the drum after the concrete has been discharged. In other trucks there is only one water compartment and the water for mixing must be measured. A water measuring device is attached for this purpose. The mixing period is usually 40 or more revolutions, and the drum is usually rotated by an independent gasoline engine mounted on the mixer frame. Generally, the drum can be operated

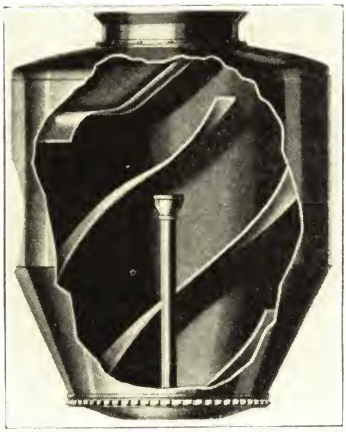


Fig. 239. Rex Modern Drum of Man-ten Steel

*Courtesy of Chain Belt Company*

at two speeds. The standard speed is used for mixing and discharging, and the slow speed for agitating the concrete. Trucks with slow speed can also be used as agitators. In the smaller size truck the truck engine is also used for mixing the concrete.

When the distance is short from the batching plant to the place where the concrete is to be deposited, the drum is started rotating on leaving the plant; if necessary, the drum can be slowed down to agitating speed. If the haul is a long one, the mixing is not started until the truck approaches the point of deposit. The concrete is discharged from the rear end of the drum. A discharge chute is part of the equipment for all trucks.

In Fig. 239, one side of the agitator cone drum is cut away, showing an interior view of a cone-shaped drum fitted with blades.

**Pumping Concrete—Pumpcrete.** In general the transportation of concrete by means of a pump and pipe line can be classed as one of the new developments in the concrete industry. This method of transporting concrete has been used for buildings four and five stories in height, for dams, locks, tunnel linings, bridge abutments, etc. This method of placing concrete has often been used advantageously to reach places found difficult by other methods; for example, when constructing the San Francisco-Oakland Bay Bridge, it was used in

Fig. 240. Rex Pumpcrete, Railroad Bridge, Milwaukee, Wisconsin
*Courtesy of Chain Belt Company*

concreting caissons for piers, in lining the vehicular tunnel on Yerba Buena Island and encasing the cables on the center anchorage.

This concrete pump is a heavy duty, single acting, horizontal piston pump with a 4-cylinder motor especially fitted for this work. The concrete is placed in the hopper, from there is drawn into the cylinder, and is then discharged into the pipe. Each new charge forces the previous charge forward and a flow of concrete is secured. Steel pipes, 6 to 8 inches in diameter, are used to convey the concrete. Pipe sections with special ends are joined by couplings, thus making the connecting and disconnecting a simple operation.

After the last batch of concrete has been pumped out of the Pumpcrete hopper, the pipeline is broken and the Pumpcrete washed

out. The Pumpcrete is converted into a water pump by inserting a special water valve. This can be done quickly and easily. A special casting is inserted into the pipeline and the pipeline is again connected to the Pumpcrete. The hopper is filled with water, the Pumpcrete is started, and the special casting begins to travel through the pipe, pushing the concrete ahead of it at about the same speed as during pumping operations. Air, instead of water, is sometimes used for cleaning out the pipeline.

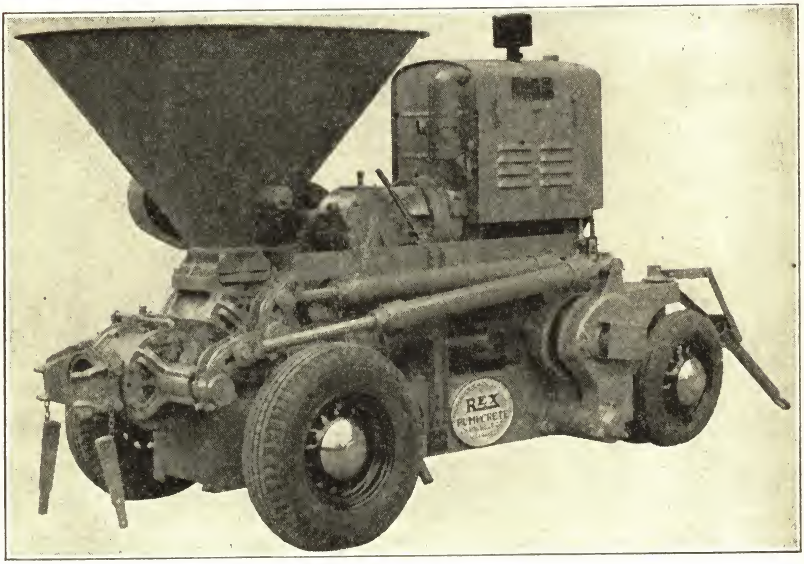



Fig. 241. Rex 160 Pumpcrete
*Courtesy of Chain Belt Company*

In Fig. 240 is shown a Rex Pumpcrete operated by a 25-horse power, 4-cylinder gasoline engine. This unit can pump concrete 800 feet horizontally or 100 feet vertically, or any combination between these limits. It has a rated capacity of 15 to 20 cubic yards per hour. In the figure, the pump is shown placed so that the mixer will discharge directly into the hopper of the pump. A 6-inch pipe extends horizontally from the pump for a short distance, then up the embankment and to the point where the concrete is wanted.

Fig. 241 shows the Rex 160 Pumpcrete mounted on pneumatic tires. This pump is made in larger sizes and also in double units

which have a rated capacity of 50 to 65 cubic yards. Any workable concrete mix can be transported by means of these pumps, and they can be operated either by gasoline or by electric motor.

**Cement Gun—"Gunite."** In Fig. 242 is shown a series of *cement guns* used for placing cement mortar under pressure, which product is commonly known as *gunite*. These machines are operated by compressed air and consist of two compression chambers, one placed above the other. The materials, which consist of sand and cement mixed dry, are placed in the upper chamber. This chamber is

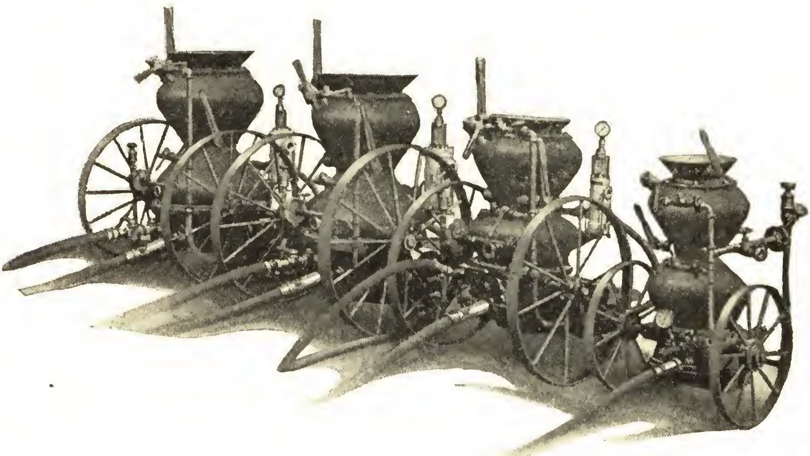


Fig. 242. Cement Guns
*Courtesy of Cement Gun Company*

alternately under pressure and free of pressure. The valve between the two chambers opens and the materials pass into the lower chamber, which is a constant pressure chamber. The feed wheel in the bottom of the lower chamber sends the material to the outlet where it is forced by air through the outlet valve to the nozzle. In a separate hose, the water, under pressure, is conducted to the nozzle where it is sprayed radially into the sand and cement. This flow is regulated by the workman. The nozzle man directs the jet of cement, sand, water, and air on the surface to be gunited. Fig. 243 illustrates the operation of the pump. The nozzle velocity meter, illustrated in Fig. 243, is a recent improvement, the use of which results in better operation and a more uniform product.

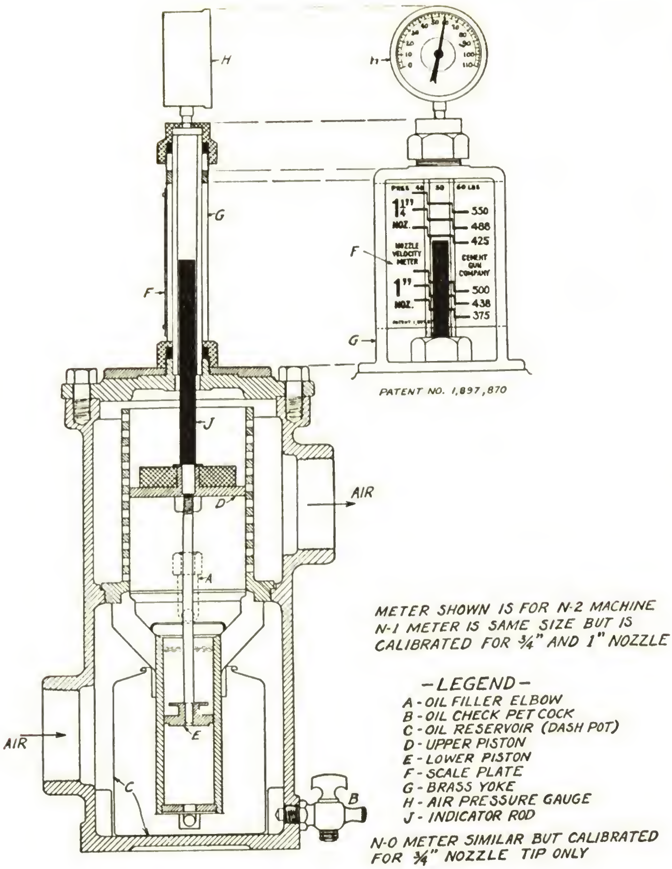



Fig. 243. "Cement Gun" Nozzle Velocity Meter
*Courtesy of Cement Gun Company*

## CONCRETE BLOCK MACHINES

**General Description.** Many improvements have been made in concrete block machines in recent years. Most of the blocks are now made on power machines. This is largely due to the great number of cinder-concrete blocks being made and used at this time. Some of the machines are entirely automatic, but for the smaller plants hand-control machines are used. Special mixers are now made for mixing the concrete and in some of the larger cinder block factories the concrete is tamped with a special vibrating machine.

Fig. 244 shows a block machine of the smaller size. The blocks are made with the face down and may be tamped either by hand control or by the use of a power tamper. This machine is equipped with three oval cores which are inserted and withdrawn mechanically

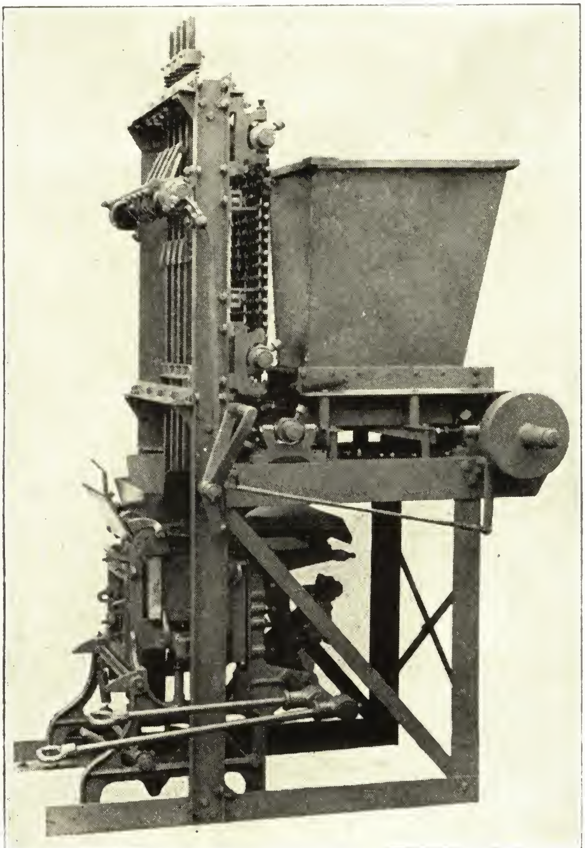


Fig. 244. Block Machine
*Courtesy of Miles Manufacturing Company*

by the lever shown at the right of the mold. Either wood or steel plates can be used. The machine can be adjusted to various heights, widths, or lengths. The thickness of the walls of a block can be made to suit the requirements of the blocks. The air space may vary from 33%, 40% to 50%, the standard being 40%. Fractional blocks and

also blocks with special designs, as shown in Fig. 245, may be made in this machine.

Fig. 246 shows a cinder block machine which may be had either fully automatic, semi-automatic, power-operated with hand controls, or hand-operated. These machines vary in capacity from 300 to 4,000 units per day. They are equipped with plain pallet strippers, usually made of steel (although wood may be used) which are about two inches longer than the blocks so that they can be handled easily. The core plates are placed lengthwise, the sides of the stripper are

Fig. 245. Concrete Blocks with Designed Faces

made of one piece, and the face liners project above the mold. When the tamping is completed, the stripper head enters, and a close fit of the stripper head inside the mold liners gives the top, edges, and corners a good finish.

The machine shown in Fig. 246 is operated by power tamping and stripping, hand feeding and strike off, but can be made to operate entirely by power.

**Mixers.** Special machines are made for mixing concrete used in the manufacture of blocks. Fig. 247 shows such a machine. These machines are made in various sizes varying from 3 to 42 cubic feet.

Unlike the ordinary concrete mixer, they are open on top and the concrete is mixed by curved blades secured to a longitudinal shaft. Paddles may be substituted for the curving blades, and they are

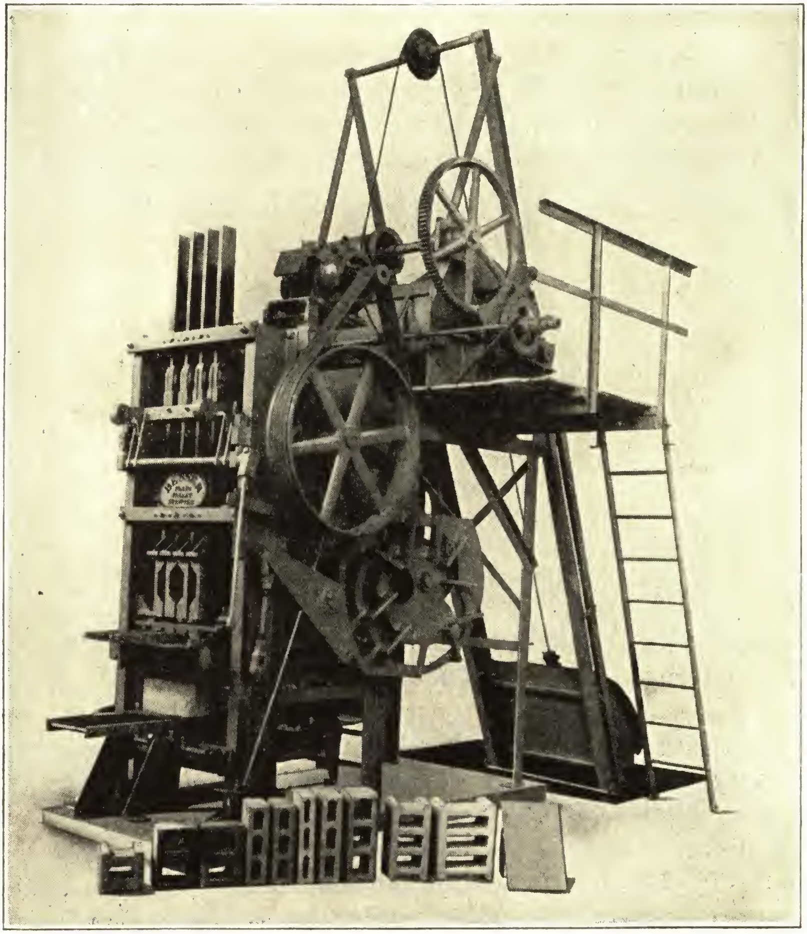


Fig. 246. Besser Champion Plain Pallet Stripper
*Courtesy of Besser Manufacturing Company*

standard in some machines. These mixers are easily charged and discharged. They are driven by geared-head motors. The gears, jackshaft, clutch, and pulley are concealed.

**Cinder Block Plants.** The cinder block industry is growing very fast and there are now many plants devoted entirely to their manu-

facture. These plants have a capacity of several thousand blocks per day.

For large plants the materials are usually raised to storage bins located several feet above the ground. From the storage bins the materials are fed to the batching scales or volume measuring tanks. The batching floor is located under the storage bins, and the mixer is placed just below this floor. From the mixer the concrete passes into the block machine, which is operated at the ground level. The blocks are then conveyed to the steam cylinders. After curing in the steam

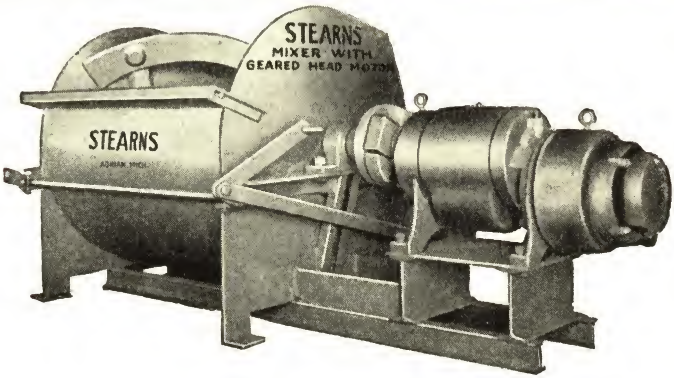



Fig. 247. Concrete Block Mixers
*Courtesy of Stearns Manufacturing Company*

for a period of 24 to 48 hours, they are placed in the seasoning shed or yards. Concrete blocks must be well cured before being used.

In Fig. 248 is shown a sectional outline for a small block manufacturing plant as suggested by the Besser Manufacturing Co. At the right of the plant is located 4 bins for the storage of aggregates, with a capacity of 42 cubic yards per bin. The aggregates are trucked up an incline and dumped into the bins through manholes in the floor. The bins and the walls of the building are constructed of reinforced concrete. When the plant is operating the aggregates are permitted to pass through the bottom of the bin, this opening being controlled by a man-operated gate, are weighed or measured by volume and dumped into the bucket shown in the pit. Cement is added and then these materials are raised and automatically dumped into the mixer. When mixed, the concrete passes to the block machine. After the blocks are tamped they are stripped and conveyed to the steam room or to the curing sheds.

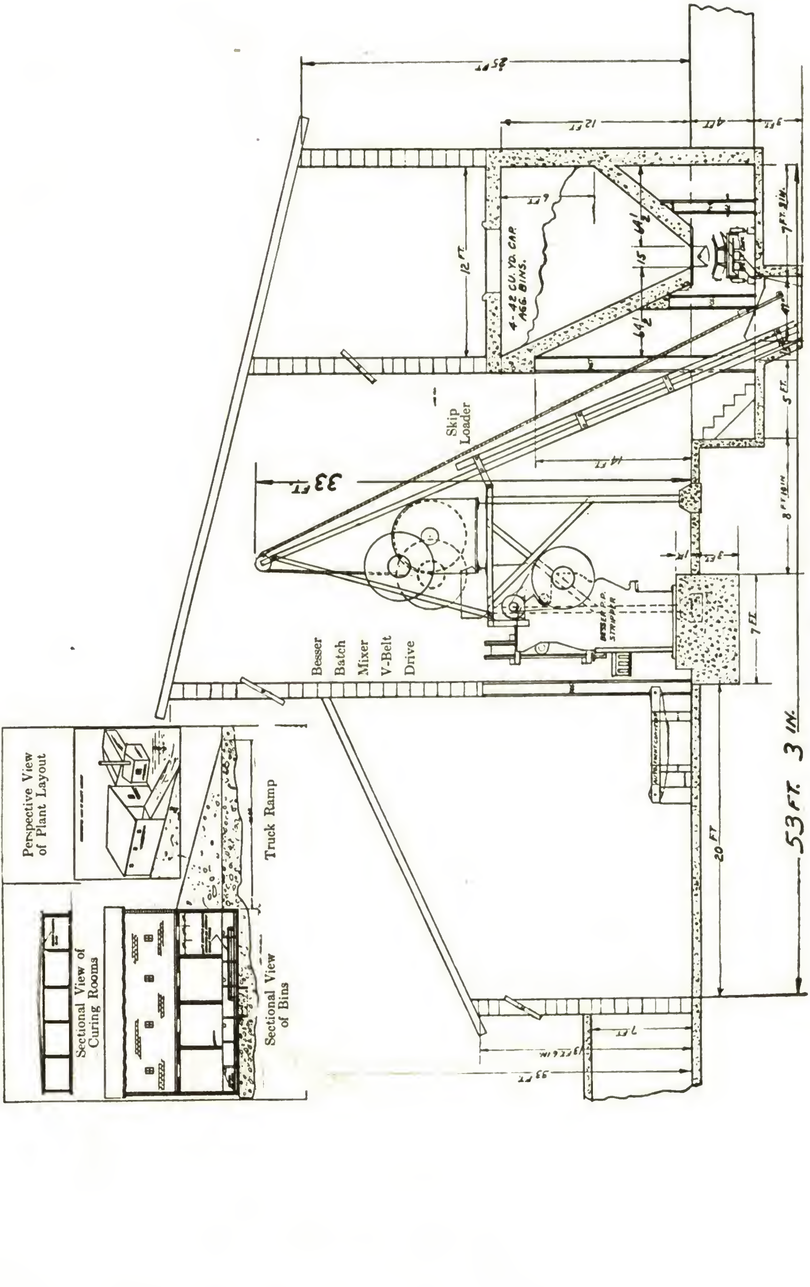



Fig. 248. Besser Simplified Plant Layout
*Courtesy of Besser Manufacturing Company*

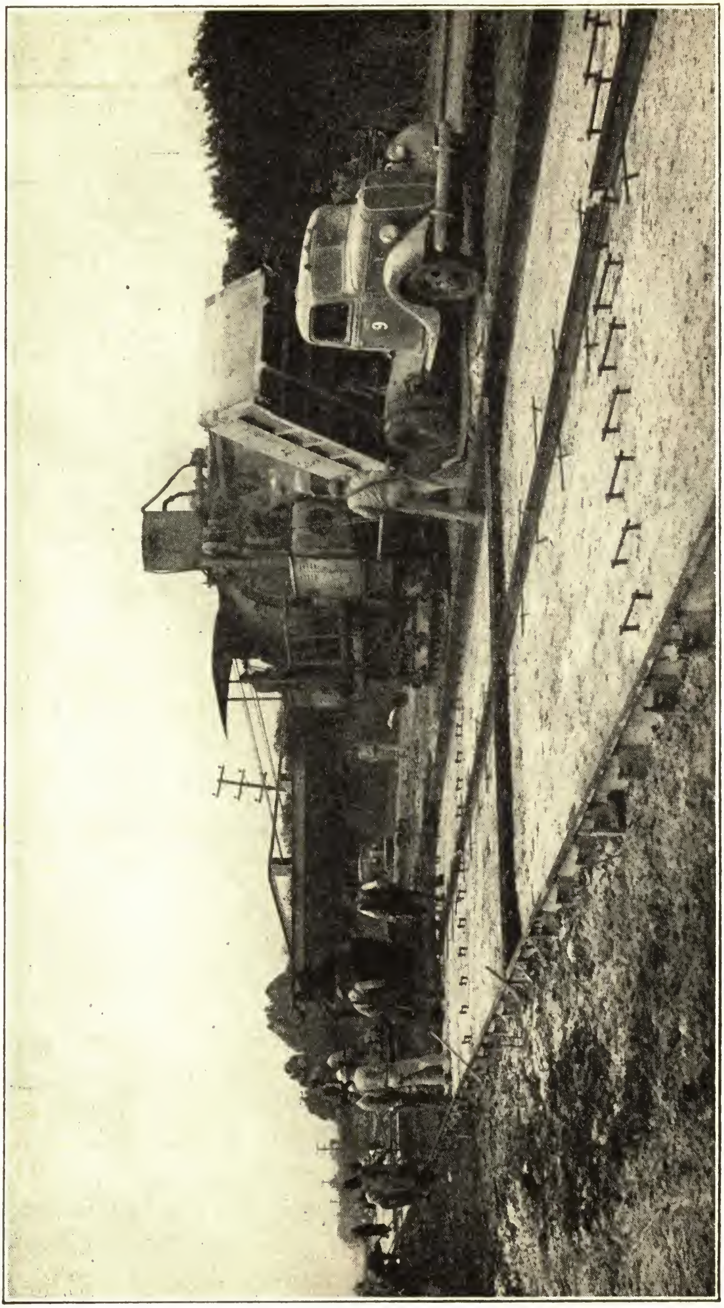


Fig. 249. Koehring Paver
*Courtesy of Koehring Company*

## CONCRETE PLANT FOR PAVING

In the highway departments of many states much has been done to secure a better working knowledge of concrete for paving purposes. The Federal Government has aided in this work. Not only has it investigated and tested materials and methods of doing work, but it has caused many important improvements to be made in machinery for highway construction, etc. The concrete now being used in highway construction is of a much better grade than that formerly used.

The paving machinery in use today is a great improvement over that manufactured a few years ago. It must be strong and well pro-

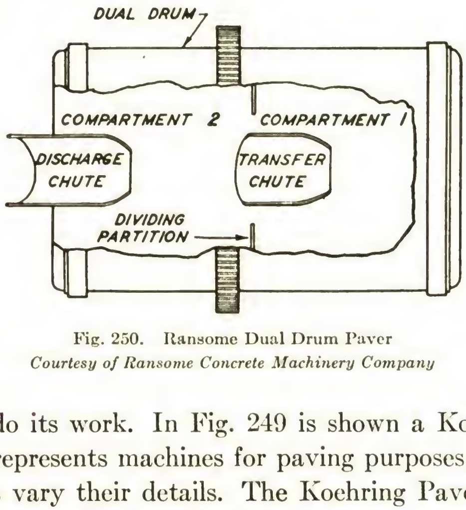



Fig. 250. Ransome Dual Drum Paver
*Courtesy of Ransome Concrete Machinery Company*

portioned to do its work. In Fig. 249 is shown a Koehring Paver. In general it represents machines for paving purposes, but different manufacturers vary their details. The Koehring Paver is equipped with a power hopper for charging the mixer, a water measuring tank, a gasoline or Diesel engine, a delivery boom, self-spreading bucket, etc. It is mounted on crawlers and has a speed of about 1.4 miles per hour at high speed and .91 miles per hour at low speed.

Road pavers are also made with a second drum. The concrete is partly mixed in the first drum and transferred to the second drum, where the mixing is finished and concrete discharged. The advantage gained here is that two batches of concrete are being mixed at the same time, and the production of concrete is, therefore, more continuous. Fig. 250 shows the drum of a Ransome Dual Drum Paver. Compartment No. 1 is charged with cement, aggregates and water in

the usual way. When partly mixed, these are transferred to the second compartment, where the mixing is completed and the concrete dis-

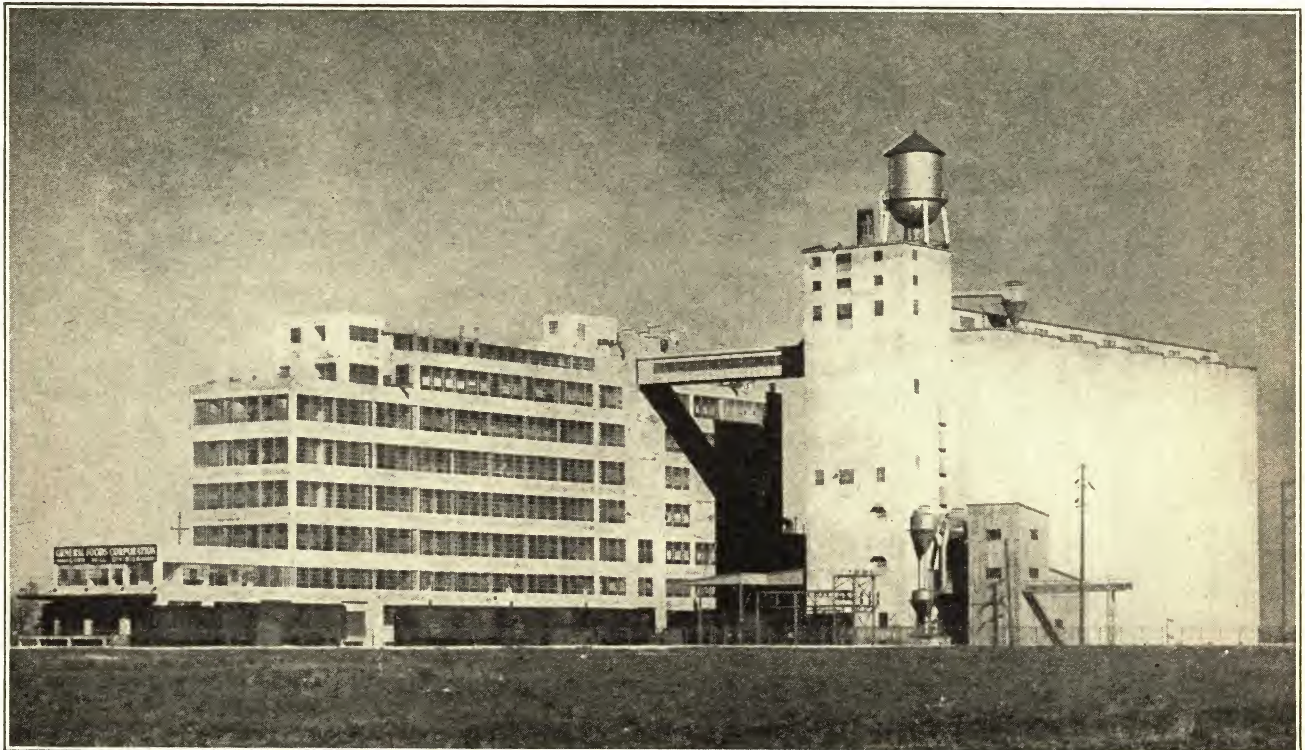
Fig. 251. Smith Paver
*Courtesy of T. L. Smith Company*

charged. In the meantime, compartment No. 1 has been recharged.

Fig. 251 illustrates a tower paver which includes a mixer, the entire unit being self-propelled. The tower is hinged so that it can be

lowered to a horizontal position while being moved from job to job. This unit may be used for paving, retaining walls, sea walls, buildings, bridges, etc. The upper chute can swing 90° on either side of the paver and the lower chute has a swing of 360°. The receiving hopper is mounted on a sliding frame, is raised or lowered by power, and is clamped in position when in operation. The hoist bucket has a capacity of 36 cubic feet.

The concrete can be placed several feet above the ground. This height depends on the height of the tower and on the angle of slope of the chutes. The chutes must be erected at such a slope that the stone will not separate from the mortar while being chuted.




REINFORCED CONCRETE INDUSTRIAL BUILDING, KANKAKEE, ILLINOIS

*Courtesy of Portland Cement Association*

# CHAPTER XXII

## SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

**Work Covered.** This specification is intended to cover the general conditions for the use of concrete and reinforced concrete in a building operation. The data given here covers only concrete and reinforced concrete and, therefore, would be a part of the general specifications for a building. To use this information in connection with the contract, it will be necessary for the engineer or architect to include all special features connected with the work. Also, it will be necessary to change qualifications for sand and stone to suit the local conditions, etc.

**General Conditions of the Contract.** The contractor for this part of the work shall carefully examine the general conditions of the contract for this work. Insofar as the general conditions refer to this part of the work, he shall be bound by all the general conditions, as they are applicable in the opinion of the engineer. These general conditions shall be made part of the contract for the concrete work covered by this part of the specification. The contractor shall make a careful examination of the site of the work before submitting his bid.

**Work Included.** The contractor for this work shall furnish all materials, tools, and labor necessary for the completion of the work, all of which shall be satisfactory to the engineer. All plain and reinforced concrete used in the construction of the footings, walls, columns, floors, lintels, areaways, stairways, pits, foundations for machinery, fireproofing of structural steel, cinder fill, floors on the ground, together with all other work shown on the structural plans and supplemented by the architectural drawings, shall be included by this contractor.

**Type of Building.** A complete framework of reinforced concrete, including exterior walls, is required for this building. All walls above the ground level shall be finished concrete painted with

Portland cement paint. The floor construction will be of the slab, beam and girder type; and the exposed surfaces will be finished ready for painting. The floors will be finished, at the time the structural floor is poured, ready for the placing of linoleum. The roof will be slightly pitched for drainage, and gussets will be formed for leading water to the downspouts. A suspended ceiling will be constructed 4 feet below the roof. The interior side of all exterior walls will be furred with 3 inches of cinder blocks for plastering. All interior partitions shall be constructed of cinder blocks and plastered. The painting will be done by another contractor.

**Exterior Walls.** Basement walls will be constructed of reinforced concrete as shown on the plans. The walls in the deep portion of the basement will be reinforced to support the earth pressure on the exterior side; but walls having a depth of 8 feet, as well as all exterior walls above the ground, will be reinforced only for temperature stresses.

The walls above the ground level will require careful workmanship and good materials, etc. These items are specified under their respective headings.

**Finished Concrete.** In general, concrete surfaces shall be finished by removing material rather than adding it. When it is necessary to patch exposed surfaces, the patching shall match the color of the original work. This may be done by substituting white Portland cement for part of the regular Portland cement, also by using a lighter colored sand. The patching, when applied, shall project about ⅛ inch and, after hardening, shall be ground down to the proper level. The mortar used for patching should never be richer than 1 part cement to 3 parts sand.

The exterior face of the basement walls below the ground level will not require any special finish, except that rough places shall be neatly filled and rubbed. The interior face of the basement walls shall have all fins or other projecting surfaces removed by grinding with a suitable machine, and all rough places shall be rubbed over.

The exterior face of all walls above the ground level must be made ready for painting, and special care will be required in this preparation. All projecting fins, form marks, etc., shall be removed carefully, and the entire surface rubbed with carborundum brick

or a grinding machine and water. All loose material shall be removed and all surfaces washed down with clean water. The interior surfaces of the walls above the first will not require a special finish as they will be furred with cinder blocks. All finished work shall be satisfactory to the architect.

The soffits of all slabs, beams, girders, and the sides of the beams and girders exposed in all floors (except the basement and top floors) shall be finished ready for painting. After removing the forms, all fins or other projecting surfaces, as well as all form marks, shall be removed. Special care is to be given to the soffits of all beams and girders. Such grinding as is necessary shall be done to secure a satisfactory finish. The finish for the basement and top floor will consist of rubbing over the rough places, and such other work as is necessary to give the surfaces workman-like appearance.

All floors shall be finished ready for the application of linoleum. This shall be accomplished by finishing the top surface the same day that the structural slab is poured. The top of the slab shall be screeded and troweled to a smooth level surface. All high spots found in the finished floor shall be ground down to the proper level.

**Stairways.** All stairways shall be constructed of reinforced concrete, including all landings and platforms. The treads shall be finished at the time the structural floor is poured. This finish shall consist of one inch of cement mortar, composed of one part cement and two parts of well-graded sand, which shall be troweled to a smooth finish. As soon as the forms are removed, the risers shall be finished by a coat of cement mortar of sufficient thickness to make them smooth.

Any imperfections of the treads at this time shall be rubbed over and made satisfactory, but no material shall be added to the treads. As soon as the forms are removed, the beam along the end of the risers and treads shall be thoroughly rubbed with sufficient mortar to make them smooth.

NOTE. If it is desirable to use metal, stone, or safety treads, the description should be added here.

**Roof Finish.** The top of the slab will be finished (at the time the structural floor is poured) with sufficient cement mortar composed of one part cement and four parts of sand, which shall be

troweled smooth ready for the roofing. This contractor will include and construct all gussets, valleys, etc., required for draining the roof to the openings, as shown on the plans.

**Basement Floor.** An excavation should be made to the depth of 12 inches below the finished basement floor. Cinders shall be tamped to a depth of 8 inches and on this surface shall be placed 4 inches of stone concrete, composed of 1 part cement, 3 parts sand, and 5 parts stone, which shall be well tamped. Immediately after the base is placed, the finish shall be applied, which shall consist of one part Portland cement and three parts of well-graded sand. The entire surface shall be floated to an even, smooth surface. The concrete shall then be protected with sand or canvas and kept well soaked with water for a period of at least two weeks.

If the material being excavated from the basement is sand or gravel, the cinder fill may be omitted and the concrete poured directly on the sand or gravel.

**Provision for Pipes.** The contractor for the reinforced-concrete work shall work in conjunction with the contractors for the plumbing, heating, lighting, etc., in laying out this work so that the strength of the floors will not be weakened by the holes required in the floors for pipes, outlets, etc.

Build in all sleeves required for piping or other mechanical work. Build in all inserts required for the support of shafting, etc.

**Design.** In designing this work the continuous beam theory has been used. The working stresses for the steel and concrete are as follows:

| | Lbs. per sq. inch |
|---|---|
| Extreme fiber stress on concrete in compression | 800 |
| Concrete in direct compression | 500 |
| Shearing stress in concrete when properly reinforced | 150 |
| Bond stress between concrete and plain reinforcing steel | 80 |
| Bond stress between concrete and deformed bars | 100 |
| Tensile stress in steel | 20,000 |
| Compression in steel | 7,500 |

**Reinforcing Steel.** Deformed bars should be used throughout the work with the exception that plain round bars may be used for stirrups and column bands. The steel may be made by either the open-hearth or electric-furnace process, and shall be rolled from billets. All chemical and physical properties of the reinforcing

steel shall be according to the Specifications for Concrete Reinforcing Bars, Serial Designation A15-35, intermediate grade, as adopted by the American Society for Testing Materials.

NOTE. These specifications cover the different grades of steel commonly used in reinforcing concrete. The intermediate grade is rolled especially for reinforcing steel and has a higher tensile strength than the structural grade but not so high as the hard grade. The bars rolled to this grade of steel can be bent more readily than the bars rolled from the hard grade.

This Society also issues a Specification Serial Designation A16-35 for rail-steel concrete reinforcing bars. The rail steel has been much used in the Middle West while in the East the billet steel is generally used. The rail steel is cheaper than the billet steel.

**Placing Steel, Drawings, etc.** All steel bars shall be accurately placed in the forms and fastened there so that they will not be displaced while the concrete is being placed and spaded. Mechanical devices, approved by the engineer, may be used for holding the steel in place or it may be securely wired together and placed on cement blocks of the proper thickness.

All bars should be bent as shown on the detail drawings or as directed by the engineer. The end span of all beams, girders and slab bars shall have hooks on the ends of the bars of the non-continuous end.

All detail drawings shall be sent to the engineer for his approval. These drawings will be criticized and returned to the contractor for corrections, if necessary; and after all corrections are made, three sets of the finished detail drawings will be sent to the engineer.

**Inspection.** The contractor furnishing the reinforcing steel shall supply the engineer with certified copies of the manufacturer's tests of all the reinforcing steel used in the work.

All reinforcing steel shall be accurately placed in the forms as shown on the drawings, and before any concrete is poured, the steel shall be inspected by the engineer. The contractor shall notify the engineer well in advance of the time of pouring the concrete so that the engineer will have ample opportunity to inspect all reinforcing steel, materials for concrete, and the pouring of the concrete.

**Cement.** Portland cement shall be used throughout the work. It shall be of a standard brand approved by the engineer and shall

conform to the standard specifications for Portland Cement (Serial Designation C77-37) of the American Society for Testing Materials. All cement shall be tested by a testing laboratory approved by the engineer. The cement shall be delivered to the work in bags with the brand of the cement and the name of the manufacturer plainly marked thereon. It shall be stored and sealed while the tests are being made. The building in which it is stored shall be watertight and the floor located at such a height that the cement shall not draw dampness from the ground. Each shipment of cement shall be kept separate and none of it used until it has at least successfully passed the seven-day test.

**Sand.** The sand for this work shall consist of a well-graded natural or artificial sand that has clean, hard, strong, durable, uncoated grains free from all injurious amounts of dust, lumps, soft or flaky particles, such as shale, alkali, organic matter, loam, or other injurious substances. When washed, the sand shall not contain over three per cent of loam or other fine material.

NOTE. If is is desirable to give the range of size of grains of sand from fine to coarse, the following table published by the American Society for Testing Materials may be used:

**Grading of Fine Aggregate**

| | Per Cent by Weight |
|---|---|
| Passing through No. 4 sieve | not less than 85 |
| Passing through No. 50 sieve | not more than 30<br>not less than 10 |
| Weight removed by decantation | not more than 3 |

Many engineers, in writing specifications for sand, specify that it shall be washed, and also, the sand from a new source of supply should always be carefully tested before being used.

**Stone.** The crushed stone, washed gravel, or other inert materials for this work shall have clean, hard, strong, durable, uncoated particles, free from soft, pliable, elongated, or laminated pieces, alkali, organic, or other injurious matter. In size, it shall range from fine to coarse. When being used for reinforced concrete, it shall be graded from one-fourth inch to one and one-fourth inches; and when used for plain concrete, it shall be graded from one-fourth inch to two inches.

**Water.** The water for the concrete shall be clean and free from

injurious amounts of oil, acid, organic matter, or other injurious substances.

**Proportions.** The unit of measure shall be the cubic foot. Ninety-four pounds of cement (one bag or ¼ bbl.) shall be considered as one cubic foot. The method of measurement shall be such as to secure the specified porportions in each batch. The aggregates shall be measured separately by volume. The fine and coarse aggregate shall be measured loose as thrown into the measuring device and struck off. The water shall be so measured as to insure the desired quantity in successive batches.

The concrete used in this work must produce a certain strength at the age of 28 days, as specified in the following table. The proportions given in this table are only to serve as a guide in the mix, but no mix shall be leaner than the proportions specified in this table.

**Strength and Proportions for Concrete**

| Grade of Concrete | Proportions by Volume | | | Concrete Strength, Lb. per Sq. Inch at 28 Days |
|---|---|---|---|---|
| | Cement | Fine Aggregate | Coarse Aggregate | |
| Plain concrete | 1 | 3 | 5 | 1500 |
| Beams, girders, slabs—reinforced | 1 | 2½ | 3½ | 2000 |
| Columns | 1 | 1½ | 3 | 2200 |

Variations in the grading of aggregates, on which the proportions are based, are to be made upon the approval of the engineer, but no claim shall be made for any extra compensation for such changes.

The quantity of water to be used shall be the minimum necessary to produce a concrete that is workable. The amount of water for the 1:2½:3½ concrete shall not exceed 7½ gallons (U.S. Standard Measure) for each one-bag mix of concrete, and the slump shall not amount to more than five inches when made according to the standards of the American Society for Testing Materials. The 7½ gallons of water must include all moisture in the materials.

**Mixing.** All concrete shall be mixed in an approved type of batch mixer that will produce a uniform and homogeneous concrete. The mixer shall be equipped with a sutiable charging hopper, water storage and water-measuring device controlled from a case that

can be kept locked and so constructed that the water can be discharged only while the mixer is being charged. It shall also be equipped with an attachment for automatically locking the discharge lever until the batch has been mixed the required time after all materials are in the mixer. The entire contents of the drum shall be discharged before the drum is recharged. The mixing of each batch shall continue not less than one minute after all materials are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200 feet per minute. The retempering of cement which has partially set will not be permitted.

**Depositing Concrete.** The concrete shall be handled from the mixer to the place of final deposit as rapidly as possible by methods which will prevent the separation or loss of the different materials. It shall be deposited in the forms as nearly as practicable in its final position to avoid rehandling.

As soon as the concrete is deposited, it shall be thoroughly compacted by means of suitable tools for assisting in spading, to avoid stone pockets and to secure the maximum density. Special care shall be taken in placing concrete in columns. The forms shall be continuously tapped, hammered or vibrated to assist in getting a dense concrete.

Special care will be required when pouring the exterior walls. This concrete shall be placed through canvas or metal spouts. Concrete shall not be permitted to splash against the forms. The forms must be kept clean.

If the concrete is conveyed by chuting, the tower shall be of such height that the angle of the chute with the horizontal will allow the concrete to flow without separation of the ingredients. The minimum angle for the chute shall be 27 degrees, or one vertical to two horizontal. The delivery end of the chute shall be as close as possible to the point of deposit, and it is preferable that the spout discharge into a hopper and the concrete be conveyed from the hopper to the floor. The chute shall be thoroughly flushed with water before and after each run. The water for this purpose shall be discharged outside of the form.

**Tests.** For each day's pouring or fraction thereof the contractor shall make up four test cylinders, which shall be 6 inches in diameter and 12 inches high, and they shall be delivered to an ap-

proved laboratory for testing. These tests will be made for 7 and 28-day periods. The testing laboratory shall make prompt reports to the engineer. All tests shall be paid for by the contractor.

**Construction Joints.** All columns shall be poured at least four hours in advance of the pouring of the floor. Likewise, all walls poured to support concrete construction must stand at least four hours before the floor is poured. All joints in columns and walls shall be horizontal and the surfaces shall be cleaned of all laitance before the concrete above is poured.

All construction joints in beams and slabs shall be vertical and shall be located as near the center of the span as possible. Whenever the structural layout prevents joints being made at the center of the span, these joints shall be made with a key at the center of the depth and shall be reinforced near each face of the member, which in length shall be at least 70 diameters of the bar. Also, inclined bars shall be put through this joint.

Construction joints for exterior walls above the ground level will require special care. These joints must not show in the finished work. The architect shall be consulted in reference to the location of all vertical and horizontal joints and a diagram shall be made showing these joints. This diagram must be approved by the architect and, when concreting, the contractor shall rigidly follow these diagrams. In general, joints will be located at window sills, rustications, belt courses, etc. When it is necessary to make a joint in a plain wall a stop strip shall be nailed to the form at the top of the pour and the concrete worked tight under it. All laitance is to be removed from the top of the previous pouring.

Wall joints, both horizontal and vertical, shall be made with a groove; straight joints will not be permitted. This groove shall be made by placing a $2'' \times 4''$ or $2'' \times 6''$ timber, slightly beveled and oiled, in the center of the wall at the point where concreting is to be stopped. The timber and all laitance shall be removed as soon as the concrete has set.

**Curing of Concrete.** Concrete poured in warm weather shall be protected from drying out for a period of at least 7 days. Concrete shall be protected from the direct rays of the sun and shall be kept moist at all times.

Special attention shall be given to the walls above the ground

level and the floors. They shall be protected by means of burlap, canvas, etc., and kept wet until the concrete is cured.

**Cold Weather Protection.** Concrete poured when the temperature is at the freezing point or below shall receive special attention. The materials used in the concrete shall be heated and care shall be taken that no material is delivered to the mixer that contains frozen lumps. The concrete shall be delivered to the forms at a temperature not less than 60 degrees and on being placed in the forms shall not be permitted to drop below this temperature for any length of time, for a period of at least six days. If necessary, the structure shall be inclosed and artificial heat used to maintain a temperature of not less than 60 degrees nor over 80 degrees. Salt or other chemicals shall not be used in the concrete to prevent the concrete from freezing.

**Fireproofing.** All slab bars shall be protected by 1 inch of concrete on the bottom and also on the top. The bars in the beams shall be covered by not less than 1½ inches on the sides and the bottom. In girders there shall be 1½ inches of concrete on the sides and 2 inches on the bottom. The longitudinal bars in the columns shall be protected by 2 inches of concrete.

All structural steel beams shall be protected by 1½ inches of concrete on the sides and bottom, and girders shall be protected by 1½ inches of concrete on the sides and 2 inches on the bottom. All steel columns shall be protected by 3 inches of concrete. Continuous clips shall be placed on the bottom flange of all beams and girders to assist in holding the concrete in place. Four vertical bars shall be placed just outside of the steel column and tied together by ¼-inch round bars spaced 8 inches on centers.

**Forms.** Forms shall be required for all concrete and reinforced concrete, except for floors laid on the ground.

Forms shall be substantial and unyielding so that the concrete shall conform to the designed dimensions and contours and shall be tight to prevent the leakage of mortar. Special care shall be taken to have the forms for all beams and girders straight and true to line. No warped or splintered lumber shall be used for forms. Sufficient sets of forms shall be provided so that the work can be carried on continuously.

The lumber shall be surfaced on the side exposed to the concrete.

The forms for the exterior face of all walls and all floor construction, including columns, shall be lined with plywood or hardprest Masonite. The forms shall be constructed so that satisfactory surfaces shall be secured. Special care shall be given to forms for cornices and other architectural features. The plywood or Masonite shall be oiled when received at the job and kept in good condition.

Wall forms may be bolted. Special bolts made for this purpose shall be used. They shall be removed from the finished work and the holes carefully filled and rubbed. Wiring of wall forms together will not be permitted.

Openings shall be left at the bottom of column forms so that any shavings, sawdust, etc., can be readily removed from the forms before the concrete is poured. All openings shall be closed before the concrete is poured. The forms must be constructed so that they can be taken down without spoiling the corners or damaging the concrete. Before any concrete is placed in the forms they shall be thoroughly soaked with water. No forms shall be removed until the concrete is hard set. Before the removal of any forms, the contractor shall satisfy himself that the concrete has progressed to the point of safety and he shall be responsible for any damage resulting from the removal of forms before the concrete has attained the proper strength. No forms shall be removed in a shorter time than given in the following schedule:

| | |
|---|---|
| Bottom of slabs, span 6′ | 5 days plus one extra day for each additional foot of span |
| Bottom of beams and girders, span under 20′ | 14 days |
| Bottom of beams and girders, span of 20′ or over | 21 days |
| Sides of lintels, beams, and girders | 3 days |
| Columns | 3 days |
| Walls | 3 days |

Shores shall be kept under at least two floors below the floor that is being poured, except when the first floor above the ground is being poured.

The contractor shall allow such additional time as may be deemed necessary to insure absolute safety and shall always consult the engineer before removing any forms. All forms, including forms for the footings, shall be removed from the work.

**Test Loads.** If required by the engineer, two tests shall be made by the contractor at the expense of the contractor. The load

for floor tests shall be twice the live load for which the floor is designed. The test shall remain on the floor for twenty-four hours and shall not cause a deflection exceeding $\frac{1}{600}$ of the span, shall show no signs of crack, and shall leave no permanent deflection.

In making the tests, should any work be damaged or unduly strained, the contractor shall rebuild at his own expense all work damaged.

**Defective Work.** Should any voids or other defects be discovered when the forms are taken down or before, the defective work shall be made satisfactory to the engineer by either rebuilding the defective section or any adjacent work connected with it, or if only a small section and in a place of no great importance is defective, that section alone will be cut out and replaced. In any case, all defective work shall be made satisfactory to the engineer, in amount of work to be rebuilt, materials, and manner of doing the work.

# QUESTIONS AND PROBLEMS

## PERTAINING TO CONCRETE DESIGN AND CONSTRUCTION

### CHAPTER I

1. What is the difference between cement mortar and concrete? Page 3.
2. Explain the "Grab Method" of sampling cement. Page 9.
3. Explain the steps required in crushing stone. Page 6.
4. What is low heat cement? Page 4.
5. How does high-early strength cement differ from ordinary cement? Page 5.
6. How can temperature stress cracks be prevented in concrete? Page 3.
7. Explain the difference between plain and reinforced concrete. Page 2.
8. Explain the function of "knobs" on reinforcing steel. Page 2.
9. What calcareous materials are used in making cement? Page 3.
10. Explain the difference between natural and Portland cements. Page 4.
11. Explain the use of Vicat apparatus. Page 11.
12. Explain the soundness test for concrete. Page 12.
13. What is a briquet? Page 19.
14. How many pounds of cement are contained in a barrel? Page 23.
15. What does the symbol "C9-38" mean? Page 8.

### CHAPTER II

1. What is the purpose of the decantation test? Page 27.
2. Explain how to compare two different sands. Page 29.
3. Explain how to make and approximate moisture calculation for sand. Page 30.
4. What does the word "Craze" mean relative to concrete? Page 51.
5. What important consideration is necessary for expansion joints? Page 50.
6. How much material is required for one cubic yard of 1:3:5 concrete? Page 49.
7. Explain the slump test. Page 41.
8. On what does the compressive strength of concrete depend? Page 40.
9. What is necessary in order to secure the best results when making concrete? Page 36.
10. Explain what is meant by the water-cement ratio. Page 35.
11. Is the sulphur found in cinders injurious to cinder concrete? Page 32.
12. Why must concrete be dense? Page 33.
13. What kind of a mix should be used for concrete exposed to water? Page 36.

14. Find the number of bags of cement, and the cubic yards of sand and stone required for a cubic yard of concrete if a mix of 1:3:6 is used. The sand

weighs 105# and gravel 110# per cubic foot; specific gravity of sand and stone is 2.65 and of cement, 3.1; and the water-cement ratio of 7 gallons per bag is used.

15. Find the number of bags of cement, and the cubic yards of sand and stone required for a cubic yard of concrete if a mix of 1:2:4 is used; with the sand weighing 100# and trap rock 120# per cubic foot; if the specific gravity of sand is 2.60, stone 3.0, and cement 3.1; and if the water-cement ratio of 6 gallons per bag of cement is used.

## CHAPTER III

1. How can sand and stone be measured for ordinary concrete? Page 53.
2. Why does concrete resist the passage of heat through it? Page 70.
3. Explain how to protect steel in retaining walls. Page 69.
4. Explain how concrete can be waterproofed using felt and asphalt. Page 67.
5. What kind of lime can be used to make concrete impervious? Page 65.
6. Explain how transit-mixed concrete is made. Page 54.
7. How can one tell when hand-mixed concrete is sufficiently mixed? Page 55.
8. What is the purpose of puddling? Page 55.
9. What advantages are gained by vibrating concrete? Page 57.
10. How are straight joints avoided in bonding construction joints? Page 58.
11. What temperature is required for the best chemical combination in concrete? Page 59.
12. Explain how concrete is made and mixed in very cold weather. Page 60.
13. Explain how a concrete floor can be well cured. Page 62.
14. What is "controlled concrete"? Page 55.
15. What advantage is gained by placing concrete during cold weather? Page 61.

## CHAPTER IV

1. Explain "mechanical bond." Page 72.
2. How is the yield point determined? Page 77.
3. Explain what and how many tests are given new bars. Page 78.
4. What minimum yield point should cold twist bars have? Page 77.
5. How is expanded metal designated? Page 75.
6. How many classes of steel are used for reinforcing bars? Page 71.

## CHAPTER V

1. Explain how concrete is deposited under water. Page 83.
2. What is the leanest mixture that can be used for grouting stone under water? Page 84.
3. What is a semi-fluid soil? Page 85.
4. How are the sides of a foundation excavation kept from sliding in a moderately wet soil? Page 89.
5. What size and shape of sections are generally used for steel piles? Page 96.
6. How is a load distributed uniformly over a group of piles? Page 113.
7. What is the advantage of cast-in-place concrete piles? Page 112.
8. What is the "actual length" of a wood pile? Page 111.
9. How are piles driven in sand? Page 109.

10. What supporting material should be present for Gow caisson piles? Page 105.

11. What is a spur pile? Page 98.

12. A rubble concrete is composed of a $1:2\frac{1}{2}:3\frac{1}{2}$ mix. The rubble stone used averages 0.33 cubic yards for each cubic yard of concrete. How much cement, sand, and stone would be required for one cubic yard of masonry?

13. Find the thickness of a plain concrete footing to support a load of 16,000# per lineal foot of wall; thickness of wall above the footing to be 16″; allowable soil pressure of the ground to be 5,000#□′; concrete to be 1:3:5 mix.

14. Design a Gow caisson to support a load of 550 tons, on rock of sufficient compactness to support a load of 20 tons. Find the minimum section of the caisson and the diameter of the bell if the concrete is to support a safe load of 500#″.

15. A footing is composed of 9 piles driven by a steam hammer, the last five blows on each pile having a mean penetration of ¼″, weight of hammer 3,000#, drop 3′. What load should the footing safely support?

## CHAPTER VI

1. Describe the three ways in which a masonry wall may fail. Page 115.

2. Explain the recommended procedure for designing a retaining wall. Page 117.

3. How can an engineer obtain information about retaining walls other than by theoretical or graphical study? Page 118.

4. Where is the force $E$ applied to a surcharged wall? Page 119.

5. Why is it that an abutment for an arch bridge does not have to be designed as a retaining wall? Page 127.

6. What is meant by "flaring wing walls"? Page 128.

7. Why is it advisable to compute the elements which affect stability in a pier? Page 125.

8. Where are "weep holes" used in retaining walls? Page 123.

9. How can stability against sliding be accomplished in a retaining wall? Page 115.

10. Find the stresses produced in a concrete retaining wall; that is, the resultant and its relation to the middle third, as well as pressure on the toe and the heel. The wall is to be 16′ in height, 8′ wide at base; the front to have batter of 1½″ per foot in height. The fill behind the wall is to be the same height as the wall. Make a diagram showing all details, resultant stresses, etc. Weight of concrete is 150#; earth 100# per cubic foot.

## CHAPTER VII

1. Explain how expansion joints are made in sidewalks and what their purpose is. Page 133.

2. Explain how and why a concrete floor is cured. Page 147.

3. How is an old concrete floor prepared for resurfacing? Page 145.

4. Explain the Fluosilicate treatment. Page 143.

5. What is meant by "rebound" and where or when does it occur? Page 142.

6. Why do cinder concrete blocks provide better insulation in a wall than brick or stone? Page 137.

7. Why must fine aggregate be used in making concrete blocks? Page 140.

8. What materials are used to make the two types of lightweight concrete? Page 142.

9. Describe the preparation of a wearing surface for a concrete structural slab. Page 144.

10. What water specification should govern the mixing of concrete for resurfacing old concrete floors? Page 145.

## CHAPTER VIII

1. What minimum ultimate strength (28 days) is used in designing the concrete for the majority of buildings? Page 175.

2. What happens when the discrepancy between the ratio of steel assumed and the theoretical value is very great? Page 171.

3. How can the strength of a beam of given size be increased without increasing its dimensions? Page 170.

4. Does the modulus of elasticity for concrete vary? Explain. Page 167.

5. Explain what is meant by "transformed section." Page 165.

6. What constitutes the resisting moment for a concrete beam? Page 150.

7. How can the shear at the end of a beam be determined? Page 151.

8. How is the maximum moment at the center of a beam figured? Page 153.

9. Find the values of $k$ and $j$ when conditions are such that **(a)** $p=.01$ and $n=15$; **(b)** $p=.015$ and $n=12$. Use Equations (**14**) and (**15**) in the solution of the problem.

10. With allowable stresses of $f_c=1{,}000\#/\square''$ and $f_s=20{,}000\#/\square''$, and $n=12$, find **(a)** the theoretical value of $p$; **(b)** the value of $k$ and $j$ and **(c)** the value of $K$ in the expression $M = Kbd^2$.

11. A simple reinforced concrete beam 12″ wide and having a depth of 24″ to the center of the steel is reinforced with four 1″$\phi$ bars, $A_s=3.14\square''$. Using the method of the "transformed section" determine **(a)** the location of the neutral axis if $n=15$; **(b)** the value of $jd$ for this position of the neutral axis; **(c)** check **(a)** using Equation (**14**). To aid in the solution, draw a section of the beam as described in the problem.

12. A reinforced concrete beam is 12″ wide and has a depth of 19½″ to the steel. If the allowable fiber stresses are $f_c=800\#/\square''$ and $f_s=20{,}000\#/\square''$, and $n=15$, **(a)** what is the size of the steel bars which are required for a balanced design, and **(b)** what is the resisting moment of the beam in foot-pounds? Solve **(b)** using the method of transformed section.

13. A reinforced concrete beam which is 12″ wide carries a uniform load of 1850# per lineal foot (including the weight of the beam) on a simple span of 20′. If the concrete is stressed to $800\#/\square''$ and the steel is stressed to $20{,}000\#/\square''$ and $n=15$, what is the amount of reinforcing steel required; and what is the total depth of the beam, allowing approximately 2½″ from the center of the steel to the bottom of the beam?

14. What is the resisting moment of a simple reinforced concrete beam 10′ wide, 22″ deep to the center of the steel, reinforced with $2.08\square''$ of steel (two ¾″$\phi$ and two ⅞″$\phi$ bars) if 2500# concrete is used so that $n=12$, $f_c=1{,}000\#/\square''$ and $f_s=20{,}000\#/\square''$. Use method of transformed section.

15. Why is a beam constructed of plain concrete not very strong? Page 159.

## CHAPTER IX

1. If conditions require the use of $Wl \div 8$ how is Table XX used? Page 181.

2. Why are concrete floor slabs usually 2 inches above the associated steel beams? Page 183.

3. What is the minimum thickness for floor slabs when used with beam and girder construction? Page 182.

4. Are the slab bars in thin slabs straight or otherwise? Page 182.

5. Under what circumstances is it common practice to consider that the entire load on a slab is carried in the short direction? Page 184.

6. What is the coefficient of expansion for concrete? Page 185.

7. How are temperature cracks avoided in slabs which structurally require reinforcing in only one direction? Page 186.

8. Under what conditions may the amount of fire protection be limited to 1 inch? Page 193.

9. How much steel area should bearing walls have vertically and horizontally? Page 186.

10. (a) A floor slab is 5″ thick; what should be the spacing of ⅜″φ bars for temperature stresses if deformed bars are used? (b) A roof slab is 3½″ thick and reinforced with plain round bars, what should be the spacing of ⅜″φ bars for temperature stresses?

11. Design a floor slab for a panel 10′x12′ to be supported in two directions. Live load is to be 100#/□′, and slab is to be continuous in all directions so that $M=\frac{Wl}{12}$; $f_c=800\#/\square''$; $f_s=20{,}000\#/\square''$ and $n=15$. What would be the thickness of the slab to the center of the steel and what would be the spacing of ⅜″φ bars?

12. (a) To assist in making concrete watertight in tanks, etc., the stresses are often made lower than for regular structural work. If it is decided to make $f_c=600\#/\square''$, $f_s=14{,}000\#/\square''$ and $n=15$, what moment factor would be used as determined by Fig. 82? (b) What would be the steel ratio?

13. Explain why a beam having four 1″ square bars, which are anchored at ends and placed in one row, should be 11½″ wide, allowing 1½″ of fireproofing as shown in Table XXII.

14. Find the spacing of ⅜″φ bars for a 4″ slab, effective depth 2¾″, live load 100#/□′, span 6′, semicontinuous construction; $f_s=20{,}000\#/\square''$, $f_c=800\#/\square''$, $n=15$.

15. A fully continuous slab with a span of 12′ is required to support a live load of 175#/□′, in which $f_s=20{,}000\#/\square''$, $f_c=800\#/\square''$, and $n=15$. Find the thickness of the slab and the spacing of bars if ½″φ bars are used. One inch of fireproofing is required.

## CHAPTER X

1. At what point in a beam does the tension change most rapidly? Page 195.

2. Why should hooks be provided at the ends of bent up bars? Page 206.

3. If a group of bars are bent up in a single plane what formula is used to compute the area of the bars? Page 205.

4. Are the centers of gravity of trapezoids generally near the centers? Page 204.

5. When designing stirrups for a reinforced concrete beam which is designed first, the spacing or the size of the stirrups? Page 203.

6. A reinforced concrete beam, 14″ wide and a depth of 24″ to the center of the steel, supports a total load of 3,000# per lineal foot on a span of 21′. Using ⅜″ϕ for stirrups, calculate the number of stirrups required, and their spacing, if the concrete carries the diagonal tension equivalent to a vertical shear of 40#/□″.

7. A reinforced concrete beam, 12″ wide and 16½″ deep to the center of the steel, is reinforced with four ¾″ϕ bars, and with a span of 18′ supports a uniform load of 700# per foot of beam. Allowing 2″ from the center of the steel to the bottom of the beam, what is the bond stress in the bars at the support? The weight of the beam is to be added to the load and included in the calculations.

8. A concrete beam 8″ wide and 22″ to the center of the steel is reinforced with six ⅞″ϕ bars. The beam supports a total load of 2,100# per lineal foot on a clear span of 18′. Two of the bars are bent up 20″ from the support and one bar is bent up for 40″ from the support. Assuming that the concrete carries diagonal tension equivalent to a shear of 40#/□″, will stirrups be required in the part of this beam where the bent up bars are used? Draw a diagram illustrating problem.

9. A reinforced concrete cantilever beam projects 10′ from a concrete pier and carries a total load of 1,500 lb. per foot, including the weight of the beam. The beam is 12″ wide, 24″ deep to the center of the steel, and reinforced with three 1″ϕ bars. (**a**) What is the bond stress in the bars at the support and (**b**) how much length is required beyond the face of the support to properly anchor the bars, assuming that they are stressed to 20,000#/□″ and the bond stress is that calculated in (**a**)?

10. A beam 8″ wide has a depth of 16″ to the center of the steel and an end reaction of 16,000#. If the concrete takes care of the diagonal tension to the amount measured by a unit shear of 60#/□″, what is the theoretical end spacing of ⅜″ϕ stirrups?

## CHAPTER XI

1. What type of formulas are used to calculate resisting moments in **T**-beams? Page 210.

2. Can the steel ratio in a **T**-beam be so chosen that the compression in the concrete and the tension in the steel will simultaneously have certain definite values? Page 212.

3. Should concrete girders be deeper than the beams they support? Page 215.

4. Should the reinforcing steel for a girder be above or below the steel in the beam it supports? Page 234.

5. Under what condition does it become difficult to have sufficient area of concrete in compression? Page 221.

6. If a beam is uniformly loaded where does the maximum shearing force occur? Page 219.

7. A continuous **T**-beam on a span of 16′ has an effective flange width of 4′, flange thickness of 4″ and stem width of 12″. The beam is limited in depth to 18″. Allowing 2″ from the bottom of the beam to the center of the steel, (**a**) what is the amount of reinforcing steel required to support a uniform load of 3100# per lineal foot (including the weight of the beam) if $n=15$, $f_c=800$#/□″,

and $f_s = 18{,}000\#/\square''$ and $M = \dfrac{Wl}{12}$? Use approximate equations, page 216. (b) What is the maximum stress in the concrete?

8. A reinforced concrete **T**-beam with a flange 36″ wide and 4½″ thick, has a 12″ stem, depth to steel of 23″, and is reinforced with five 1″ square bars. The beam is semicontinuous so that $M = \dfrac{Wl}{10}$ and it carries a load of 69,600# on a span of 24′. What are the maximum fiber stress (concrete and steel) in the beam if $n = 12$? (Omit the compression of the concrete in the stem.)

9. A reinforced concrete **T**-beam has a flange of 60″ over all, and is 4″ thick. The stem is 12″ wide, the depth to the center of the steel is 24″ and it is reinforced with four 1″ square bars. Omitting the compression in the stem below the flange, locate the position of the neutral axis of the beam and calculate the resisting moment for both the concrete and the steel, if $n = 15$, $f_c = 800\#/\square''$ and $f_s = 20{,}000\#/\square''$, using equation.

10. A rectangular concrete beam is limited to a size of 14″x28″ over all. On a simple span of 25 feet this beam carries a total load of 48,000#. If $n = 15$, $f_c = 800\#/\square''$ and $f_s = 18{,}000\#/\square''$, design the beam, allowing 2″ from both top and bottom to the center of the steel. Consider the beam as a simple beam with balanced reinforcement and add the necessary steel to provide for the excess moment.

11. A rectangular concrete beam 12″ wide and 22″ deep to the center of the tensile steel is reinforced with four 1″$\phi$ bars top and bottom. If $n = 15$, $f_c = 800\#/\square''$ and $f_s = 20{,}000\#/\square''$, what is the resisting moment of the beam, allowing 2″ from the top of the beam to the center of the compression steel?

## CHAPTER XII

1. A building plan requires that columns be spaced 22′ center to center each way. If the live load is $175\#/\square'$, what thickness of slab is required, using a 2-way flat slab without a dropped panel, $c = 5'$, and the concrete used is such that $f_c = 800\#/\square''$, $f_s = 20{,}000\#/\square''$ and $n = 15$? Check the shearing stresses.

2. If the slab of Problem *1* has a live load of $150\#/\square'$ imposed on it, what is the value of the maximum moments which must be resisted by the slab in both the column strip and the middle strip of an interior panel? Use Table XXV. Also, what is the area of steel required to resist these moments? Assume an average value for $d = t - 1.5''$ and $f_s = 20{,}000\#/\square''$ and $j = 0.87$.

3. Determine the slab and panel thickness required for a flat-slab floor supporting a live load of $200\#/\square'$ when the columns are spaced 15′ center to center each way. Diameter of capital $= 0.225l$. Use the four-way system with a dropped panel 6′ wide, 2000# concrete, $f_c = 800\#/\square''$, $f_s = 18{,}000\#/\square''$ and $n = 15$. Check the section for shear (interior panel only).

4. In a flat-slab floor 8″ thick with a live load of $100\#/\square'$, the columns of which are 18′ center to center, there is a marginal beam with depth greater than 1½ times the thickness of the slab. If the floor height is 15′ and there is a 12″ wall around the building, what is the load for which the marginal beam is designed if the wall weighs 130# per cubic foot?

## CHAPTER XIII

1. When resultant of forces are oblique to the axis of a column what stresses are developed? Page 253.

2. When is the steel of one face or another of a column alternately in tension and compression? Page 253.

3. How is the moment of inertia of a rectangle about its base found? Page 255.

4. Why are arches and columns designed to have symmetrical sections? Page 256.

5. When is a beam freely supported? Page 261.

6. If a load is applied to one member of a reinforced concrete structure or frame what causes all other members to deform to some extent? Page 263.

7. Describe the theorem of Three Moments. Page 264.

8. Describe the method of moment distribution called the Cross Method. Page 268.

9. What is the method of model analysis chiefly used for? Page 270.

10. Calculate the moment of inertia of the section of a beam 20″ wide, 28″ total depth, reinforced top and bottom with seven 1″ square bars, the center of the steel at the top and bottom being 2½″ from the surface.

## CHAPTER XIV

1. Give three reasons why tied columns are extensively used.

2. (a) Explain the function of the spiral reinforcement in a spiral column. (b) When does it come into use?

3. A tied column is 12″x16″ and is reinforced with four 1″$\phi$ bars. Find the load that it will support if the column is 13′-0″ in height and 2,000# concrete is used.

4. What load will a tied column 14″x20″ support when reinforced with four 1″ square bars, if the height is 11′ and a concrete of 2,500# is used?

5. A spiral column is 33″ in diameter, core 30″, reinforced with fourteen 1″ square vertical bars, ⅜″$\phi$ spiral with a 1¼″ pitch, 2,500# concrete, height of column 12′. What is the maximum load that it will support?

6. An 8″ **H** 35-lb. steel column is encased in a 22″ diameter column having an 18″ core. The longitudinal reinforcement consists of eight 1″$\phi$ bars; spiral is ⅜″; pitch, 2″; concrete, 2500#; height of column, 12′. What load will it support? The radius of gyration of an 8″ **H** 35# column is 2.03.

7. A reinforced concrete tied column 25″ square is reinforced with eight 1¼″ square bars so that there are three bars on each side of the column. The center of the bars is 2½″ in from the face of the column. The column carries a direct concentric load of 250,000# and in addition supports two beams on opposite faces of the column. These beams are framed 3″ off the center line of the column and have a reaction of 87,500# each. What are the maximum and minimum fiber stresses in the concrete if $n=10$?

8. A reinforced concrete tied column 20″x24″ is reinforced with eight 1″ square bars, the center of the bars being 2½″ from the edges of the column. The column receives a direct concentric load of 150,000# and in addition carries a load of 100,000# which is 3″ to the right of the center of the 20″ side, and 3″ from the center of the 24″ side. What are the maximum and minimum fiber

stresses if the concrete is such that $n=12$? To aid in the solution, draw a diagram showing the different conditions.

9. A column 22″x22″ supports a load of 250,000#. If the soil has a bearing value of 8,000#/□′, $f_s=20{,}000$#/□″, $f_c=800$#/□″, $n=15$ and $d=13$″, what are the dimensions of the footing, what is the area of steel required, and how many ½″φ bars would be required for this area? The number of bars must be an even number so that there will be the same amount of steel in each direction.

10. Find the length, depth to center of the reinforcing steel, area of tension steel, and the unit shear, for a footing supporting two columns. The columns are placed 20′ on centers, one being located one foot from the property line, the footing extending to that line. The load for the property column is 275,000# and for the other one is 350,000#. The width of footing is to be 5′, and it is to be designed as a rectangular beam, assuming that the center of gravity is located at the center of the span (which is not there) and the weight of the concrete is omitted. The bearing on the earth is 5,000#/□′; $f_c=800$#/□″; $f_s=20{,}000$ #/□″; $n=15$.

11. If 2,000# concrete is used and the tension in steel is 18,000#/□″, what would be the spacing of ½″φ bars in a wall footing 4′ 6″ wide, 12″ in thickness to the center of the steel, and bearing on the earth 6,000#/□′? The wall above is 18″ thick.

## CHAPTER XV

1. In designing reinforced concrete walls how is the weight of the supported material used to make the wall less liable to overturn? Page 300.

2. What relationship should there be between resistance and pressure to make a wall safe against sliding? Page 304.

3. What must be done to properly anchor the bars of a vertical wall in the base plate? Page 305.

4. What purpose do longitudinal bars in a wall serve? Page 306.

5. How are cracks in concrete and deterioration of steel prevented in the construction of tanks? Page 318.

6. Is the bursting pressure in a concrete tank proportional to the depth of the water? Page 314.

7. Under what conditions can a wall be called a curtain wall? Page 312.

8. How are concrete walls anchored when they are not stable against sliding? Page 312.

9. Upon what does the hold of bars in the back of the counterforts depend? Page 311.

10. How are the requirements of counterforts determined? Page 308.

11. When designing the footing for a reinforced concrete wall where should the resultant force intersect the base? Page 302.

12. What is the purpose of counterforts? Page 306.

13. A reinforced concrete retaining wall is required to support an earth bank, level on top, 22′ high including the base. The base will be 10′ 6″ wide and 21″ thick. The face of the vertical part of the wall will be 2′6″ from the toe. Find the thickness of the vertical wall to the center of the steel when $f_c=800$#/□″, $f_s=20{,}000$#/□″, $n=15$, and the weight of the earth is 100# per cubic foot. Make a sketch of the wall and base, the top of the wall being 15″ in thickness.

14. If $f_c=800$#/□″, $f_s=18{,}000$#/□″, and $n=15$, what would be the spacing of ⅞″φ bars in the vertical shaft of a reinforced concrete retaining wall 15′

high (above the base) if the earth behind the wall weighs 90# per cubic foot, the earth being level at the top?

15. A reinforced concrete tank has an inside diameter of 16′ and a height of water of 36.5′. (a) What area of steel is required to resist the bursting pressure on the lowest foot of the tank, with $f_s = 15{,}000\#/\square''$? (b) If the walls of the tank are 12″ thick, what is the maximum fiber stress in the concrete due to a wind pressure of 30#/□′, allowing 2′6″ from the top of the water to the top of the tank?

## CHAPTER XVI

1. Explain how to design the floor slab for the school building shown in Fig. 129. Page 320.
2. Explain how to determine the size and spacing of bars for the slab in Question 1. Page 321.
3. Explain how to design the joists shown in Fig. 130. Page 322.
4. Where beams are supported on brackets how should the columns be designed? Page 324.
5. Explain how to design an interior column for a building such as a school. Page 326.
6. Under what conditions will a 12″x12″ reinforced concrete column (4 bars ¾″) support 75,816# safely? Page 327.
7. Explain how to investigate the supporting capacity of a concrete column 14″x14″ reinforced with four 1″ round bars. Page 328.
8. Explain how to determine the load to be supported by basement columns. Page 328.
9. Explain how to determine the area of a footing for an interior column. Page 346.
10. Explain how to determine the amount of required steel in a column footing. Page 347.
11. How is the shear in a reinforced concrete beam resisted? Page 335.
12. What kind of a mix should be used for swimming pools? Page 394.

## CHAPTER XVII

1. What makes the variation in color on a surface of a poured concrete wall? Page 357.
2. Explain how the acid treatment is applied to the surface of a poured concrete wall. Page 364.
3. What is the maximum percentage of coloring material that can be used in concrete? Page 365.
4. Explain how the Quimby finish is applied on a concrete surface. Page 363.
5. Explain how the pebble finish can be secured on a concrete wall. Page 362.
6. Explain the procedure when a smooth concrete ceiling is desired. Page 359.
7. If mortar made with ordinary Portland cement and sand produces a darker finish when applied to concrete in which the same cement and sand has been used, what can be done in order to lighten the color of the mortar? Page 360.
8. Explain the procedure in using a cement paint on a concrete wall. Page 361.

9. Explain how the size and number of expansion joints are determined in concrete work. Page 367.

10. Explain how satisfactory floor joints are made, thinking in terms of expansion. Page 368.

11. Explain how reinforcing bars are used when making a construction joint in a floor. Page 369.

12. Explain how to make a construction joint when pouring concrete walls. Page 370.

13. What is the cause of rough places or stone pockets on the exteriors of poured concrete walls? Page 357.

14. Why is it that the same brand of cement should be used throughout on any one job? Page 359.

15. Explain the proper preparation of concrete walls where plaster is to be applied to the concrete. Page 360.

16. Why is it necessary to have reinforcing bars in cast-concrete-slab veneer? Page 364.

## CHAPTER XVIII

1. Explain the type of outfit used for bending the bars cold. Page 383.

2. Why do architects and engineers generally require that the ends of all bars, beams and girders shall be hooked at the end? Page 384.

3. Are bars used for temperature stress, straight or bent? Page 385.

4. What is the advantage of a continuous welded stirrup? Page 386.

5. What is the purpose of spacers as far as the reinforcing rods in concrete are concerned? Page 387.

6. What are hi-chairs and what is their purpose? Page 388.

7. Who should detail a few of the typical beams and girders in order to show in a general way what length of bars will be required, the number of turned up bars, etc.? Page 383.

8. Explain the manner in which the designing engineer should detail a typical beam so that the constructing engineer can develop these details. Page 383.

9. Why are one-half of the bars in a continuous slab turned up over all supports? Page 384.

10. Explain how rust marks can be avoided in concrete due to the stirrups. Page 385.

## CHAPTER XIX

1. Why are two or more complete sets of forms necessary when constructing buildings of more than three stories in height? Page 391.

2. Why is dry lumber preferable to green lumber in making forms? Page 391.

3. Are stronger forms necessary where concrete is to be vibrated than when spaded? Page 392.

4. In what types of form work are double headed nails used? Page 393.

5. Where form work is supported by steel, such as I-beams which are to be fireproof, explain the use of wire or **U**-shaped bars. Page 394.

6. Why is an opening always left at the bottom of a column form? Page 395.

7. What are shores, and how are they used? Page 399.

8. What is the minimum time for removal of forms used for columns? Page 399

9. What are rustications, and how are they used? Page 406.

10. What can be done to the planks used in the forms for concrete walls in order that they may be taken down more easily and used again? Page 391.

## CHAPTER XX

1. Why is it that when a contractor is purchasing concrete plant materials he should be sure that duplicates of all parts of the machinery are carried in stock by the manufacturer? Page 417.

2. Which type of mixer gives the better and quicker results—one having a complicated motion, or one having a simple motion? Page 418.

3. What is the usual speed or number of revolutions per minute for the drum of a concrete mixer? Page 419.

4. How is the water tank on a concrete mixer regulated as to the amount of water required per batch? Page 420.

5. Where are electric motors most generally used as far as mixing concrete is concerned? Page 426.

6. What three general sources of power are used for operating a concrete plant? Page 424.

7. Explain how to find the depreciation on a concrete mixing plant, and how it is handled. Page 418.

8. Is it cheaper to handle sand, cement and stone before or after the mixing process? Page 417.

9. Should a large or a small mixer be used on jobs where there is a small amount of concrete work to be done at scattered points on the job? Page 423.

10. What disadvantage has the use of steam engines as power for mixing concrete? Page 427.

11. Explain the hoisting procedure for concrete in a retaining wall. Page 429.

12. Why does a wheelbarrow having a pneumatic tire serve the purpose better than the old style wheelbarrow having a metal wheel? Page 430.

## CHAPTERS XXI AND XXII

1. Explain why it is that a concrete can be produced with a richer mix when the aggregates are controlled properly with respect to size, measurements, etc., and when the correct amount of mixing water is used and the concrete is well mixed. Page 439.

2. Write a typical specification for form work on an average reinforced concrete building job. Page 478.

3. Write a typical specification for finished concrete. Page 470.

4. Write a typical specification for concrete stairways. Page 471.

5. Explain why some road pavers have two drums. Page 465.

6. Explain the procedure in a cinder block plant. Page 461.

7. Explain how mixers are made for concrete blocks. Page 460.

8. Explain the amount of air space generally found in standard concrete blocks. Page 459.

9. Explain the principle of the cement gun and the product called "Gunite." Page 457.

10. Explain the principle and operation of the concrete pumps. Page 455.

11. Explain the principle and operation of a truck mixer. Page 453.

12. Explain the operation of a batching plant. Page 451.

# INDEX